Arithmetic 1

Click this to start.

1. Arithmetic Basics 1

What do we do doing math?

Doing math, we do *mathematical operations*,

often just quickly called *operations*.

There are many kinds in math operations.

Among those many, the most often used are *arithmetic operations*, often just called *arithmetic* for short. And oftentimes, we call such an operation a calculation, too, which has more meanings, though. What then are those operations in arithmetic?

In arithmetic, we do four kinds.

Additions and subtractions

Multiplications and divisions

Adding numbers together, we get a number called the sum.

Subtracting a number from another, we get a number called the difference.

Multiplying a number by another, we get a number called the product.

Dividing a number by another, we get a number called the quotient.

And we can put those four kinds in a table the way as follows.

Arithmetic	**Operator**	**Operation**
Addition	+	3 + 2
Subtraction	–	3 – 2
Multiplication	x, ·	3 x 2 = 3·2
Division	÷, —, /	$3 \div 2 = \frac{3}{2} = 3/2$

So doing the operations, we use operators. And we use math signs as the operators. So doing each operation, we use a math sign as an arithmetic operator.

What then about the numbers used in the operations?

We call them operands, and each operation needs two operands as in this: $3 + 2$.

Doing an addition, we use a cross like this: +, and call it the addition operator, called a plus sign, too.

Doing a subtraction, we use a short horizontal bar like this: –, and call it the subtraction operator, called a minus sign, too.

Doing a multiplication, we use an x, and call it the multiplication operator.

Oftentimes, though, we use a tiny dot this way: $3 \cdot 2$, which means, thus, 3 x 2, and use it as the multiplication operator, too.

And doing a division, we use a horizontal bar separating two dots like this: ÷, and call it the division operator. Oftentimes, though, we use a horizontal bar separating two numbers like this: — as in $\frac{3}{2}$, or we use a slash like this: / as in 3/2.

What do we mean by a multiplication, and when and how do we use it?

Adding together many of the same numbers as 7 of 3s, that is, taking the sum of seven threes, we do this: 3 + 3 + 3 + 3 + 3 + 3 + 3. Though it takes some time, we can add one number at a time to get the sum. What if we need to take the sum of 175 of 3s?

To begin with, multiplying a number by another, we get a number called the product. So taking the product of two numbers, we multiply one number by the other.

For instance, multiplying 2 by 3, we get 6, and can call 6 the product of 2 and 3. So taking the product of 2 and 3, we multiply 2 by 3. Then, the product is 6. And multiplying 7 by 3, what do we get?

We get the product, which is 21.

If keeping the multiplication table in your memory, you can quickly get it.

2. Arithmetic Basics 2

… –5 , –4, –3, –2, –1, 0, 1, 2, 3, 4, 5 …

Doing math, we use numbers, of course, which are most often used among all the things in math. And of all the numbers we use, the most basic and the most often used are integers. Using integers in fact, we make numbers of other kinds, too, together with signs or symbols.

What is an integer though?

The word ‘integer’ is probably a composite word made of ‘integral’ and ‘number’.

That is to say that

integer = integral + numb**er**.

So an integer can be called an integral number, and can be a whole number as 1, 2, 3, etc. which are called natural numbers.

Thus, an integer can indicate an amount as one or three, and does not indicate any amount with a fraction as a half, three halves or one and a half, two thirds, or seven fifths.

And we have three kinds in integers.

What then, are the three kinds?

In integers, we have positive ones and negative ones, together with 0.

So we have these: 0, positive integers, and negative integers.

Indicating a positive integer, we can put a plus sign in front of a whole number. For instance, +1 is a positive integer, and is read as a positive one or plus one.

We can use a whole number as a positive integer, too. So for instance, a positive integer can be 2 or 9. Though it is positive, we don’t usually use the plus sign, so we take 2 as +2.

What then do we mean by a negative integer?

A negative integer is a product of –1 and an integer positive, and can be taken as the number opposite of the integer positive, because the two have opposite signs.

For instance, a negative integer can be: –2, and we can get it doing this: (–1) x 2, or doing this, of course: 2 x (–1), and we can get –9, doing this: (–1) x 9.

So we have –2 = (–1) x 2, and –9 = (–1) x 9. And the sign of 2, that is, the sign of +2 is the opposite of the sign of –2.

By the way, doing a multiplication, we don’t usually use the operator x, and instead, we just use a dot, or nothing if no ambiguity is expected.

So we often just put the expression above this way: $-2 = (-1)\cdot 2$, or $-9 = (-1)\cdot 9$.

And –2 can be taken as the number opposite of 2, and –9 can be taken as the number opposite of 9.

Formally though, a negative integer is called the additive inverse of a positive integer. And vice versa. So a positive integer is the additive inverse of a negative integer.

What do we mean by the additive inverse though?

The sum of an integer and its additive inverse is 0.

So if two integers add up to 0, what is one of the two to the other?

It’s the additive inverse of the other. So the two are additive inverse of each other.

So for instance, 2 and –2 are additive inverse of each other, and we get 2 + (–2) = 0, which is thus, the same as 2 – 2,
so we have 2 + (–2) = 2 – 2.

What does then, the additive inverse have to do with a negative integer?

The additive inverse of an integer is the negative of the integer.

So is the negative of an integer, negative?

Not necessarily
The negative of an integer can be positive, 0, or negative.
If an integer is positive, the negative of the integer is negative. If an integer is negative, its negative is positive. And if it is 0, its negative is 0, too.

So for instance, the negative of 5 is –5, the negative of –5 is –(–5) = 5.

And the negative of 0 is 0, so the negative of an integer is not always negative.

How then can we make a positive integer negative?

Multiplying or dividing an integer by -1, we get the negative of the integer.

And if the integer is positive, the negative of the integer is negative.
So for instance,
the negative of 23 is -1 x 23 = -23,
or 23/(-1) = -23, which is a negative integer,
and is the opposite of 23, that is, the additive inverse of 23.

Thus, we get -23 + 23 = 0, and 23 + (-23) = 0, that is, 23 + (-23) = 23 – 23 = 0.

How then, can we make a negative integer positive?

We know multiplying or dividing an integer by

–1, we get the negative of the integer.

And if the integer is negative, the negative of the integer is positive.

So for instance,
the negative of -34 is -1 x (-34) = -(-34) = 34, or (-34)/(-1) = 34, which is a positive integer, and is the opposite of -34, that is, the additive inverse of -34.

Thus, we get $34 + (-34) = 0$,
that is, $34 + (-34) = 34 – 34 = 0$,
and $(-34) + 34 = 0$, too.

So multiplying or dividing an integer by –1,
we change the sign of the integer.

Thus, multiplying or dividing a positive integer
by –1, we make the integer negative.

And multiplying or dividing a negative integer by –1, we make the integer positive.

So how do we make a negative integer?

We can make it multiplying a positive integer by –1, and of course, dividing the positive integer by –1, too.
In fact, every nonzero integer has its negative.

And taking the sum of an integer and its negative, we get 0, so both are equal in magnitude and are opposite in sign.

And we call the magnitude, the absolute value, too, which is positive or 0.
So the magnitude of an integer is the absolute value of the integer, which is greater than or equal to 0, that is, ≥ 0.

And indicating the absolute value of an integer, we use as a sign a pair of vertical bars, between which the integer is placed.

For instance, the absolute value of –7 is indicated by |–7|, which is thus, the magnitude of –7, and is 7. And the magnitude or the absolute value of 7 is 7, too, of course.

That is to say that $|-7| = |7| = 7$. And of course, we have: $|0| = 0$.

And the same is true for any other kind in numbers, too. So what do we mean by a negative number?

A negative number is a product of –1 and a number positive, and can be taken as the number opposite of the number positive.

So for instance, a negative number can be –2, which is –1 times 2, that is, –2 = (–1)·2, and more examples can be: –0.5 = (–1)·0.5, $-\sqrt{3} = (-1)\sqrt{3}$, and $-\frac{2}{3} = (-1)\cdot\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$.

Thus, –2 can be taken as the number opposite of 2, and 0.5 can be taken as the number opposite of –0.5.

So in sum, integers are whole numbers as 2, 0, and –5, and we have three kinds, one is positive, another is 0, and the other is negative.

And we can change the sign of a number multiplying (dividing) by –1.

And adding together two numbers opposite of each other, we get 0.
The two are equal in magnitude, but opposite in sign.
One of the two is positive, and the other is negative. So 0 can be said to be neutral.

Formally though, the two are said to be additive inverse of each other.

And in fact, subtracting a number, we add its negative, that is, the additive inverse of it.

So for instance, subtracting 2, we can add –2, and subtracting –3, we add –(-3), that is, 3.

So doing a subtraction, we actually do an addition adding the additive inverse, that is, the negative of the number subtracted.

And we know 0 is nether negative nor positive, that is, neutral.

So adding together numbers negative and positive, we just keep neutralizing using the idea of the additive inverse.

For instance, we can do a subtraction the way below.

$5 - 2 = 3 + 2 - 2 = 0$, and the idea behind is

$5 - 2 = 5 + (-2) = 3 + 2 + (-2) = 3 + 0 = 3$.

And we can put it this way, too:
$5 - 2 = -2 + 5 = -2 + 2 + 3 = 0 + 3 = 3$.

So in the calculation above, -2 gets neutralized with 2, and vice versa.

And the additive inverse of -9 is 9, that is, the negative of -9 is 9.

So for instance,
we can get 5 – (-9) = 5 + 9 = 14.

And for another instance,
we know $7a = 5a + 2a$, so we can get this:

$7a – 2a = 5a + 2a – 2a$, and the idea behind is as follows.

$7a – 2a = 7a + (-2a) = 5a + 2a + (-2a)$
$= 5a + 0 = 5a$.

And we can put it this way, too:
$-2a + 7a = -2a + 2a + 5a = 0 + 5a = 5a$.

So in the calculation above, $-2a$ gets neutralized with $2a$, and vice versa.

And of course, the additive inversed of $-a$ is a, that is, the negative of $-a$ is a.
So for instance,
we can get $4a - (-a) = 4a + a = 5a$.

It’s a good idea therefore, to get used to the idea of the additive inverse, that is, the negative of a number or an expression.

So in the next section, we will go over the basics above, the idea of the additive inverse, and then, add some more basics, after doing some examples in the next several pages.

What then, about multiplications and divisions?

The idea applies to subtractions applies to divisions, too.

So doing a division, we can do a multiplication.
When we do a division in fact, what we actually do is a multiplication.

Without doing a multiplication, we cannot do a division.

Doing a division, we can multiply by an inverse, which is thus, called the multiplicative inverse. Usually though, we just call it the reciprocal.

So dividing by a number, we multiply by the reciprocal.

Thus, for instance, the reciprocal of 2 is $\frac{1}{2}$.

So dividing 6 by 2, we can multiply 6 by $\frac{1}{2}$.

Thus, we get $6 \div 2 = 6 \times \frac{1}{2} = 3$.

And we will get to cover the idea of the reciprocal in the section, The Reciprocals.

3. Addition Basics

Addition basics:

Doing an addition, we add a number to another. Then, we get a number called the sum.
And when doing it, we add a digit to a digit for each place value, and then, get the sum of all the individual sums.

Let's now begin with the idea as follows.

Doing an addition, we add a number to another. Then, we get a number called the sum.

So adding a number to another, we do an addition, and then, get a number called the sum.

An addition is a math operation, and doing it, we use an *operator*, together with two numbers called the *operands*.

And as the operator, we use this: +, which is put between the numbers called the operands.

So for instance, adding 5 to 3, we do this: 5 + 3, read as 5 added to 3, or as 5 plus 3. Then, we get 8 called the sum.

And using math language, we can simply put all the ideas above this way: 5 + 3 = 8, read as 5 added to 3 equals 8. And for short, we can just read it this way: 5 plus 3 is 8.

So 5 + 3 = 8 is saying that adding 5 to 3, we get 8, or that 8 is the sum of 5 and 3.

What then about this: 5 + 3 + 1?

It’s made of two additions, and we can do it this way: 5 + 3 + 1 = 8 + 1 = 9.

It’s because 5 + 3 = 8, so 5 + 3 + 1 = 8 + 1, and 8 + 1 = 9. Then, 9 can be called the sum of the three numbers 5, 3, and 1.

Those numbers in the additions above are single digit numbers and are not many. So doing mental math, we can get the sum fast.

What then about multi-digit numbers? So, for instance, what if we add 23 to 45?

Then, we get to use math basics as follows.

Math basics:

Add a digit to a digit for each place value,

and then, add up all the individual sums.

So doing an addition, we add a digit to a digit for each place value in the numbers, and then, add up all the individual sums.

For instance, adding 23 to 45, we add 20 to 40, add 3 to 5, and then, get the sum of the two individual sums.

So in the number 23, 2 is not just 2. It’s 20, 2 of 10s, since 2 is in the 10’s digit.

Thus, since 23 = 20 + 3 and 45 = 40 + 5, if doing 23 + 45, we can do it this way:

23 + 45 = 20 + 3 + 40 + 5

= 20 + 40 + 3 + 5 = 60 + 8 = 68

And doing some mental math, we can get the sum this way:

23 + 45 = (2 + 4)0 + (3 + 5) = 60 + 8 = 68

The metal math parts are in brackets.

What then is this: (2 + 4)0?

In this: $(2 + 4)0 + (3 + 5)$,

$(2 + 4)0$ is short for $(2 + 4) \times 10$

That is, $(2 + 4)0 = (2 + 4) \times 10$

And $(2 + 4) \times 10 = 20 + 40$, which is the sum of the 10's digits in 23 and 45.

So when doing 23 + 45, we can do mentally these two: 20 + 40 and 3 + 5. And what we actually write can be this: 60 + 8 = 68.

And putting the process of the mental math in a sequential manner, you can put it the way as follows.

You are now doing this: 23 + 45.

So do mentally this first: 20 + 40, and write this: 60 +

Next, do mentally this: 3 + 5, and you have written this: 60 + 8.

And next, do mentally this: 60 + 8, and then, you'll have written this: 60 + 8 = 68

It’s mental math by parts. Getting fully used to mental math, you can get 68 at once. And taking now, another instance, we can do an addition this way:

123 + 456 = 100 + 20 + 3 + 400 + 50 + 6

= 100 + 400 + 20 + 50 + 3 + 6

= 500 + 70 + 9 = 579

So when you do this: 123 + 456, what's happening in your mind are these:

(1 + 4)00,   (2 + 5)0,   (3 + 6)

And putting together what happens in your mind and your actual writing, you can put them the way as follows.

$$123 + 456 = (1 + 4)00 + (2 + 5)0 + (3 + 6)$$

$$= 500 + 70 + 9 = 579$$

So, if doing mental math by parts, your actual writing can be as follows.

$$123 + 456 = 500 + 70 + 9 = 579$$

And if you get much used to mental math, your writing can happen the way as follows.

First off, $\underline{1}23 + \underline{4}56 = 5$

Next, $1\underline{2}3 + 4\underline{5}6 = 57$

And finally, $12\underline{3} + 45\underline{6} = 579$

That is to say that if getting much used to mental math, you can just do it this way:

123 + 456 = 579

So if much used to mental math and if putting together what happens in your mind and your actual writing, you can put them the way as follows.

$123 + 456 = (1 + 4)00 + (2 + 5)0 + (3 + 6)$

$= 579$

Note that (1 + 4)00 is not an official math expression, so such an expression cannot be used at school or at a test, exam, etc.

Now, what about this: 25 + 47?

$$25 + 47 = 20 + 40 + \underline{5 + 7} = 60 + \underline{12}$$

The sum of the two 1’s digits in 25 and 47 is more than 9, since it is 12. Then, 1 in 12 is called the carry over. We don’t need to worry about it.

60 + 12 is just another addition, so we just keep using the addition basics the way as follows.

25 + 47 = 20 + 40 + 5 + 7 = 60 + 12

= 60 + 10 + 2 = 70 + 2 = 72

And you can do some mental math this way:

25 + 47 = (2 + 4)0 + (5 + 7) = 60 + 12

(6 + 1)0 + 2 = 70 + 2 = 72

The mental parts are in the brackets.

Then, your actual writing goes this way:

25 + 47 = 60 + 12 = 72

What then about 75 + 47?

We have these: $75 = 70 + 5$ and $47 = 40 + 7$

So doing some mental math, you can get

$75 + 47 = (7 + 4)0 + (5 + 7) = 110 + 12$

$= 100 + (1 + 1)0 + 2 = 100 + 20 + 2 = 122$

The mental math parts are in the brackets.

Then, your actual writing goes this way:

75 + 47 = 110 + 12 = 100 + 20 + 2 = 122

And let’s now do some more examples.

Do this addition: 995 + 47

Doing some metal math, we can get this:

$995 + 47 = 900 + (9 + 4)0 + (4 + 7)$

$= 900 + 130 + 12 = (9 + 1)00 + (3 + 1)0 + 2$

$= 1000 + 40 + 2 = 1042$

Then, the actual writing can be as follows.

$995 + 47 = 900 + 130 + 12$

$= 1000 + 40 + 2 = 1042$

What about this: 999 + 99?

Doing some metal math, you can get this:

999 + 99 = 900 + (9 + 9)0 + (9 + 9)

= 900 + 180 + 18 = (9 + 1)00 + (8 + 1)0 + 8

= 1000 + 90 + 8 = 1098

Then, your actual writing can be like this:

999 + 99 = 900 + 180 + 18

= 1000 + 90 + 8 = 1098

What then about 24 + 35 + 67 + 91?

24 + **3**5 + <u>6</u>7 + <u>9</u>1

= (**2** + **3**)0 + (<u>6</u> + <u>9</u>)0 + (4 + 5) + (7 + 1)

= 50 + 150 + 9 + 8

= 100 + (5 + 5)0 + (9 + 1) + 7

= 100 + 100 + 10 + 7

= (1 + 1)00 + 10 + 7

= 200 + 10 + 7 = 217

Then, your actual writing can be

24 + 35 + 67 + 91 = 50 + 150 + 9 + 8

= 100 + 100 + 10 + 7 = 200 + 10 + 7 = 217

And if getting much used to mental math, you can do the mental math the way as follows, too.

24 + 35 + 67 + 91

= (2 + 3 + 6 + 9)0 + (4 + 5 + 7 + 1)

= 200 + 17

Then, your actual writing can be

24 + 35 + 67 + 91 = 200 + 17 = 217

What about this: 87 + 95 + 67 + 98?

$$87 + 95 + 67 + 98$$

$$= (8 + 7)0 + (6 + 9)0 + (7 + 5) + (7 + 8)$$

$$= 170 + 150 + 12 + 15$$

$$= (1 + 1)00 + (7 + 5)0 + (1 + 1)0 + (2 + 5)$$

$$= 200 + 120 + 20 + 7$$

$$= (2 + 1)00 + (2 + 2)0 + 7$$

$$= 300 + 40 + 7 = 347$$

And your writing can go this way:

$$87 + 95 + 67 + 98 = 170 + 150 + 12 + 15$$

$$= 200 + 120 + 20 + 7 = 300 + 40 + 7 = 347$$

And if getting much used to mental math, you can do the mental math the way as follows, too.

87 + 95 + 67 + 98

= (8 + 9 + 6 + 9)0 + (7 + 5 + 7 + 8)

= 320 + 27

= 347

Then, your actual writing can be

87 + 95 + 67 + 98 = 320 + 27 = 347

What then about additions like this?

87 + 95 + 67 + 98 + 79 + 89 + 97 + 99 + 83

Way too many to add up for most of us.
So we may want to use a calculator. What if
it's broken or the battery is dead, but we've
still got to get the work done?

Well then, we've got to do it by hand.
It's gonna be a bit too lengthy.

$87 + 95 + 67 + 98 + 79 + 89 + 97 + 99 + 83$

$= (8 + 9)0 + (6 + 9)0 + (7 + 8)0 + (9 + 9)0 +$

$80 + (7 + 5) + (7 + 8) + (9 + 9) + (7 + 9) + 3$

$= 170 + 150 + 150 + 180 + 80 + 12 + 15 + 18$

$+ 16 + 3$

$170 + 150 + 150 + 180 + 80 + 12 + 15 + 18 +$

$16 + 3$

$= (1 + 1 + 1 + 1)00 + (7 + 5)0 + (5 + 8)0$

$+ (8 + 1)0 + (1 + 1 + 1)0$

$+ (2 + 5) + (8 + 6) + 3$

$= 400 + 120 + 130 + 90 + 30 + 7 + 14 + 3$

$$400 + 120 + 130 + 90 + 30 + 7 + 14 + 3$$

$$= (4 + 1 + 1)00 + (2 + 3)0 + (9 + 3)0$$

$$+ (7 + 3) + 14$$

$$= 600 + 50 + 120 + 10 + 14$$

$$= (6 + 1)00 + (5 + 2 + 1 + 1)0 + 4$$

$$= 700 + 90 + 4 = 794$$

And your writing goes the way as follows.

87 + 95 + 67 + 98 + 79 + 89 + 97 + 99 + 83

= 170 + 150 + 150 + 180 + 80 + 12 + 15 + 18

+ 16 + 3

= 400 + 120 + 130 + 90 + 30 + 7 + 14 + 3

= 600 + 50 + 120 + 10 + 14

= 700 + 90 + 4 = 794

So now, how then do we add?

When adding, add a digit to a digit for each place value.

If getting used to mental math, though, try adding together all the digits for each place value when adding several numbers together.

So the bottom line: Each place value matters. Adding thus, several numbers together, and if getting used to mental math, add together all the digits for each place value.

So for instance, add all 1’s together, all 10’s together, all 100’s together, … and get the sum sequentially.

And, for more instances, we can have these:

23 + 54 = (2 + 5)0 + (3 + 4) = 70 + 7 = 77

23 + 54 = 70 + 7 = 77

48 + 79 = (4 + 7)0 + (8 + 9) = 110 + 17

= 100 + (1 + 1)0 + 7 = 100 + 20 + 7 = 127

48 + 79 = 110 + 17 = 127

$24 + 12 + 53 = (2 + 1 + 5)0 + (4 + 2 + 3)$

$= 80 + 9 = 89$

$24 + 12 + 53 = 80 + 9 = 89$

$27 + 35 + 84 = (2 + 3 + 8)0 + (7 + 5 + 4)$

$= 130 + 16 = 100 + (3 + 1)0 + 6 = 146$

$27 + 35 + 84 = 130 + 16 = 146$

Note, however, that you don’t have to do additions the way above. It’s just one of many ways. In math, there are usually many ways to get the same.

You can do additions vertically, that is, the way you learn them at school, or other way as the way you learn them in other books.

However, if you take a test, exam, or quiz at school, or do homework, and have to show your work, you should use the way you learn at school.

4 Subtraction Basics

Subtraction basics:

Doing a subtraction, we subtract a number from another. Then, we get a number called the difference.

And when doing it, we subtract a digit from a digit for each place value.

So for instance, subtracting 2 from 3, we do this: 3 – 2, read as 3 minus 2. Then, we get 1 called the difference.

And we put the idea this way: 3 – 2 = 1, which is saying that 1 is the difference between the two numbers 3 and 2.

What then about this: 9 – 4 – 2?

It’s made of two subtractions, and we can do it this way: 9 – 4 – 2 = 5 – 2 = 3

And we call 3 the result or solution. And of course, it’s the difference between 5 and 2.

The numbers in the subtractions above are single digit numbers and not many. So doing mental math, we can quickly get the result.

What then about multi-digit numbers? So, for instance, what if we subtract 23 from 45?

Doing a subtraction, we subtract a digit from a digit for each place value.

For instance, subtracting 23 from 45, we subtract 2 from 4, and put the difference in 10’s digit, and then, subtract 3 from 5 and put the difference in 1’s digit.

So when doing a subtraction, subtract a digit from a digit for each place value.

Thus, since 23 = 20 + 3 and 45 = 40 + 5, if doing 45 – 23, we can do it this way:

45 – 23 = 40 + 5 – (20 + 3) = 40 + 5 – 20 – 3

= 40 – 20 + 5 – 3 = 20 + 2 = 22

And doing some mental math, we get

45 – 23 = (4 – 2)0 + (5 – 3) = 20 + 2 = 22

The mental math parts are in the brackets.

What then is this: (4 – 2)0?

In this: (4 – 2)0 + (5 – 3),

(4 – 2)0 is short for (4 – 2) x 10

That is, (4 – 2)0 = (4 – 2) x 10

And we have this: (4 – 2) x 10 = 40 – 20,

which is the difference between the 10's

digits in the two numbers 45 and 23.

So now again, doing 45 – 23 by mental math in parts, we can do it this way:

45 – 23 = (4 – 2)0 + (5 – 3) = 20 + 2 = 22

Thus, when doing 45 – 23, we can do mentally these two: 40 – 20 and 5 – 3. And what we actually write is this: 20 + 2 = 22.

So when we subtract 23 from 45, our actual writing can be this: $45 - 23 = 20 + 2 = 22$

And putting the processes of the mental math in a sequential manner, you can put them the way as follows.

You are now doing this: 45 – 23
So do mentally this first: 40 – 20, and write this: 20 +

Next, do mentally this: 5 – 3, and you have written this: 20 + 2
And next, do mentally this: 20 + 2, and then, you’ll have written this: 20 + 2 = 22

It’s mental math in parts. Getting fully used to mental math, you can get 22 at once. And taking now, another instance, we can have

$$479 - 236 = 400 + 70 + 9 - (200 + 30 + 6)$$

$$= 400 + 70 + 9 - 200 - 30 - 6$$

$$= 400 - 200 + 70 - 30 + 9 - 6 = 200 + 40 + 3$$

And doing some mental math, you can get

479 – 236 = (4 – 2)00 + (7 – 3)0 + (9 – 6)

= 200 + 40 + 3 = 243

So when you do this: 479 – 236, what's

happening in your mind are these:

(4 – 2)00, (7 – 3)0, (9 – 6)

And putting together what happens in your mind and your actual writing, we can put them the way as follows.

$$479 - 236 = (4 - 2)00 + (7 - 3)0 + (9 - 6)$$

$$= 200 + 40 + 3 = 243$$

And if you get used to mental math, your writing actually happens the way as follows.

First off, $\underline{4}79 – \underline{2}36 = 2$

Next, $4\underline{7}9 – 2\underline{3}6 = 24$

And finally, $47\underline{9} – 23\underline{6} = 243$

Note that (4 – 2)00 is *not an official math expression*, so such an expression *cannot* be used at school or at a test, exam, etc.

Let's now move on to the next example.

We can have a situation as follows.

65 – 29 = (6 – 2)0 + 5 – 9 = 40 + 5 – 9

The 1's digit in 65 is less than the 1's digit in

29. And at the moment, we have 40 + 5 – 9

So 5 – 9 is not a normal subtraction.

We can do it two ways.

Both are about the same, though. In one, we split the higher digit when the split is needed, and can do it the way as follows.

$65 - 29 = 60 - 20 + 5 - 9 = \underline{40} + 5 - 9$

$= \underline{30} + 5 + \underline{10} - 9 = 30 + 5 + 1 = 36$

And doing some mental math, we get this:

$65 - 29 = (6 - 2)0 + 5 - 9 = 40 + 5 - 9$

$= (30 + 5) + (10 - 9) = 35 + 1 = 36$

The mental math parts are in the brackets.

So your actual writing can go this way:

65 – 29 = 40 + 5 – 9 = 35 + 1 = 36

And in the other way, we split the higher digit first, so the subtraction can be done the way as follows.

6**5** – 29 = 50 + 10 – 20 + **5** – 9

= 50 – 20 + 5 + 10 – 9 = 30 + 5 + 1 = 36

And doing some mental math, we get this:

65 – 29

= (5 – 2)0 + 5 + (10 – 9) = 30 + 5 + 1 = 36

Then, your actual writing can be this:

65 – 29 = 30 + 5 + 1 = 36

So, which way do you prefer?

The two ways are about the same. If you get more used to, though, mental math in parts, you might prefer the second way, since it takes less steps, so can be faster.

And you can do more examples in Example Set 1.

5. Negative Numbers

A number can be positive, negative, or 0.

If a number is positive, it is greater than 0,

and if it's negative, it is less than 0.

In math language, if $\boldsymbol{P}$ is positive, we can put it this way: $\boldsymbol{P} > \mathbf{0}$, saying thus, $\boldsymbol{P}$ is positive, which means it's greater than 0.

And if N is negative, we can put it this way:

$N < 0$, saying thus, N is negative, which means it's less than 0.

So if a number is positive, it's greater than every negative number.

Thus, we have this: $P > N$

And the two are opposite in nature.

One is opposite of the other in nature. And we can indicate the opposite nature using a math sign.

And the math sign is a minus sign we use in a subtraction.

So it’s a short horizontal bar like this: **–**.

And indicating a negative number, we use a minus sign this way: –5, called minus five.

What then about a positive number?

We don’t need to use a math sign indicating a positive number. A number with no math sign is positive if it’s not 0, of course.

We can use a plus sign, +, for emphasis purposes.
So 5 is positive. And we can explain the opposite nature the way as follows.

If numbers are positive, the bigger the magnitude, the bigger the number, but the opposite is true if numbers are negative.

So among negative numbers, the bigger the magnitude, the smaller the negative number.

For instance, $5 > 2 > 0 > -2 > -3 > -5 > -10$

And we can explain the opposite nature the way as follows, too.

In math, we have a line of numbers, called a number line. At the center, we see 0. On one side of 0, we see positive numbers, and on the other side, we see negative numbers.

And a number line begins at a point where 0 is located, and grows in two directions *opposite* of each other.

Thus, it grows up and down if it’s vertical, and if it’s horizontal, it grows left and right the way as follows.

… -4, -3, -2, -1, 0, +1, +2, +3, +4 …

On the left of 0, we see negative numbers,

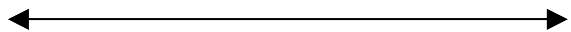
and on the right, we see positive numbers.

… -4, -3, -2, -1, 0, +1, +2, +3, +4 …

Plus signs are used for emphasis purposes.

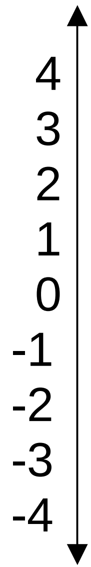
So for instance, 3 = +3, 7 = +7, and 5 = +5

Thus, normally, in a number line, if it’s horizontal, we put the numbers the way as follows.



… -4, -3, -2, -1, 0, 1, 2, 3, 4 …

If it’s vertical, positive numbers are above 0, so we can get it rotating about 0 the line above 90 degrees counterclockwise.


4
3
2
1
0
-1
-2
-3
-4

And in math, a number on the left of 0 and a number on the right of 0 are said to be *opposite in sign*.

… -4, -3, -2, -1, 0, 1, 2, 3, 4 …

So for instance, -2 and 2 are opposite in sign. And 4 and -3 are opposite in sign.

In a number line, the distance from 3 to 0 is the same as the distance from –3 to 0.

… -4, -3, -2, -1, 0, 1, 2, 3, 4 …

So 3 and –3 share the same distance from 0. What then do we mean by the distance?

The *distance* is the *magnitude* of the number.

It can be called the *absolute value*, too.

So the absolute value of a number is its

magnitude, and both are positive, of course.

Thus, 3 and –3 share the same magnitude.

And we have math basics as follows.

Math basics:

Adding together two numbers equal in magnitude but opposite in sign, we get 0.

For instance, what is the number equal to 5 in magnitude but opposite in sign?

It’s –5, called minus five or negative five.

And we can say that 5 and –5 are *equal in magnitude* but are *opposite in sign*, and we have this: 5 + (–5) = 0

Why 5 and –5 are equal in magnitude?

Both are the same distance away from 0.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …

The same distance means the same magnitude, so 5 and –5 are equal in magnitude, and the magnitude is 5.
How then do we get the magnitude?

Taking the difference between a number and 0, we get the magnitude of the number.

And a magnitude is positive.

So taking the difference between the two, we subtract the smaller from the greater.

And 0 is smaller than 5.

So subtracting 0 from 5, we get this:

5 – 0 = 5

Thus, 5 is the magnitude of 5, which has to be the magnitude of –5, too, since 5 and –5 are equal in magnitude.

And taking the difference between –5 and 0, we subtract –5 from 0, because 0 is greater than –5. So we get this: $0 - (-5) = 5$

That is to say that we have this: $0 = 5 + (-5)$
Why, though?

It’s because of the basic principle of addition and subtraction.

So now, adding back what’s subtracted, what do we get?

We get the original. Taking, thus, the sum of what's left and what's subtracted, we get the original back.

So for instance, 3 = 1 + 2, since 3 – 2 = 1 That is to say that because we get 1 if we subtract 2 from 3, we get 3 if adding 2 to 1.

By the same token, because we get 5 if we subtract -5 from 0, we get 0 if adding -5 to 5.

Thus, 0 = 5 + (-5), since 0 – (-5) = 5

So now, 5 and –5 are equal in magnitude but opposite in sign, and we have 0 = 5 + (–5)

Thus, adding together two numbers equal in magnitude but opposite in sign, we get 0.

Negative Numbers

$5 + (-5) = 0$ $(-5) + 5 = 0$ $-5 + 5 = 0$

$9 + (-9) = 0$ $(-9) + 9 = 0$ $-9 + 9 = 0$

$2 + (-2) + 1 = 1$ $-1 + 5 + (-1) + 5 = 0$

$3 + 9 + (-3) + 7 + (-9) = 7$

And mentioned earlier that positive and negative are opposite in nature. For instance, making 5 dollars, we gain 5 dollars.

What then do we mean by –5 dollars?

Making –5 dollars, we spend 5 dollars.

So 5 dollars and –5 dollars are equal in magnitude and opposite in nature.

If therefore, making 5 dollars, and also, making –5 dollars, we end up with 0. The change in wealth is 0. So 5 + (-5) = 0

Why brackets, though, surrounding –5?

So why not just this: 5 + –5?

The use of brackets in this case is just for clarity purposes. If an expression begins with a negative number, we don't normally use brackets, since it's clear enough.
So we can have these:

$$-5 + 7 = (-5) + 7 = 7 + (-5)$$

$$-3 + (-4) = (-3) + (-4) = -4 + (-3) = (-4) + (-3)$$

$$-3 - (-4) = (-3) - (-4) = -4 - (-3) = (-4) - (-3)$$

Let’s now talk about ***negatives***, not just negative numbers. It will be about the same story, but will let you taste a bit different flavor. It’s quite important, though.

So to begin with, saying the negative in math, we don't always mean a negative number. It can be positive, too. It all depends on what number we are talking about. Thus, the question is "The negative of what?"

So what is the negative?

Let's first, go over that same story a little bit.

So again, a number can be positive, negative, or 0. If a number is positive, it is greater than 0, and if it's negative, it is less than 0.

In math language, if P is positive, we can put it this way: $P > 0$, and if N is negative, we can put it this way: $N < 0$

So if a number is positive, it’s greater than any negative number. Thus, we have $\boldsymbol{P} > \boldsymbol{N}$

And the two are opposite in nature.
If numbers are positive, the bigger the magnitude, the bigger the number, but the opposite is true if numbers are negative.

So among negative numbers, the bigger the magnitude, the smaller the negative number.

For instance, $-3 < -2 < -1 < 0 < 1 < 2 < 3$

And in math, we have a line of numbers, called a number line.

In a number line, all negative numbers are on one side of 0, and all positive numbers on the other side.

So a number line begins at a point where 0 is located, and grows in two directions *opposite* of each other.

Thus, it grows up and down if it’s vertical, and if it’s horizontal, it grows left and right the way as follows.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …

On the left of 0, we see negative numbers, and on the right, we see positive numbers.

If it’s vertical, positive numbers are above 0, so we can get it rotating about 0 the line shown below 90 degrees counterclockwise.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …

And in math, a number on the left of 0 and a number on the right of 0 are said to be *opposite in sign*.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …

So for instance, -3 and 3 are opposite in sign. And 2 and -4 are opposite in sign.

And in a number line, each and every number on one side of 0 has *its negative* on the other side.

… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …

What do we mean by its negative?

Negative is opposite of positive, and also, it means *opposite in sign*.

So, negative of negative is positive, and of course, negative of positive is negative. So for instance, we have this: $-(-9) = 9$, and of course, the negative of 9 is -9.

Thus, if we take the negative of a number, the number changes its sign if it’s not 0. And taking the negative, we change the sign only. By the way, 0 is nether positive nor negative.

So if a number is 5, its negative is –5, and if a number is -9, its negative is 9, since we have this: –(–9) = 9.

Taking, however, the negative of a number, we don’t just change its sign. So we don’t just erase a minus sign or add it, of course.

When taking the negative of a number, that is, when changing the sign of a number, we need to multiply it by –1.

So multiplying it by –1, we get its negative, that is, we change the sign of the number. For instance, 9 x (–1) = –9, and of course, we have this, too: –1 x 9 = –9

That’s not the only way, though.

Dividing it by –1, we can change the sign of it, too. So we can take the negative of 9 this way, too: 9/(–1) = –9.

By the way, we don't have –0, and we don't have +0 either. 0 has no sign. So 0 is 0.
In math called Calculus though, we can see +0 or –0, which is not 0 in that case.

It is called an infinitesimal, which means very extremely small, so it's as good as 0, but is not 0.

And 9 and –9 have the same magnitudes.

So *the negative* of a number is a number *equal in magnitude and opposite in sign*.

In other words, a number and its negative are equal in magnitude and opposite in sign.

So whether positive or negative, *the negative* of the number is a number that is equal in magnitude and opposite in sign.
What then do we mean by *the negative* in the statement above?

It means *opposite in sign*.

So for instance, the negative of –3 is 3.

And of course, the negative of 3 is –3.

So 3 and –3 are *equal in magnitude* and *opposite in sign*, and we have this:

3 + (–3) = 0.

In general, we have this: $\boldsymbol{A + (-A) = 0}$

And we can put it this way, too: $\boldsymbol{-A + A = 0}$

For instance, we can have these:

12 + (-12) = 0

-35 + 35 = 0

287 + (-287) – 5 = 0 – 5 = –5

–15 + 2 + 15 – 1 = –15 + 15 + 2 – 1 = 2 – 1

Math basics: *The negative of a negative number is positive*.

In short, negative of negative is positive.

So we have this: $-(-9) = 9$.

And in general, we have this: $-(-A) = A$

For instance,

$-(-16) = 16$, $-(-8) = 8$, $-(-328) = 328$

What then about this: $-(-(-5))$?

$-(-(-5)) = -5$, because $-(-5) = 5$