Triangle Basics A

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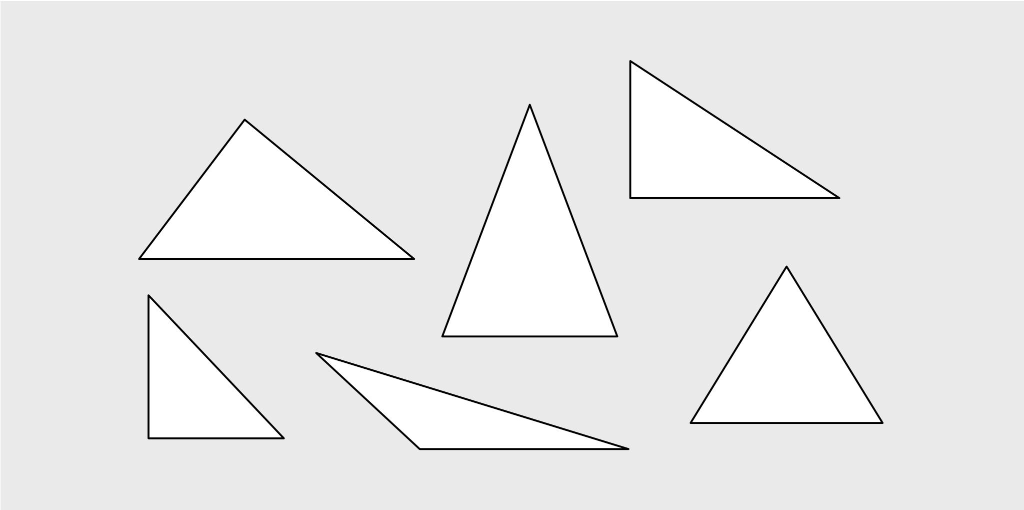
Triangle Basics 1



A triangle?　What is it in math?

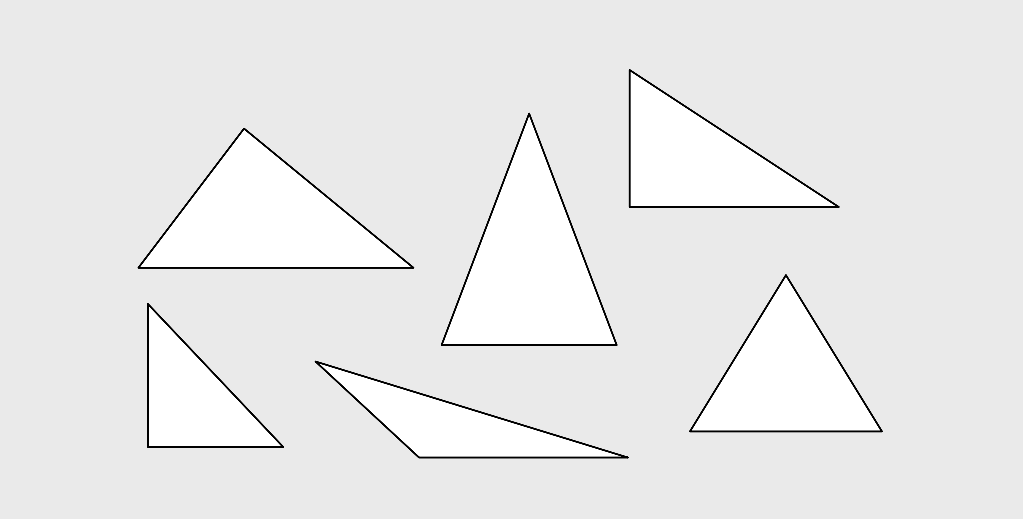
It’s a mathematical shape, and is in a plane. So a triangle is one of plane figures called polygons.　Polygon?

A polygon in basic math is said to be simple and convex, and is covered briefly in the lesson for polygons.

Such a polygon is like a loop made of ***straight edges connected end-to-end***. And those edges are often called ***sides***, and in math, we call them ***line segments***, too.

So a polygon is a closed plane figure made of line segments called sides and connected end-to-end.

In short, it’s a multi-sided closed plane figure.



So a triangle can be called a three-sided closed plane figure.

And three sides are the minimum in polygons.

So it’s the ***simplest*** polygon, and is a plane figure ***the most basic*** and ***most often used*** when we do math.

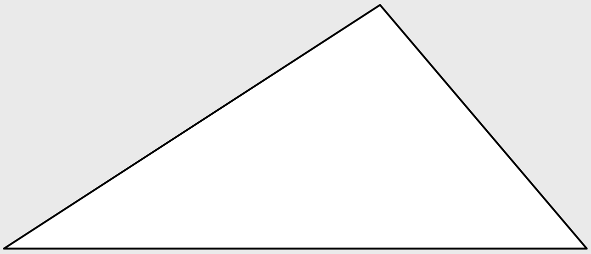
What then, do we do with triangles?

How do we work with triangles?

And what does a triangle have to do with?

Though a triangle is made of three line segments called sides, it’s not just made of those line pieces.

Fig. 1



So what else is there, in the triangle?

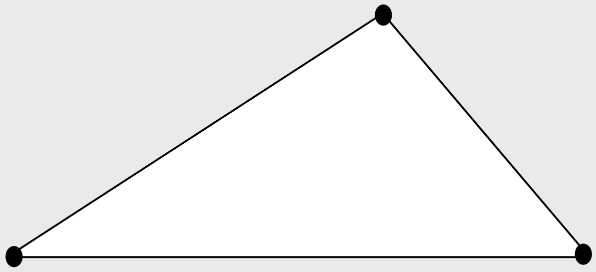
It’s rather important components. And we often work with them, too. They might not be visible right off the bat. Some of you might have already notice them, though.

So let’s now take a closer look, and see what’s in there, what it’s really made of, what’s hiding behind the sides.

First off, in a triangle, each side meets the other two sides. Where then does it meet the other two?

At their endpoints. In a triangle, each side meets the other two sides at their endpoints as shown in the figure below.

Fig. 2

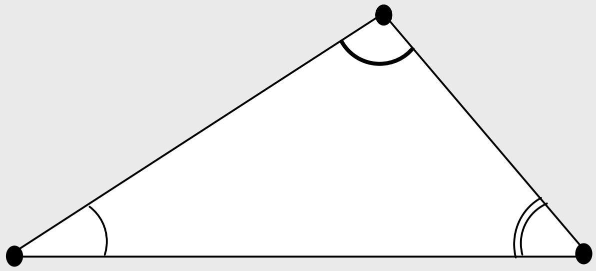


And we call each of those endpoints a vertex. A triangle has three such points, so it has three vertices.

And next, tri means three, so triangle means three angles. Where then are those three angles?

Two sides meet at a vertex, and make an angle there, at the vertex. A triangle has three vertices, so it has three angles.

Fig. 3



So, working with a triangle, we work with its angles and sides, connected end-to-end at the points called vertices.

Making therefore, a definition for triangles, we may want to put it this way:

> A triangle is a closed plane figure made of ***three angles*** and ***three sides*** connected ***end-to-end*** at the vertices.

So a triangle is composed of three angles and three sides, and thus, has six components.

And we can call the six components triangle components.

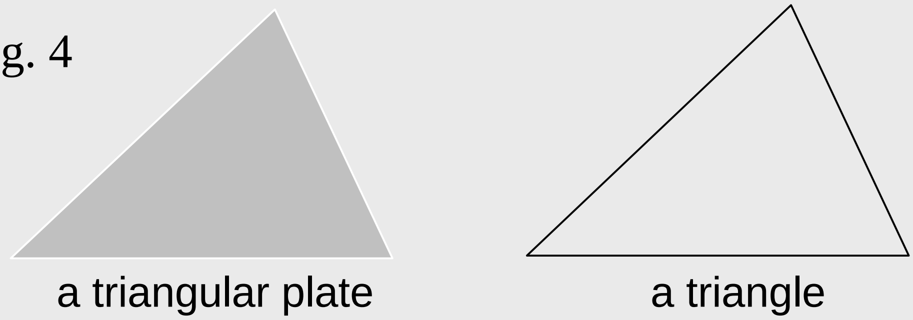
What then about the vertices?

They are the endpoints of the sides, and therefore, are parts of the components.

And note that a triangle itself is a loop of three line segments, and has nothing inside, as well as outside.

Thus, it's not a plate that is triangular. So ***nothing*** inside a triangle is a part of the triangle, which is therefore, empty inside.

Fig. 4


a triangular plate

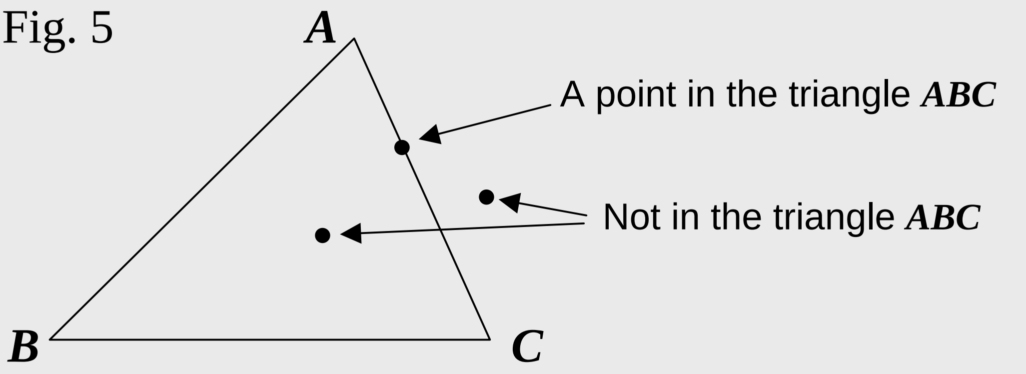
a triangle

A triangle makes a boundary, the boundary of a triangular area. And we call the amount of a triangular area the area of a triangle. In these lessons, the area is painted white or left transparent to show that it's empty.

So a point inside a triangle is not a point the triangle has. It's not in the triangle, and doesn't belong to a triangle.

What then about a point ***in*** a triangle?

Fig. 5



A point in a triangle is one of the points that make a side of the triangle. So saying a point in a triangle, we mean a point that belongs to the triangle; thus, it's a point the triangle has.

And working with a triangle, we work with its components, so we work with its angles and sides. Working with those components, we can find, for instance, the area of a triangle.

So let's see now, how we can find the area of a triangle.

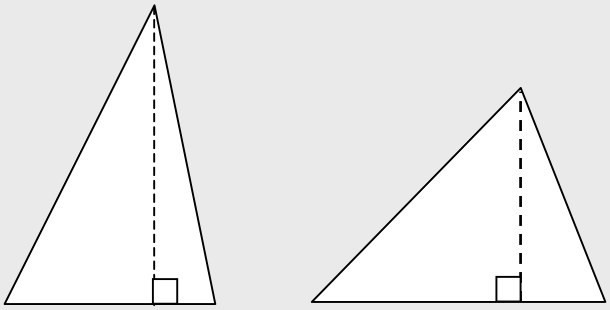
There are several ways we can find the area. Of those ways, though, we are going to cover the one most common.

To begin with, as a formula, we have this:

Taking ***the product of the base and the height divided by 2***, we get ***the area of a triangle***.

So finding the area, we need the base and the height. What then are the base and the height?

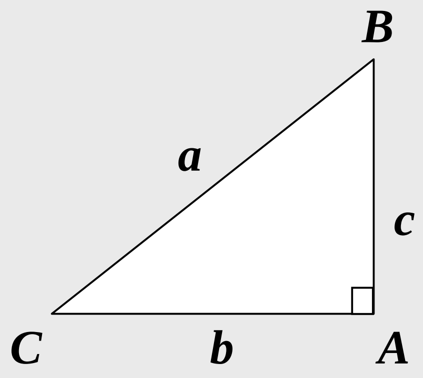
Fig. 6



In other words, in a triangle, what do we take as the base and the height?

The base is a side in a triangle, and if the triangle is a right triangle, the height is a side, too. So the area depends on the sides of a triangle.

Fig. 7



In the figure above, ΔABC is a right triangle, so not only the base but the height is a side, too.   Why, though?

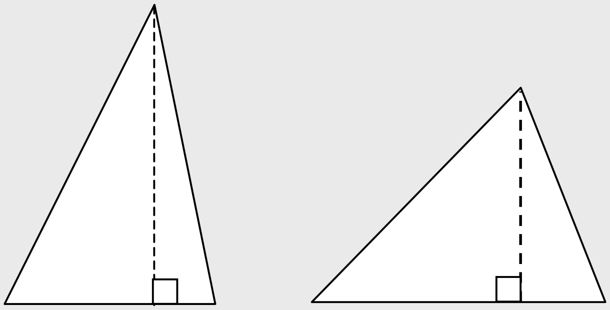
The height is perpendicular to the base, that is, the two make a right angle, 90°, so in the figure above, if taking the side c as the height, we take the side b as the base.

And, of course, instead of b, we can take c as the base; then, b is the height. So either way, in a right triangle, the base and the height both are sides, and the two are perpendicular to each other.

What if, however, it's not a right triangle?

The height is the distance from the base to the vertex away from the base.

Fig. 8



And if it’s not a right triangle and the height is not given, we can use a side to find the height. Normally though, unless taking higher courses in math, you will be given the height.

How then, do we find the area?

We can find it using the formula below, which is very basic.

> Taking ***the product of the base and the height divided by 2***, we get ***the area of a triangle***.

And the chances are you’ve been using it already.

So the question seems to be like this: why or how does the formula work. So you probably want to see how it works.

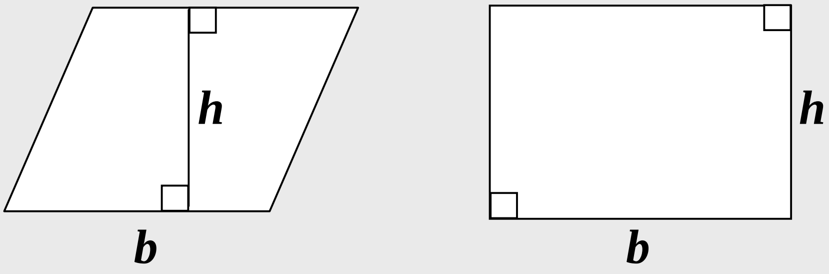
> Taking ***the product of the base and the height divided by 2***, we get ***the area of a triangle***.

Looking at closely the formula above, you can notice this:

> the product of the base and the height

So what is it?

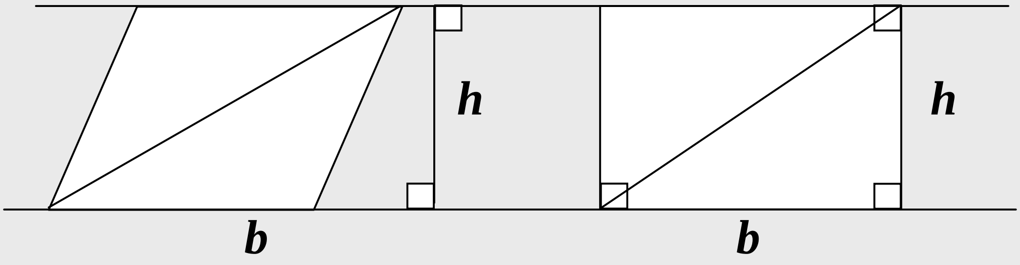
Fig. 9



It is the area of a parallelogram.
The two parallelograms above have the same areas.
So taking the half the area, what do we get?

We get, of course, half the area of a parallelogram.

Fig. A



It isn’t just the half but the area of a triangle, too.

So taking the product of the base ***b*** and the height ***h***, we get the area of a parallelogram where the base is ***b*** and the height is ***h***, and then, taking the half the product, we get the area of a triangle where the base is ***b*** and the height is ***h***.

In short:

> Taking ***the product of the base and the height divided by* 2**, we get ***the area of a triangle***.

Well then, the next question is, why the product is the area of a parallelogram?

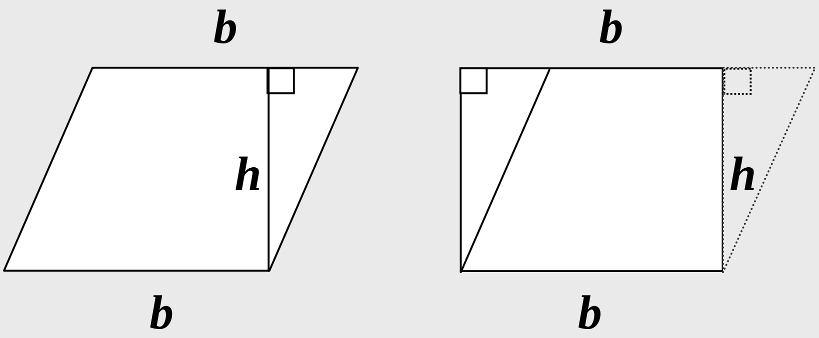


Fig. B

In the figure above, the small right triangle on the left is the same as the small one on the right.

So cutting the triangle away from the parallelogram and then, putting it on the opposite side of the parallelogram, we get the rectangle on the right side of the figure above.

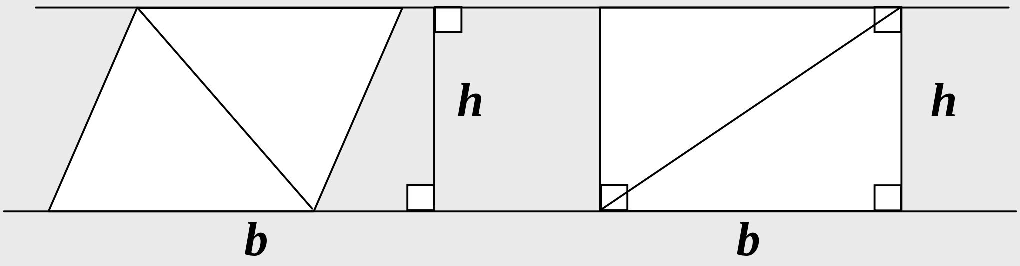
Then, the area of the rectangle is the same as that of the parallelogram. So taking the product of the base and the height of a parallelogram, we get the area of it.

Thus, taking the area of a triangle, we can use the formula as follows.

Taking ***the product of the base and the height divided by 2***, we get ***the area of a triangle***.

In short, base times height divided by 2.

Fig. C

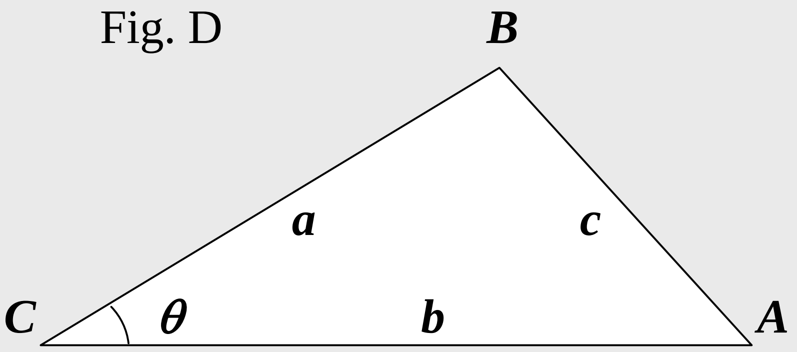


What are then the other formulas?

We have two more often used, but we need some more tools in higher math to understand them.

If given two sides and the angle between the two, we can find R, the area of ΔABC using the formula as follows.

Fig. D



$R = \frac{1}{2}ab\sin\theta$, where a and b are the two sides forming the angle θ, called ***theta***, and $0^{o} < \theta < 180^{o}$.

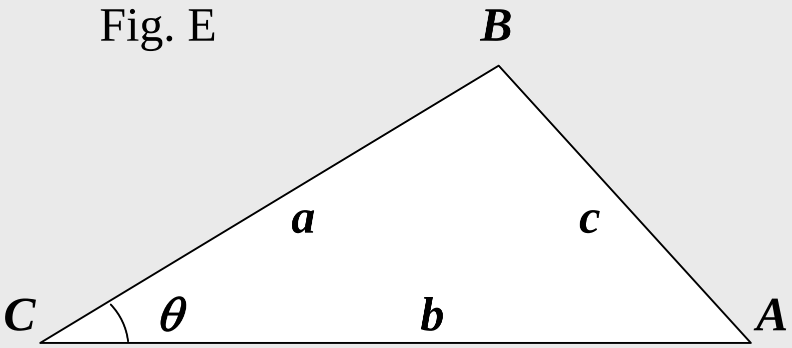
And if given neither height nor angle, we can still get the area using the one below, called Heron’s formula.

$$R = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2},$$

where ***a***, ***b***, and ***c*** are, of course, the three sides.

So finding, for instance, the area of a regular triangle where each side is of length 8, we can get it using the first one above the way as follows.

Fig. E



$R = \frac{1}{2}ab\sin\theta$, where ***a*** and ***b*** are the two sides forming the angle θ, called ***theta***, and $0^o < \theta < 180^o$.

It's a regular triangle, so every angle is 60^o; thus, we can set $a = 8$, $b = 8$, and $\theta = 60^o$, and then, get the area this way:

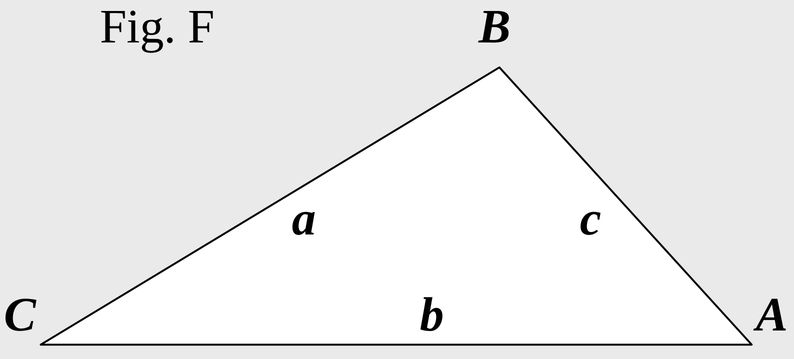
$\boldsymbol{R} = \frac{1}{2}$ x 8 x 8 x **sin** 60^{o}, where **sin** $60^{\mathrm{o}} = \frac{\sqrt{3}}{2}$, so we get this: $\boldsymbol{R}$ = 32 **sin** 60^{o} = $16\sqrt{3}$, so the area is $16\sqrt{3}$.

And let’s now, find the area using Heron’s formula, and see if both are the same.
It’s a regular triangle where each side is of length 8.

$$\boldsymbol{R} = \sqrt{\boldsymbol{s}(\boldsymbol{s}-\boldsymbol{a})(\boldsymbol{s}-\boldsymbol{b})(\boldsymbol{s}-\boldsymbol{c})}, \text{ where } \boldsymbol{s} = \frac{\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}}{2},$$

where ***a***, ***b***, and ***c*** are, of course, the three sides.

Fig. F



So setting ***a*** = 8, ***b*** = 8, and ***c*** = 8, we get this:

$$\boldsymbol{R} = \sqrt{\boldsymbol{s}(\boldsymbol{s}-8)(\boldsymbol{s}-8)(\boldsymbol{s}-8)}\text{, where } \boldsymbol{s} = \frac{8+8+8}{2} = 12.$$

So we get this:

$$\boldsymbol{R} = \sqrt{12(12-8)(12-8)(12-8)} = \sqrt{12 \text{ x } 4 \text{ x } 4 \text{ x } 4} = 16\sqrt{3},$$

which is the area we found using this: $\boldsymbol{R} = \frac{1}{2}\boldsymbol{ab}\,\mathbf{sin}\,\boldsymbol{\theta}$.

And working with the components of a triangle, together with other math tools, we can find lengths, distances, angles, etc.

And more importantly, understanding those math tools, we should be much familiar with the basics on triangles.

They all begin with basics in triangles. Well, so again, basics matter, and it’s particularly the case, they matter to students. And doing math, most of us do basic math.

What triangles then do we often use in basic math?

We’ll continue this in the next lesson.

Triangle Basics 2

Doing basic math, we use several different kinds in triangles. And they are as follows.

Right triangles (Not opposite of wrong triangles ☺)

Isosceles triangles (two equal sides)

Isosceles right triangles (two equal sides and 90^o)

Regular or equilateral triangles (three equal sides)

Scalene triangles (three different sides)

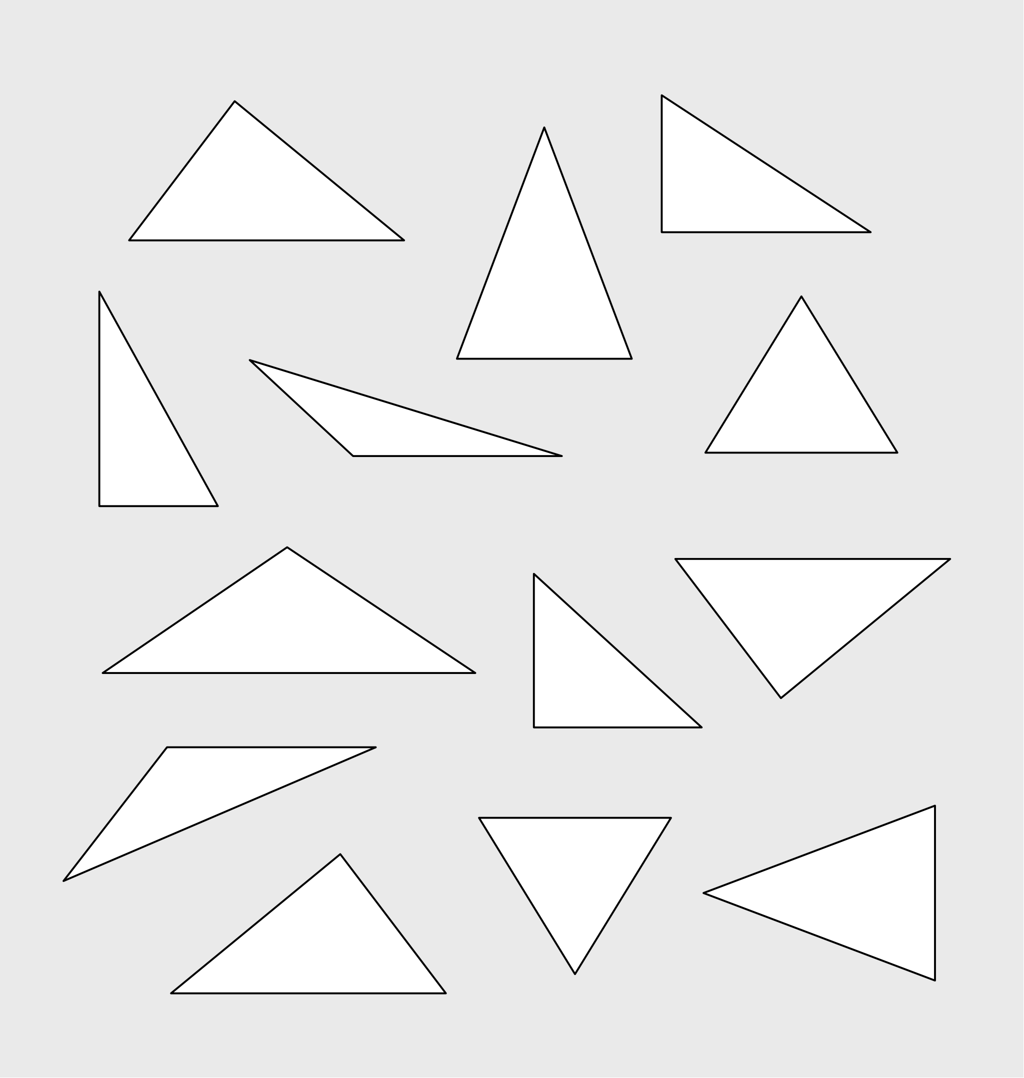
Obtuse triangles (an angle is bigger than 90^o)

Acute triangles (no angle is bigger than 90^o)

You might have noticed that they are sorted in terms of sides, angles, or both.

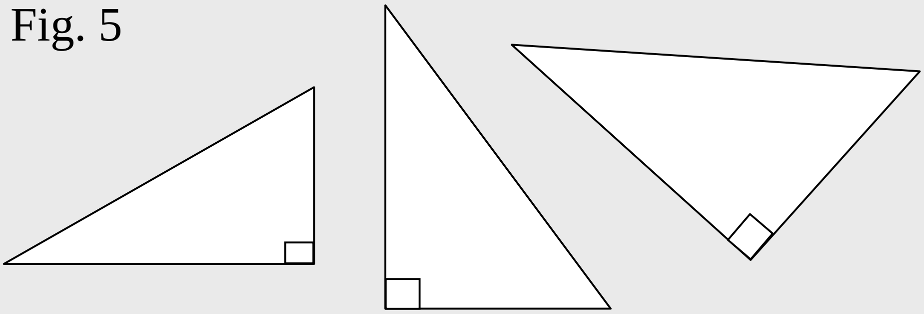
And you will notice that angles and sides go together. So, you get to notice that two equal sides mean two equal angles, three equal side mean three equal angles, and three different sides mean three different angles.

A triangle is made of three sides and three angles, along with three vertices. So we can sort triangles in terms of angles or sides the way as follows.



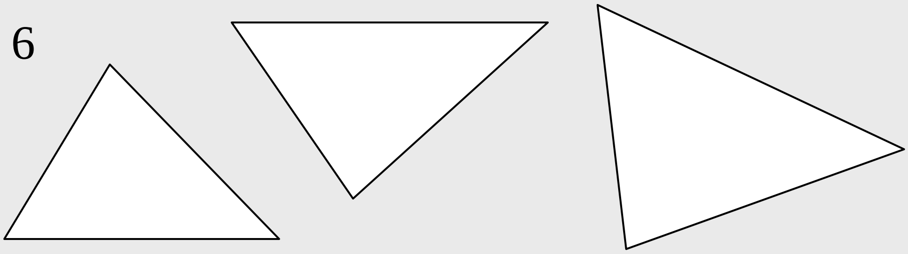
To begin with, if a triangle has an angle of 90^o called a ***right*** angle (not necessary a good angle ☺), we call it a ***right*** triangle, not an opposite of a wrong or bad triangle. ☺

Fig. 5



Next, suppose all the three sides are different in a triangle. What then about the three angles?

Fig. 6



In such a triangle, not only are the three sides, but the three angles are all different, too. So it’s made of three different angles and three different sides.

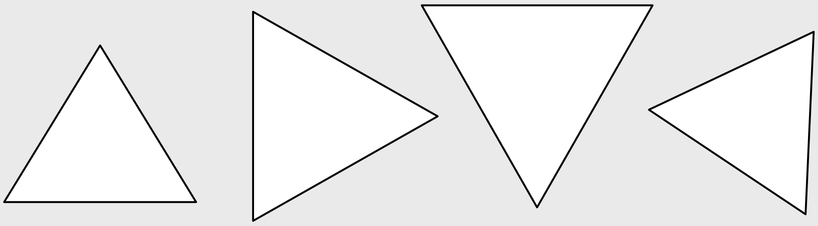
If in a triangle, all the three angles are different, all the three sides are different, too. Also, if all the three sides are different, all the three angle are different, too. And we call such a triangle a ***scalene*** triangle.

If however, all the three sides are the same, the triangle is said to be ***regular*** or ***equilateral***, so we call it a regular triangle or an equilateral triangle.

And if all the thee angles are equal, all the three sides are equal, too, and vice versa. So a regular triangle is made of the same three angles and the same three sides.

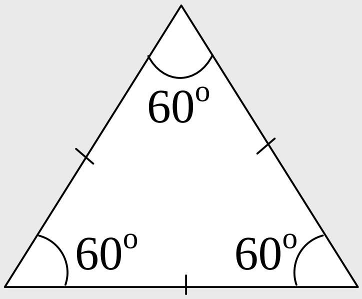
What then is each angle in a regular triangle?

Fig. 7



The sum of the three angles in a triangle is 180^o, and the three angles are equal in a regular triangle. So every angle in it is a third of 180^o, which is 60^o.

Fig. 8



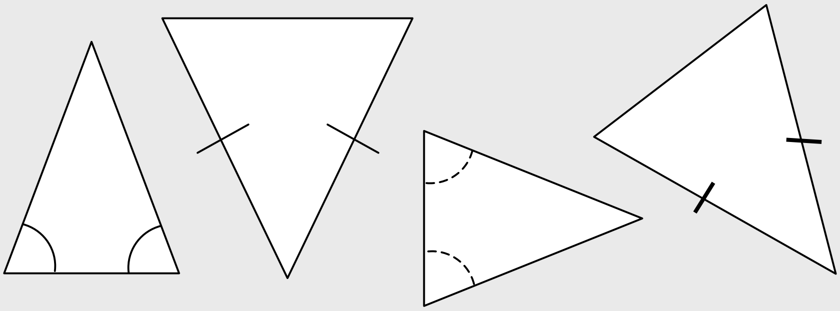
Why is the sum 180^o, though?

It is 180^o in every triangle, and we'll see shortly how it is the case, the proof.

What if ***two sides only*** are the same?

Then, two angles only are the same, too, and the triangle is said to be ***isosceles***, so we call it an isosceles triangle.

Fig. 9



And if two angles only are equal, two sides only are equal, too, and vice versa. So in an isosceles triangle, two angles are equal and two sides are equal.

What if two sides only are the same and an angle is 90^{o}, that is, a right angle?

Then, it is called an isosceles right triangle or a right triangle isosceles.

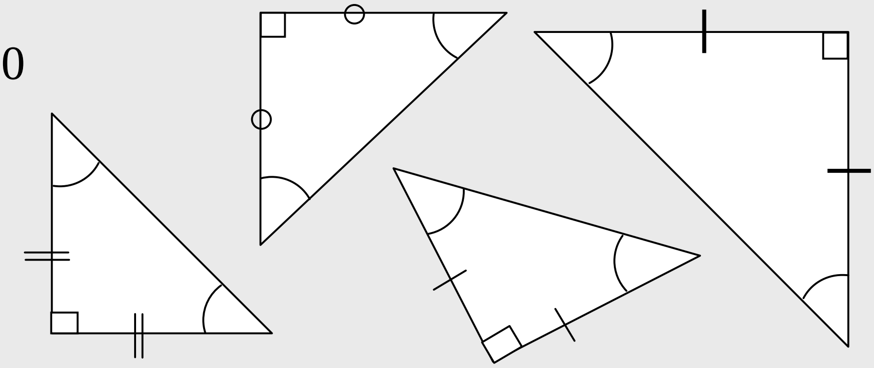


Fig. 10

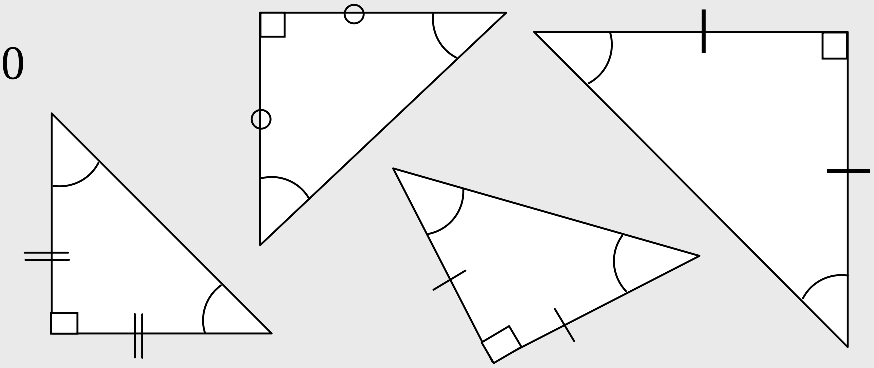
If in a triangle, two angles are the same, its two sides are the same, too, and vice versa. And it is called isosceles. So an isosceles triangle is a triangle where two sides and two angles are the same.

And a regular triangle can be taken as an isosceles triangle, too, because it has two equal sides, since all the three sides in it are the same.

What then are the three angles in a right triangle isosceles?

In a right triangle isosceles, two angles are equal, the other angle is 90^o, and all the three angles add up to 180^o. What then is the sum of the two equal angles?

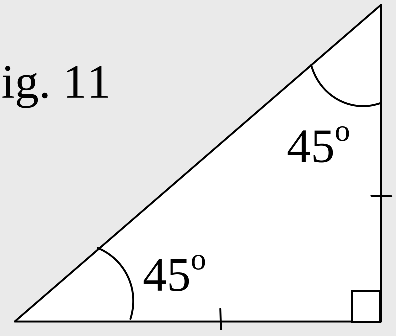
Fig. 10



We have this: $180^o – 90^o = 90^o$.

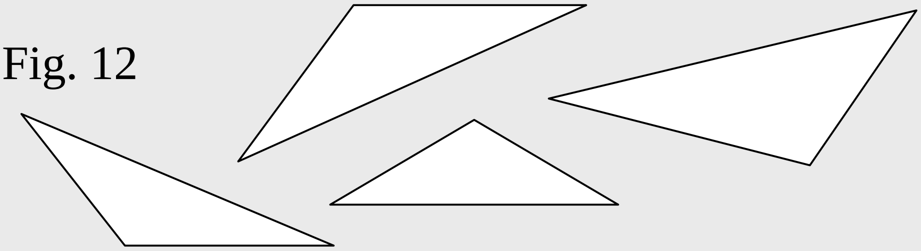
So the sum of the two same angles is 90^o, and the two angles are 45^o each. And thus, the three angles are these: 45^o, 45^o, and 90^o.

Fig. 11



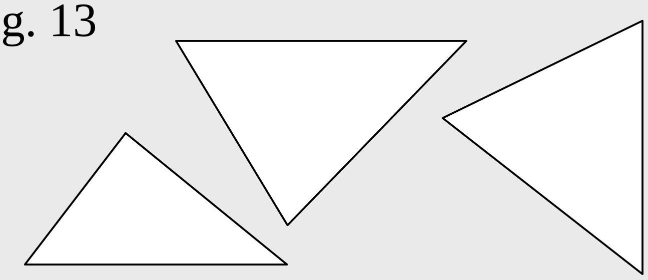
Now next, if an angle is greater than 90^{o} as 95^{o} or 100^{o}, the triangle is said to be obtuse, so we call it an obtuse triangle.

Fig. 12



If however, no angle is bigger than or equal to 90^{o}, that is, every angle is less than 90^{o}, the triangle is said to be acute, so we call it an acute triangle.

Fig. 13



We'll move on to the next step in the next lesson.

Triangle Basics 3

Doing basic math, we use several different kinds in triangles. And they are as follows.

Right triangles (an angle is 90^{o})

Isosceles triangles (two equal sides)

Isosceles right triangles (two equal sides and 90^{o})

Regular or equilateral triangles (three equal sides)

Scalene triangles (three different sides)

Obtuse triangles (an angle is bigger than 90^{o})

Acute triangles (no angle is bigger than 90^{o})

And note that angles and sides go together. So two equal sides mean two equal angles, three equal side mean three equal angles, and three different sides mean three different angles.

Understanding the concepts of and working properly with those triangles above, we can do many problems and learn many things in math.

And working with triangles, we don't just work with them. Working with a triangle, we name it first. And that's not it. We may want to name components as sides and angles, too.

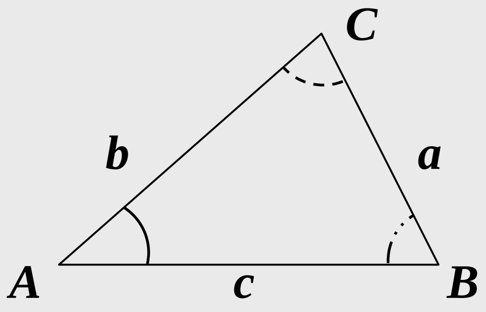
And naming them, we often use letters as alphabets. Naming them, we can work with them quickly and with ease.

And naming a triangle, we often use the names of the three vertices in the triangle.

What do we mean by the vertex, though?

A vertex in a polygon is a point where two sides meet. So a triangle has three vertices.

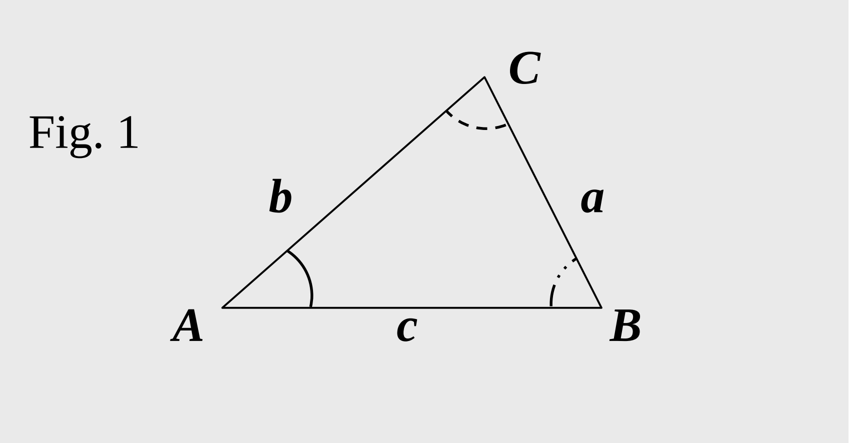
Fig. 1



If for instance, a triangle has three vertices called ***A***, ***B***, and ***C***, the triangle is usually called a triangle ***ABC***.

And for simplicity, we often use a math symbol, together with the name of the triangle. And the symbol is this: **Δ**, which is just a small triangle. So for instance, ΔABC means a triangle ABC.

Fig. 1



What then about naming angles?

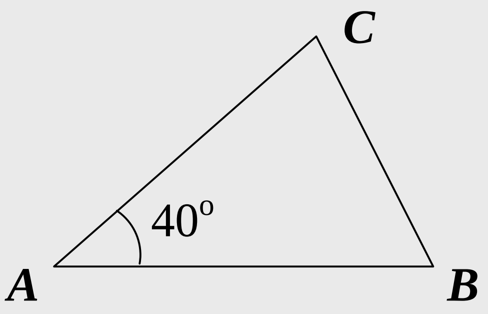
Naming an angle a triangle has, we usually use a capital letter as A, and the angle is an internal angle, of course.

That is to say that naming an angle in a triangle, we often use the name of the vertex that has the angle.

So for instance, saying the angle A in a triangle ABC, we mean the angle at the vertex A. And indicating an angle, we often use an angle sign, which is this: $\angle$.

For instance, naming an angle called A, we can put it this way: $\angle A$. And indicating that the angle at the vertex A is 40^{o}, we can put it this way: $\angle A = 40^{o}$.

Fig. 2



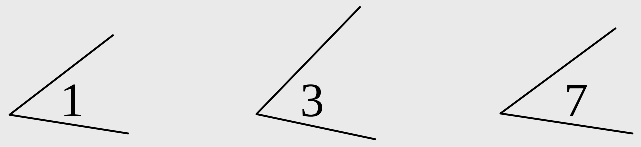
And indicating that two angles A and B are equal, we can put them this way: $\angle A = \angle B$.

Indicating also, that three angles A, B, and C are equal, we can put them the way as follows: $\angle A = \angle B = \angle C$.

There are many ways to name an angle. As long as it is clear and efficient, we can use any letter, number, or figure to name an angle.

For instance, if indicating an angle 1, an angle 3, and an angle 7, we can put them this way: ∠1, ∠3, and ∠7.

Fig. 3

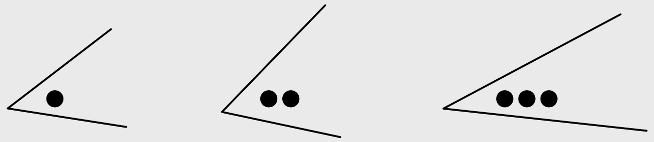


And indicating that the angles 1 and 3 are equal, we can put them this way: ∠1 = ∠3. And of course, indicating that the three angles 1, 3, and 7 are all equal, we can put them this way: ∠1 = ∠3 = ∠7.

It seems some people like to use dots as these, •, ••, •••, so when using them, they indicate angles this way:

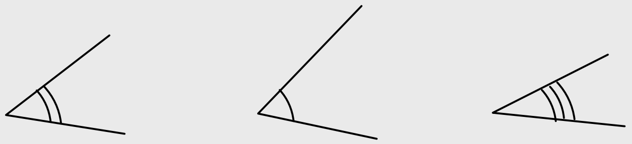
∠•, ∠••, ∠•••

Fig. 4



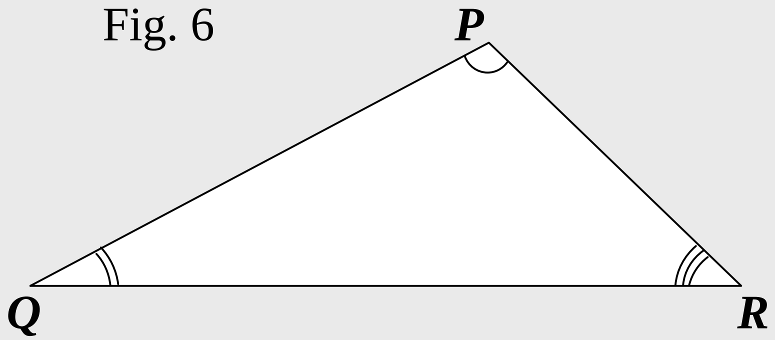
And often times, indicating angles in a polygon as a triangle, we use an arc or arcs cascading one another as shown below:

Fig. 5



Sometimes, we use three letters to indicate an angle.

Fig. 6



In the triangle above, the angle $\boldsymbol{Q}$ can be indicated the way as follows: $\angle \boldsymbol{Q}$, which may be the simplest way.

And the angle $\boldsymbol{Q}$ can be indicated either of the ways as follows: $\angle \boldsymbol{PQR}$ and $\angle \boldsymbol{RQP}$.

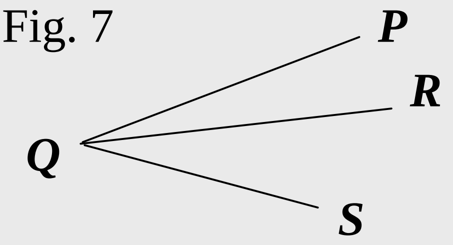
So we have this: $\angle \boldsymbol{Q} = \angle \boldsymbol{PQR} = \angle \boldsymbol{PQR}$.

And as shown above, when we use three letters, we use the letter in the middle for the angle indicated by the one letter.

Why bother using three letters, though?

Sometimes, we can have a situation as follows.

Fig. 7

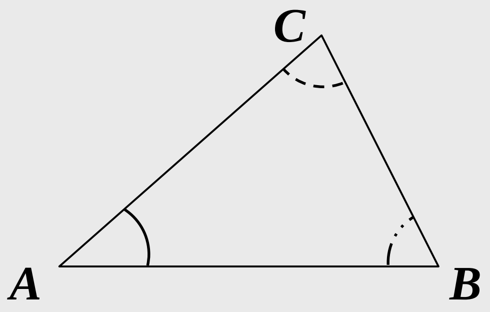


In the figure above, we have these:

$\angle Q \neq \angle PQR$, $\angle Q \neq \angle RQS$, but $\angle Q = \angle PQS$.

What then about naming the sides in the triangle ABC below?

Fig. 8



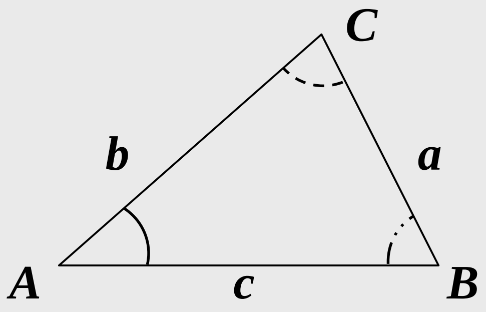
Naming each side in a triangle, we usually use a lowercase letter as ***a***.

So for instance, as shown in the triangle ***ABC*** below, we can use ***a*** as the side facing the angle ***A*** or the vertex ***A***.

That is, ***a*** is the side opposite of the angle ***A*** or the vertex ***A***.

So for instance, we can put the names of the angles, the vertices, and the sides the way below.

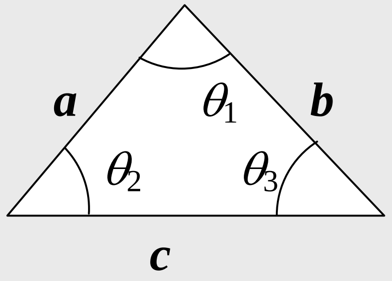
Fig. 9



So the side ***b*** is the side facing the angle ***B*** or the vertex ***B***, and ***c*** is the side facing the angle ***C*** or the vertex ***C***.

For instance, we can express a scalene triangle the way as follows.

Fig. 10

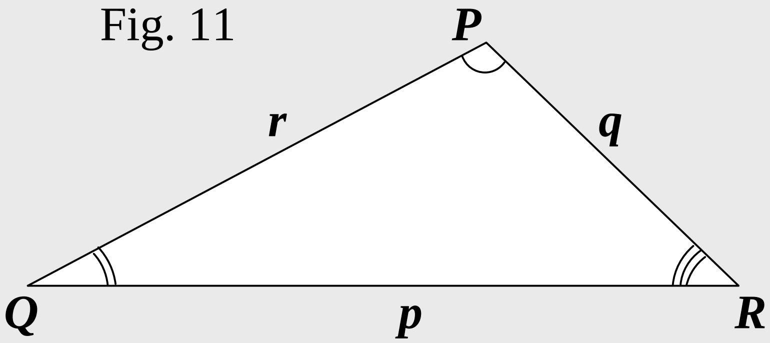


$$\boldsymbol{\theta_1 \neq \theta_2 \neq \theta_3 \Leftrightarrow a \neq b \neq c}$$

In Fig. 10, $\boldsymbol{\theta_1}$, $\boldsymbol{\theta_2}$, and $\boldsymbol{\theta_3}$ indicate the angles in the triangle, and ***a***, ***b***, and ***c*** indicate its sides. So in a scalene triangle, not only are different all the sides, but all the angles, too.

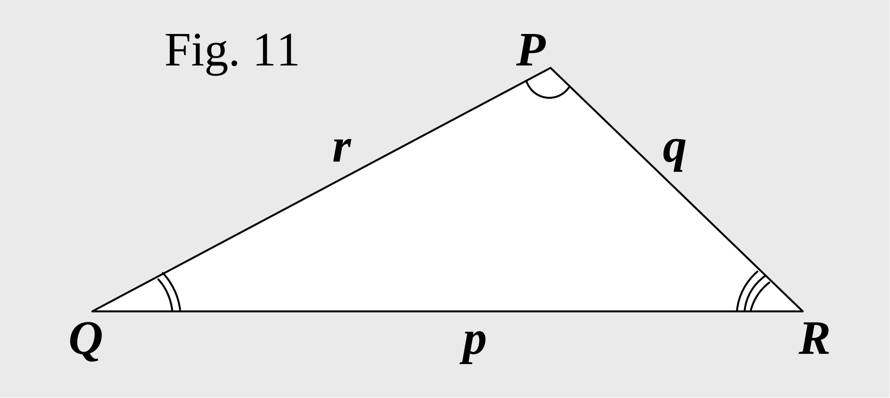
And naming each side, we can either use one letter or two letters. For instance, if naming the side facing the angle or the vertex ***R***, we use ***r***.

Fig. 11



And using two letters for a side, we use the letters used for the vertices connected by the side. For instance, we can name the side $\boldsymbol{r}$ this way, too: $\boldsymbol{PQ} = \boldsymbol{QP}$.

Fig. 11



So we have this: $\boldsymbol{r} = |\boldsymbol{PQ}| = |\boldsymbol{QP}|$.

Even if reversing thus, the names of the vertices when indicating a side, we still indicate the same side.
Why the vertical bars around the letters, though?

Using a pair of vertical bars around the letters used for a side, we indicate the length of the side.

For instance, $|\boldsymbol{PQ}|$ indicates the length of $\boldsymbol{PQ}$, and $\dfrac{|\boldsymbol{PQ}|}{|\boldsymbol{QR}|}$ equals $\dfrac{\boldsymbol{r}}{\boldsymbol{p}}$, that is, $\dfrac{|\boldsymbol{PQ}|}{|\boldsymbol{QR}|} = \dfrac{\boldsymbol{r}}{\boldsymbol{p}}$. So if the length of $\boldsymbol{PQ}$ is 3, we can put it this way: $|\boldsymbol{PQ}| = 3$, or this way: $\boldsymbol{r} = 3$.

The bars are though, often omitted for simplicity purposes if no ambiguity is worried.

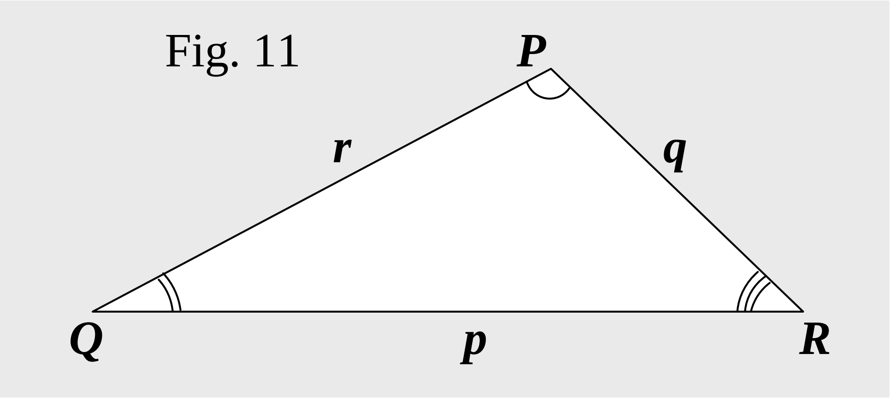
By the way, saying ***sides*** talking about triangles, rectangles, etc., we often mean ***their lengths***, as well as ***line segments themselves***.

And we can indicate the same triangle many ways.

For instance, we can have this:

$\Delta PQR = \Delta QRP = \Delta RPQ = \Delta PRQ = \Delta RQP = \Delta QPR$.

Fig. 11

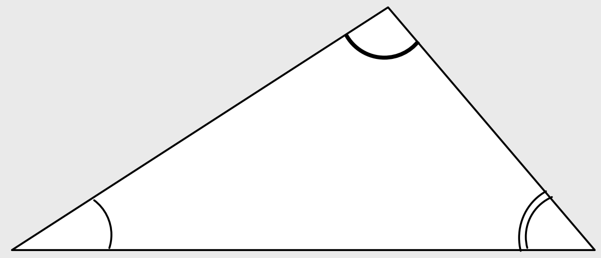


We’ll move on to the next step in the next lesson.

Triangle Basics 4

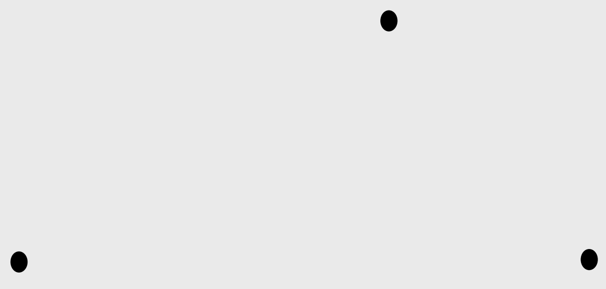
Now, again, what is a triangle made of?
What are its components?

Fig. 1



A triangle comprises three angles and three line segments called sides. How then do they make a triangle?

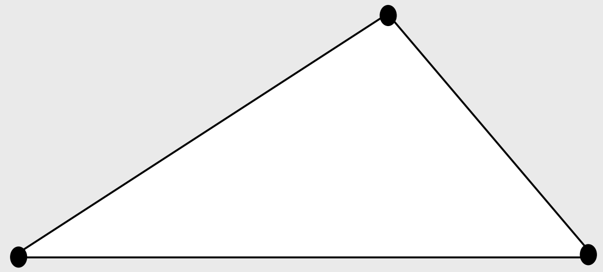
Fig. 2



Each side meets the other two sides at their endpoints. So every side shares its endpoints with the other two sides.

Then, a loop is made, and is made of three sides connected end-to-end. And we call each endpoint a vertex, and call the loop a triangle.

Fig. 3

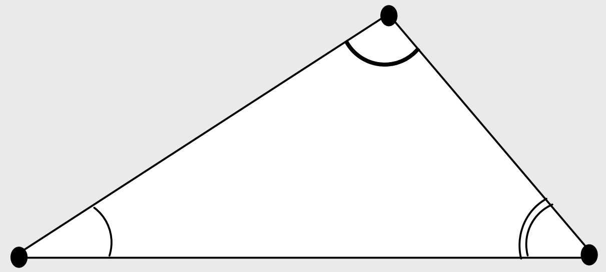


So each side share with the other two sides two points called vertices, and a triangle has three vertices.

What then about the three angles?

At each vertex, two sides meet and make an angle, so a triangle has three angles, since it has three vertices.

Fig. 4



Now, a triangle is in a plane, and a loop is a closed object.

Making thus, a definition for triangles, we can put it this way:

> A triangle is a closed plane figure made of ***three angles*** and ***three sides*** connected ***end-to-end*** at the vertices.

So if forming a triangle, do we have only to connect three points using three line segments?

We can form a triangle connecting three points, only if the three are not in a line. So given three points in a line, we cannot make a triangle connecting the three points.

Fig. 5

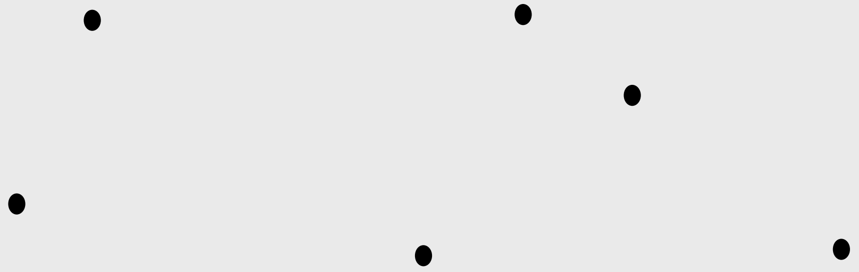
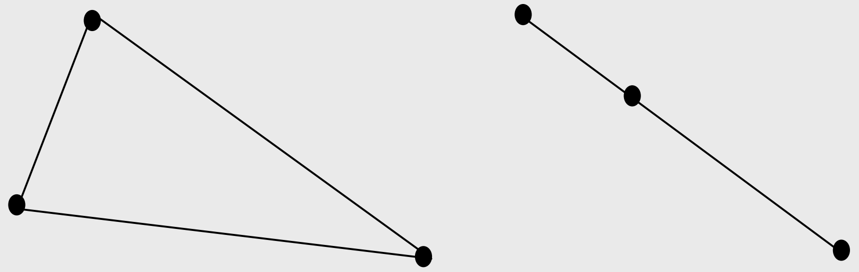


Fig. 6



So not every group of three points can make a triangle.

What then about line segments?

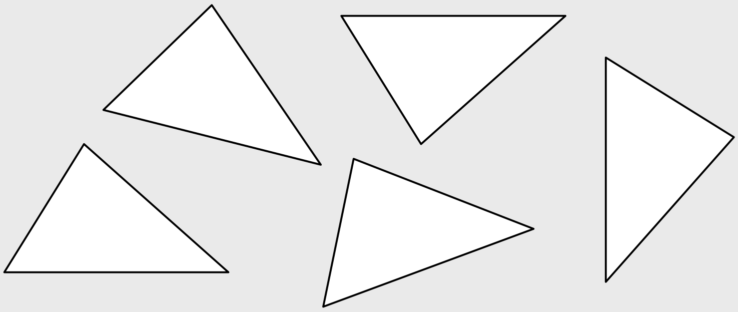
Can any three line segments be three sides in a triangle?

No, not any. So some three line segments can make a triangle. Not any three.

Only one triangle can be made of three line segments ***if the three can make a triangle***.

So if three line segments can make a triangle, every time we make a triangle using the three, we always make the same triangle. One triangle only, and no other.

Fig. 7



No matter how we may connect them end to end, we make the same triangle time and time again. Well, there is in fact, only way to connect them end-to-end, since only triangle is possible.

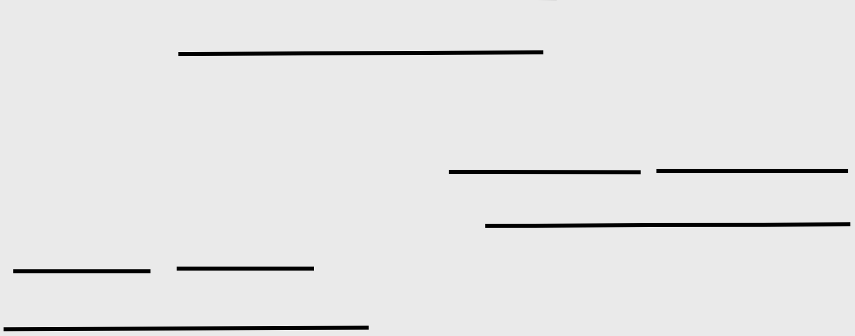
So we can make ***only one particular triangle using three line segments if the three can make a triangle***.

And thus, there are two things to note.

One is that we can make one triangle only, not two or more.

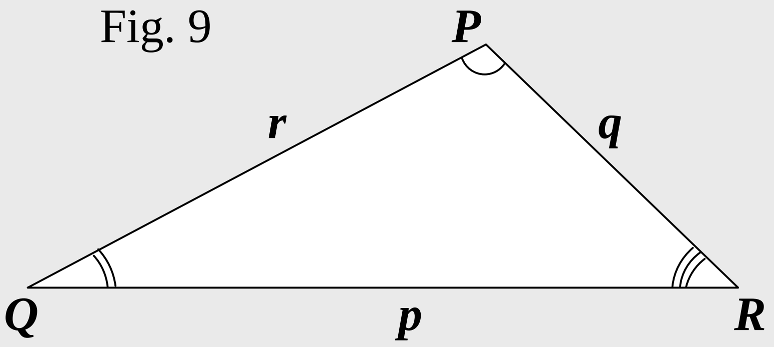
And the other is that ***not*** every group of three line segments can make a triangle. Why not?

Fig. 8



In the figure below, since the three sides ***p*, *q*,** and ***r*** make a triangle, ***each of the three is less than the sum of the other two***. It might sound quite obvious to some people, and yet, seems to be often neglected when they solve problems.

Fig. 9



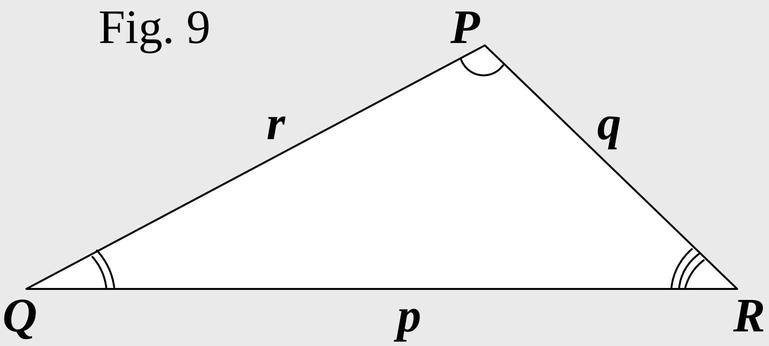
So, anyway, what do we mean by the statement below?

"Since the three sides ***a*, *b*,** and ***c*** make a triangle, ***each of the three is less than the sum of the other two***."

In every triangle, the sum of any two sides is ***grater*** than the other, and we call the fact ***Triangle Inequality***.

In other words, the ***Triangle Inequality*** is saying that any one side has to be ***less*** than the sum of the other two.

Fig. 9



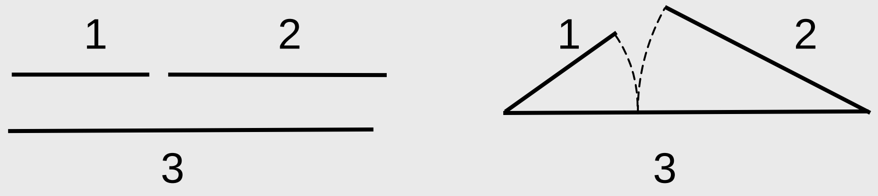
$$p < q + r, \quad q < r + p, \quad r < p + q$$

And the math property called ***Triangle Inequality*** is very important. It often helps get solutions when you solve problems related to triangles not only directly but indirectly, too. So you may want to be very familiar with that.

Let’s now take a look at some simple examples.

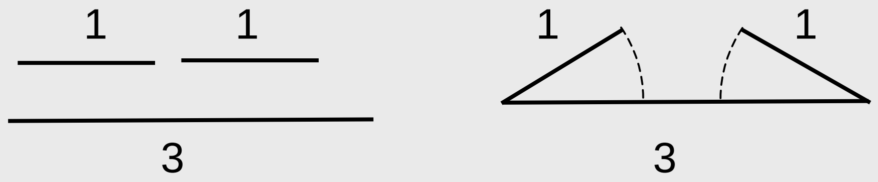
For instance, a group of three line segments (1, 2, 3) cannot make a triangle, because 1 + 2 = 3, which violates the triangle inequality.

Fig. 10



And the same is true of this, too: (1, 1, 3), because we have this: 1 + 1 < 3, which violates the triangle inequality.

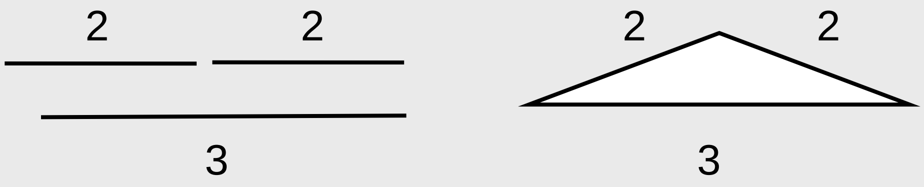
Fig. 11



What about (2, 2, 3)?

It can make a triangle, because, of the three line segments, the sum of any two is grater than the other.

Fig. 12



So, of the three sides in a triangle, each and every one is less than the sum of the other two, which is called ***Triangle Inequality***, one of the basics in triangle geometry, often used when we solve problems.

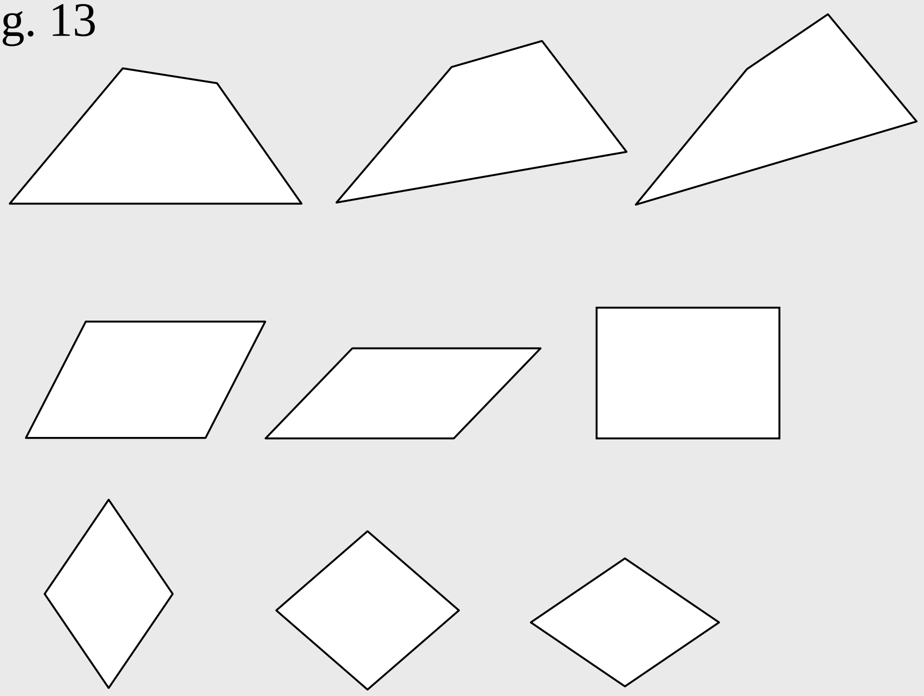
And also, we may want to keep in mind the fact below, too.

> If three line segments can make a triangle, they can make only one triangle, and no other triangle is possible.

That’s why a lot of structures that need to fight stress or deformation are made of triangles as in bridges, electricity transmission towers, etc.

By the way, connecting four rods end to end, we can make many, infinitely many, different quadrangles if the four rods can make a quadrangle.

Fig. 13



The quadrangles in each row above share all the four sides, that is, they have the same groups of four sides.

What then about a pentagon?

The same is true of a pentagon, too. So connecting five rods end to end, we can make so many different pentagons if the five rods can make a pentagon.

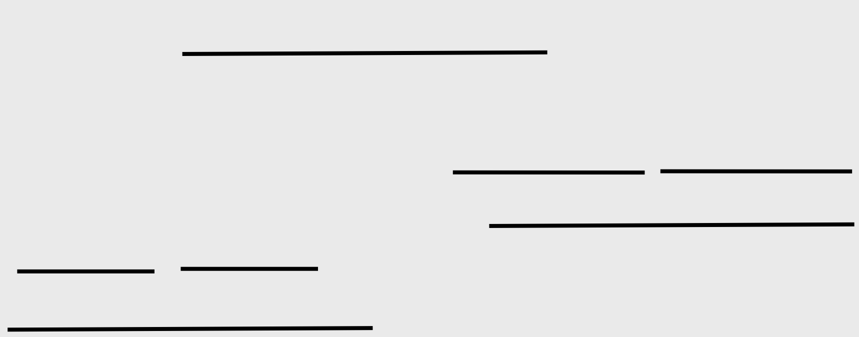
And the same is true of a hexagon, heptagon, and so forth, that is, any polygon other than a triangle.

One condition applies to every polygon, triangle or not. What's common to all polygons including triangles is this:

Rods make a polygon if they can. So not all groups of six rods can make a hexagon. What then is the condition?

We'll cover it in the next lesson. It's about the same as the one for a triangle, though.

Fig. 14



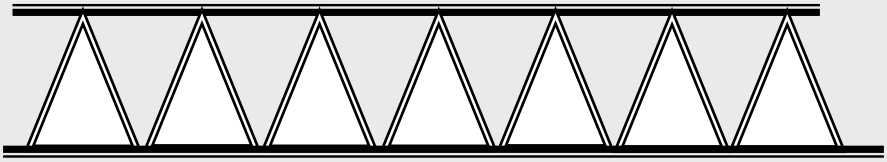
What’s important, for now, is this:

Connecting three rods end to end, we can make one triangle only, if of course, the three rods can make a triangle, that is, if the three satisfy the triangle inequality.

Thus, making a triangle not riveting but pinning together three rods, we cannot change the shape pulling or pushing the sides even if there is no friction at all at the joints.

Of course, if pulling or pushing too hard, the rods will break.

Fig. 15



What then can happen to structures with other shapes, that is, other polygons as quadrangles or pentagons?

So, on the other hand, making a quadrangle not riveting but pinning together four rods, we can change the shape pulling or pushing the sides even very slightly. And the same is true, too, of any other polygon as an octagon and nonagon.

Therefore, in the construction of bridges or towers, though the steel beams used are riveted and even welded together, the beams make triangular grids, not rectangular grids.

Even if the rivets loosen, the bridges hardly collapse if the beams make triangular grids. If however, the grids are rectangular, you know what could happen to the bridge.

Triangle Basics 5

So, what’s good about triangles?

Triangles make a structure simple, strong, and stable.
It’s because of the math basic as follows.

Only one triangle can be made of three line segments.

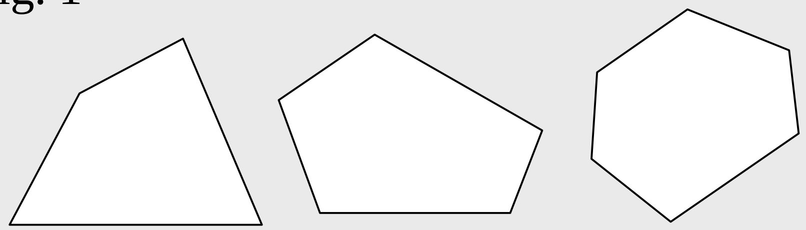
And the three do make the only triangle ***if the three can make a triangle***. That is to say that if the three satisfy the basic math rule called ***Triangle Inequality***, which says,

“Each and every side is less than the sum of the other two.”

Is there then, such a rule that applies to other polygon?

As in the case of triangles, is there any restriction on the choices of rods that are to make a polygon as a tetragon?

Fig. 1

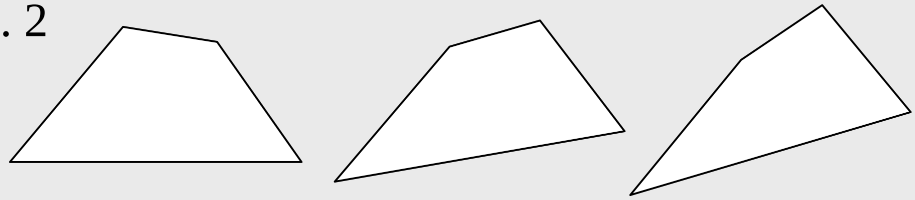


Yes, there is. Basically, the same rule applies.
For instance, we can make a tetragon using four line segments if the four can make a tetragon. And if the four make a tetragon, each of the four has to be less than the sum of the other three.

As mentioned earlier, though, it’s not the case that only one tetragon can be made.

That is to say that every time we make a tetragon using the same group of four rods, we can make a different tetragon.

Fig. 2



The tetragons above have the same groups of four sides.

And the same is true of any polygon other than a triangle. We can make one only, only in the case of triangles.

We can call the rule above polygon inequality, if you will.

And the rule says,

“If line segments make a polygon, each segment is less than the sum of all the other segments.”

We can put the idea this way, too: In a polygon, each and every side is less than the sum of all the other sides.

And after all, we can put the idea this way: In a polygon, the longest side is less than the sum of all the others.

If it’s not a triangle, every time we make it, it can be different. And if it’s a triangle, every time we make it, it is the same.

So putting together three line segments, we can make a triangle, and only one gets made if the segments satisfy the rule called Triangle Inequality.

What if now, putting together triangles?

What can we make putting together triangles?

Putting together triangles, we can form another kind of geometric object called triangle polyhedrons.

A triangle is a plane figure, so a polyhedron is made of plane figures as triangles, tetragons, etc.
And it’s said to have faces, which are plane figures.

So in a triangle polyhedron, each face is a triangle.
For instance, we can make triangle polyhedrons as follows.

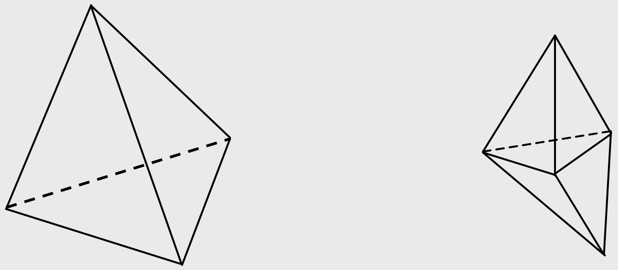


Fig. 3

tetrahedron   triangle hexahedron

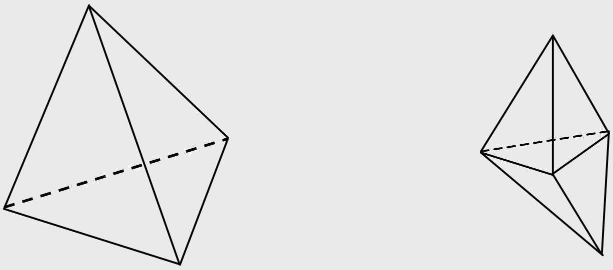
Why just tetrahedron?　Why not triangle tetrahedron?

If it’s a tetrahedron, it’s a triangle tetrahedron. There is no tetrahedron that has faces other than triangles. It’s because a polyhedron is a closed spatial figure.

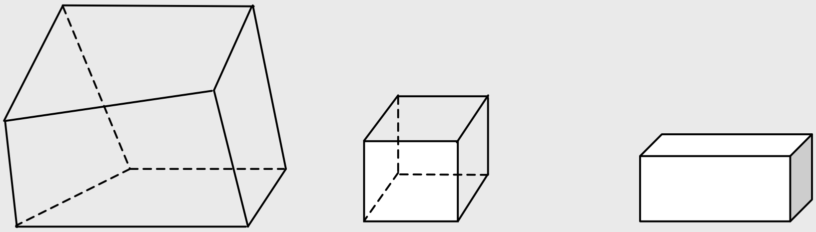
Why then triangle hexahedron?　Why not just hexahedron?

If it’s a hexahedron, it’s not only a triangle hexahedron. There are hexahedrons that have faces other than triangles. You can find some examples in the figure below.

Fig. 4



tetrahedron triangle hexahedron



tetragon hexahedron cube rectangular hexahedron

A cube can be called a square hexahedron, too. And among hexahedrons, we can have parallelepipeds, each face is a parallelogram, and among parallelepipeds, we can have rhombus hexahedrons, every face is a rhombus. And among parallelograms, we have squares, rectangles, and rhombuses, too. We’ll cover more on parallelograms later.

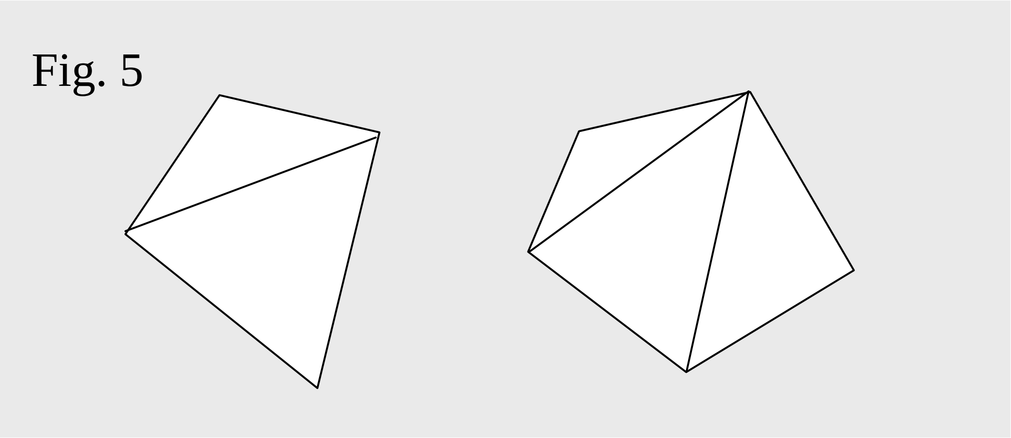
So putting together some triangles, we can make other geometric objects called polyhedrons.

And we can do the opposite, too. A triangle can be partitioned into many triangles, as many as we want.

A triangle is the simplest and the most basic polygon, and we can say that every polygon is made of triangles.

It’s because we can divide any polygon into triangles the way below.

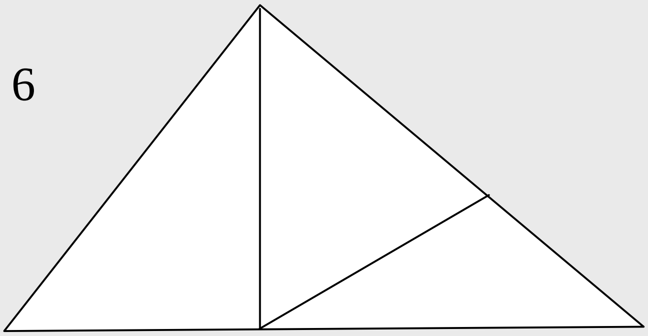
Fig. 5



No matter what the polygon may be, it can be made of triangles, and can be partitioned into triangles, too.

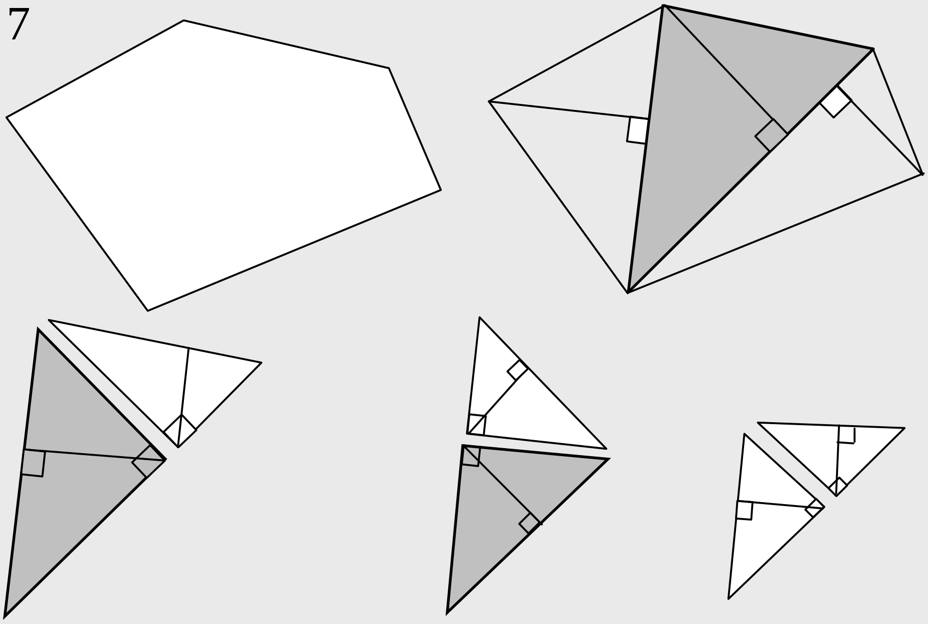
And of course, we can divide a triangle into triangles, also.

Fig. 6



So a triangle can be said to be made of many other triangles. And it can be divided into many right triangles, too. Every polygon can be therefore, said to be made of right triangles.

Fig. 7

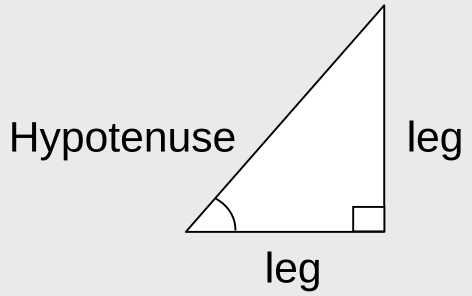


So right triangles are fundamental triangles, and thus, are important. And they are very much so. It is often the case we can't do much without right triangles solving problems.

And we can do a lot using right triangles.
Right triangles do much not only in geometry but in algebra, too. Using right triangles right, we do problems right. ☺

And thus, we want to know them very well, and use them very well, too.

Fig. 8



In a right triangle, a side is called its hypotenuse, and the other two are called its legs.

And the two legs are perpendicular to each other, so the two make 90^o. What then about the hypotenuse?

The hypotenuse is like a diagonal in a rectangle, and is facing the angle 90^o, and thus, is opposite of the right angle.

Fig. 9

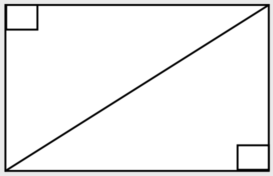
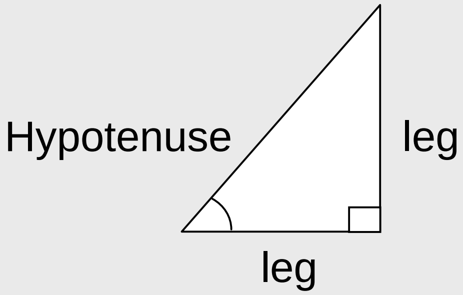


Fig. 10



And the sum of the two angles adjacent to the hypotenuse is 90^o, because the sum of all the three angles in a triangle is 180^o. Why 180^o, though?

We'll cover the proof in the next lesson.

And the next is this:
A right triangle can remind us of a famous math tool, called the Pythagorean Theorem, a.k.a. Distance Formula.

The theorem says, the square of the hypotenuse equals the sum of the two squares of the two legs.

Fig. 11

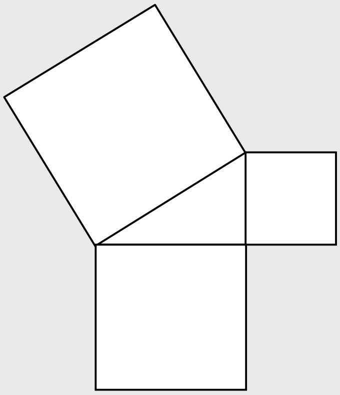
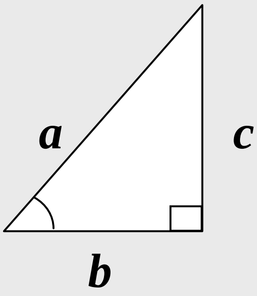


Fig. 12



So we have this: $a^2 = b^2 + c^2$.

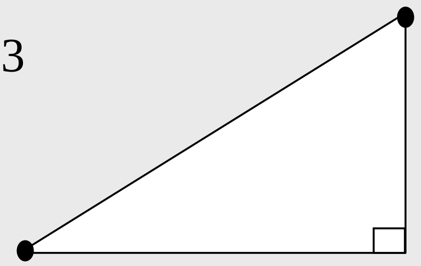
And what it says is the area of the largest square equals the sum of the two areas of the other two squares.

Why then is it, the Distance Formula?

Using it, we can find the distance between two points.

We can find the distance using the formula taking the two points as the endpoints of the hypotenuse in a right triangle.

Fig. 13

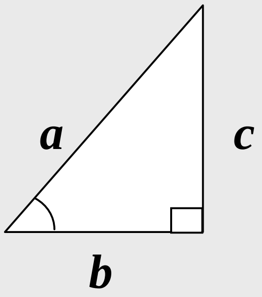


In a geometry said to be analytic, we can put the triangle above in a plane, called a coordinate plane, called specifically, Cartesian coordinate plane. And taking the differences between the coordinates, we can use the theorem.

For instance, in the x-y plane, taking as one leg the difference between x-coordinates, taking as the other leg the difference between the x-coordinates, and using the equation $a^2 = b^2 + c^2$ where b and c are the two legs, we can get a, which is the hypotenuse, that is, the distance.

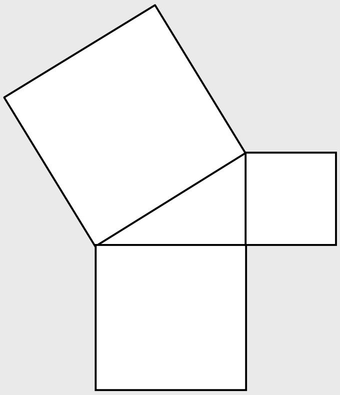
And there are tons of different versions of the proof that can show that the equation below is true for the figure below.

Fig. 12



$$a^2 = b^2 + c^2$$

Fig. 11



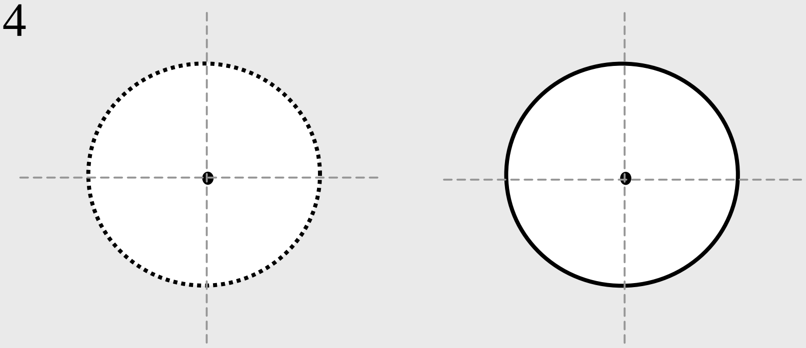
So it’s a well known theorem, and is a very useful math tool we can use doing many real world problems in real world, as well as doing test problems in school world.

For instance, as introduced above, we can find the distance between two points using the theorem. And since it can be used for such a distance, we can use it defining a circle, too.

A circle? What is it in math?

A circle is in a plane and is a collection of all those points that are the same distance called the radius away from a particular point called the center.

Fig. 14



So a radius and a center determine a circle. And knowing the radius and the center, you can recognize the circle, that is, you know what particular circle the circle is.

So if defining a circle, that is, if defining a particular circle, we specify the radius and the center, together with the name.

Also, when defining a circle, we can produce its equation using Pythagorean Theorem, a.k.a. the distance formula.

For example, we can define a particular circle using an equation this way: $(x - a)^2 + (y - b)^2 = r^2$.

And defining a circle that way, we can show that (a, b) is the center and r is the radius.

Producing the equation above, we define a circle in the x-y plane, which is a coordinate plane. Using such an equation and its graph, we work with a circle in analytic geometry, the geometry often used in a branch of math called calculus.

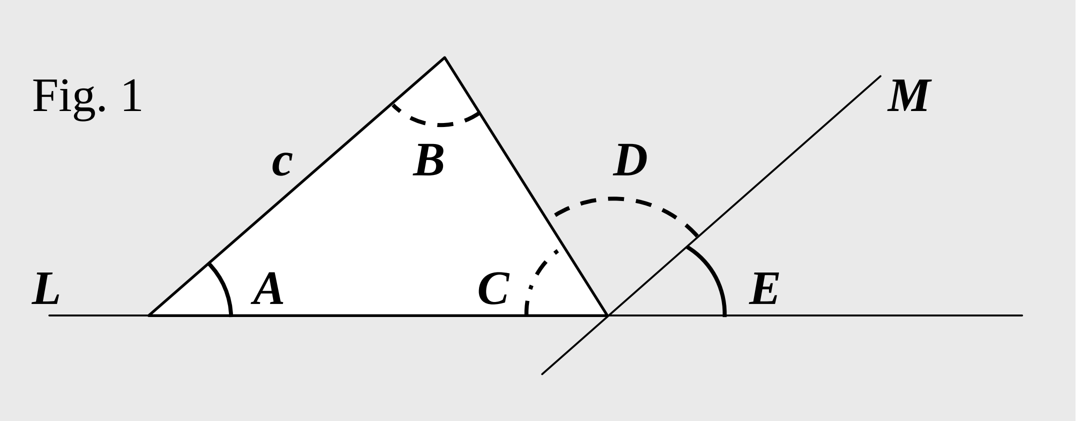
When defining a circle in math, we produce a particular circle specifying its center and radius.

And in the next lesson, we'll cover the proof that shows how the sum of the three angles in a triangle is 180°.

Triangle Basics 6

Let's see now, how the sum of the three angles in a triangle is 180^o. First, put a triangle on a line ***L*** as shown below.

Fig. 1



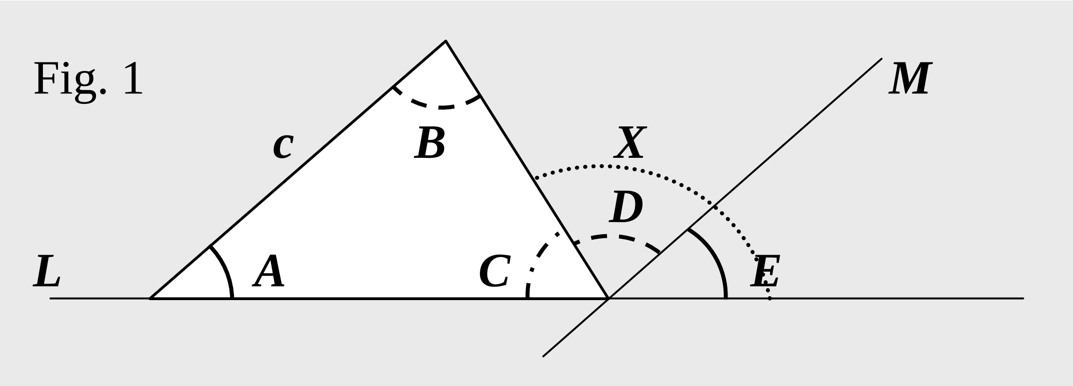
Suppose now, the side ***c*** is parallel to the line ***M*** shown in the figure above.

Then, the line ***L*** is the transversal, and we can say that the two angles $\angle A$ and $\angle E$ are corresponding angles that are equal, and the two angles $\angle B$ and $\angle D$ are alternate angles that are equal. That is, we get $\angle A = \angle E$, and $\angle B = \angle D$.

(Note: if not sure of the transversal and corresponding and alternate angles, refer to the lessons, ***Angles and Lines***.)

So next, what can we get?

So we can get this: $\angle A + \angle B + \angle C = \angle E + \angle D + \angle C$, which is a straight angle, which is 180°.



Thus, in a triangle, the sum of all the three (internal) angles is 180°. And it's worth to note that in a triangle, the sum of ***two internal*** angles equals ***an external*** angle, which is said to be ***supplementary to the other*** internal angle. Why?

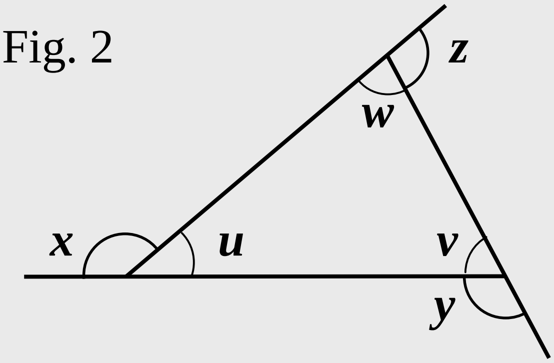
First, we have this: $\angle A + \angle B + \angle C = \angle E + \angle D + \angle C$, which is 180°, so we get this: $\angle A + \angle B = \angle E + \angle D$, which is the sum of two internal angles.

So next, assuming this: $\angle X = \angle E + \angle D$, we can say that $\angle X$ is the sum of two internal angles, and is said to be supplementary to $\angle C$, which is the other internal angle.

What then about the sum of all the three external angles?

We know if two line segments are in a line, the angle between the two is 180^o. So in the figure below, we get this: $\boldsymbol{x} + \boldsymbol{u} = \boldsymbol{y} + \boldsymbol{v} = \boldsymbol{z} + \boldsymbol{w} = 180^o$, where ***x, y***, and ***z*** are external angles, whereas ***u, v***, and ***w*** are internal angles

Fig. 2



Thus, we get this: $(\boldsymbol{x} + \boldsymbol{u}) + (\boldsymbol{y} + \boldsymbol{v}) + (\boldsymbol{z} + \boldsymbol{w}) = 3 \text{ x} 180^o$.

That is, we get this: $(\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z}) + (\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w}) = 3 \text{ x} 180^o$.

So we get this: $(\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z}) = 3 \text{ x} 180^o - (\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w})$.

And we know this: $\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w} = 180^o$.

So we get this: $\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} = 3 \text{ x } 180^o - 180^o = 360^o$.

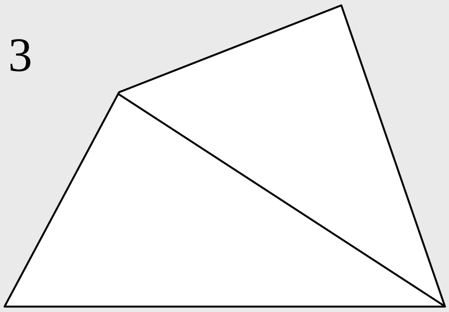
And thus, the sum of all the three external angles is 360^o.

What then is the sum of all the internal angles in a tetragon?

It is 360^o, which is twice 180^o.   Why, though?

A tetragon can be made of two triangles.

Fig. 3



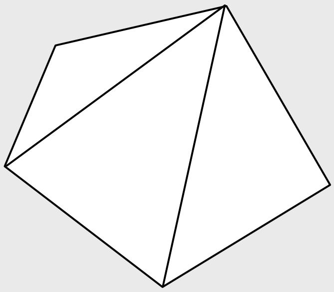
So the sum of the four angles is twice the sum of the three angles in a triangle.

Note that just saying angles in a polygon, we mean internal angles. So just saying all the angles in a tetragon, we mean all the four internal angles in a tetragon.

What then about a pentagon?

We can partition a pentagon into three triangles. So we can say that a pentagon comprises three triangles.

Fig. 4

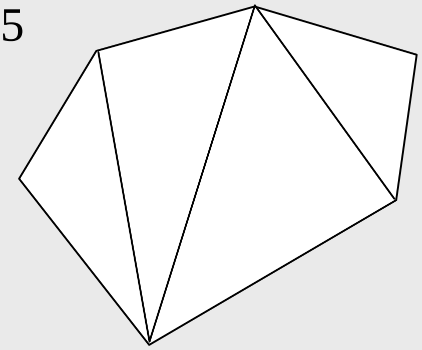


A pentagon has five angles, and is composed of three triangles, so the sum of all the five angles in a pentagon is three times the sum of the three angles in a triangle. Thus, the sum of all the five is three times 180^o.

What then about a hexagon?

We can say that a hexagon is composed of four triangles.

Fig. 5



A hexagon has six angles, and comprises four triangles, so the sum of all the six angles is four times the sum of the three angles in a triangle, and thus, it’s four times 180^o.

What then is the number of triangles a polygon is made of?

A tetragon has two triangles, a pentagon has three, and a hexagon has four. So what is the number of triangles a polygon is made of?   What is that number?

That number is two less than the number of the sides the polygon has. And the sum of all the angles in a polygon is that number times 180^o.

For instance, the sum for an octagon is six times 180^o, since an octagon is eight-sided, and the sum for a decagon is this: 8 x 180^o. A decagon is ten-sided.

So is it important to keep that in mind?

Not really. That’s just an example that shows how basics on triangles can be used. Understanding and getting used to those basics, we can grow our insight into other math ideas or objects. And the same is true of basics on other math objects or ideas, too, as numbers and arithmetic operations.

Securing the basics, we can do a lot.

Triangles are the basics in geometry, and are often used in many areas of industry, as well as in math ed.

Let’s now take another example.

Putting together two triangles, we can make a parallelogram, which is the next basic shape in geometry, and is often used, too.
So what is a parallelogram?

We can make a definition for parallelograms the way as follows.

If in a tetragon, each pair of opposite sides are parallel, the tetragon is a parallelogram, and vice versa. So if a tetragon is a parallelogram, each pair of opposite sides are parallel.

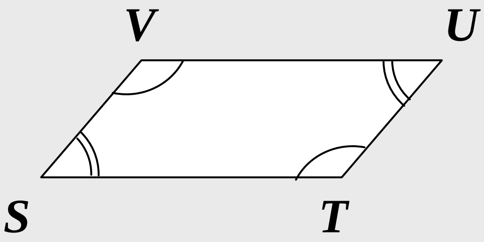


Fig. 6

In short, a tetragon is a parallelogram ***iff*** each pair of opposite sides are parallel.

Note that ***iff*** is short for ***if and only if***.

What then about their lengths?

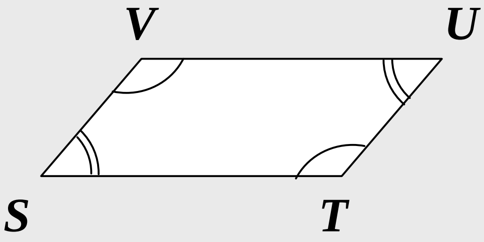


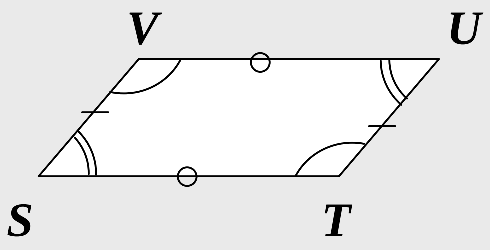
Fig. 6

Just about the same is true of their lengths, too.

So we can say that if in a tetragon, each pair of opposite sides have the same lengths, the tetragon is a parallelogram, and vice versa. So if a tetragon is a parallelogram, each pair of opposite sides have the same lengths.

Thus, in short, a tetragon is a parallelogram iff each pair of opposite sides are of the same length.

Fig. 7



And of course, we can call the statement above a definition for parallelograms, too.

So you can notice that in a tetragon, if each pair of opposite sides have the same lengths, the sides in each pair have to be parallel, and vice versa. So in a tetragon, if each pair of opposite sides are parallel, the sides in each pair have the same lengths. What then is the tetragon?

It is a parallelogram, of course. However, we don't make a definition for parallelograms that way.

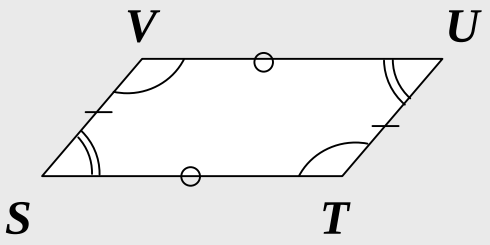
The two definitions stated earlier are not the only ones, though. We can make the definition other ways, too. And we can make two more versions the way as follows.

One is this:

If in a tetragon, each pair of angles facing each other are the same, the tetragon is a parallelogram, and vice versa. So if a tetragon is a parallelogram, each pair of angles facing each other are the same. Thus, in short:

> A tetragon is a parallelogram iff each pair of angles facing each other are equal.

Fig. 7



And the other version is as follows.

If in a tetragon, every pair of angles next to each other add up to 180^o, the tetragon is a parallelogram, and vice versa. So if a tetragon is a parallelogram, every pair of angles next to each other add up to 180^o. Thus, in short:

> A tetragon is a parallelogram iff every pair of angles next to each other add up to 180^o.

And the conditions in the definitions can be called properties of a parallelogram. So the properties are as follows.

Each pair of opposite sides are parallel.

Each pair of opposite sides have the same lengths.

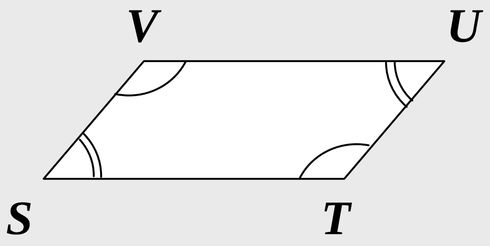
Each pair of angles facing each other are the same.

And every pair of angles next to each other add up to 180^o.

So in Fig. 6 below, we get these: $\angle \boldsymbol{S} = \angle \boldsymbol{U}$ and $\angle \boldsymbol{V} = \angle \boldsymbol{T}$, and also, this:

$\angle \boldsymbol{S} + \angle \boldsymbol{T} = \angle \boldsymbol{T} + \angle \boldsymbol{U} = \angle \boldsymbol{U} + \angle \boldsymbol{V} = \angle \boldsymbol{V} + \angle \boldsymbol{S} = 180^{o}$.

Fig. 6



How?

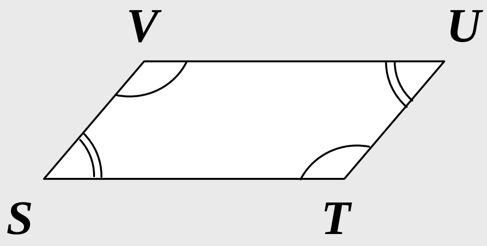
In this lesson and the next, we are going to prove it two ways.

Also, in the next lesson, we'll show that in a parallelogram, each pair of angles facing each other are equal.

And now, one of the two ways stated above is as follows.

In a parallelogram, two angles facing each other are the same. So if the quadrangle ***STUV*** is a parallelogram, we get these: $\angle V = \angle T$, and $\angle U = \angle S$ … (1)

Fig. 6



And we know the sum of all the four angles in a quadrangle is 360^{o}, so we get this: $\angle S + \angle T + \angle U + \angle V = 360^{o}$ … (2)

Now, applying (1) to (2), we get the two as follows.

$$\angle S + \angle T + \angle U + \angle V = \angle S + \angle T + \angle S + \angle T = 360^{o}.$$

$$\angle S + \angle T + \angle U + \angle V = \angle U + \angle V + \angle U + \angle V = 360^{o}.$$

So we get the two as follows.

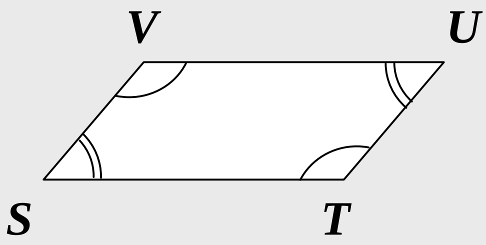
$$2\angle S + 2\angle T = 2(\angle S + \angle T) = 360^{o} \Rightarrow \angle S + \angle T = 180^{o}.$$

$$2\angle U + 2\angle V = 2(\angle U + \angle V) = 360^{o} \Rightarrow \angle U + \angle V = 180^{o}.$$

In short, we get this: $\angle S + \angle T = \angle U + \angle V = 180^{o}$.

Now, back to this: $\angle V = \angle T$, and $\angle U = \angle S$ … (1)

Fig. 6



And we now have this:

$\angle S + \angle T = \angle U + \angle V = 180^{o}$ … (3)

So applying (1) to (3), we get this:

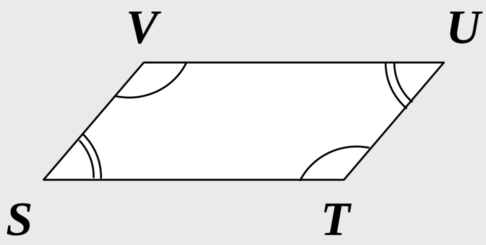
$\angle S + \angle V = \angle U + \angle T = 180^{o}$, which means,

$\angle V + \angle S = \angle T + \angle U = 180^{o}$ … (4)

And thus, putting (3) and (4) together, we get this:

$\angle S + \angle T = \angle T + \angle U = \angle U + \angle V = \angle V + \angle S = 180^{o}$.

Fig. 6



Triangle Basics 7

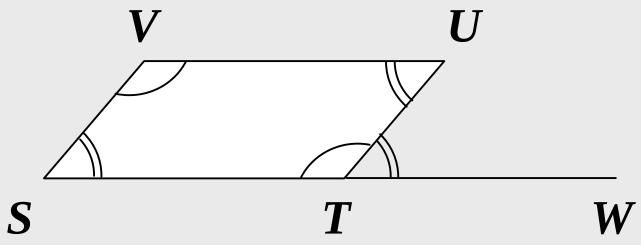
We are now going to show that in a parallelogram, each pair of angles facing each other are equal.

We can get the proof using the basics as follows.

- Corresponding angles with parallel lines are equal.
- Alternate angles with parallel lines are equal.

Now, first off, extend the side ***ST*** the way as follows.

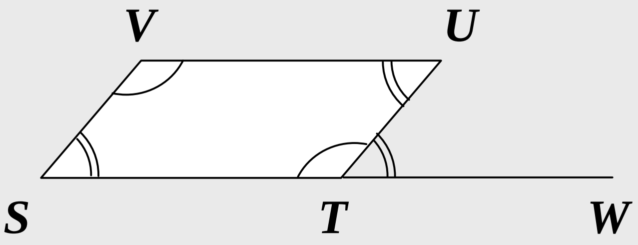
Fig. 1



Then, using the first in the basics above, we can say that since ***VS*** is parallel to ***UT***, that is, we have this: ***VS*** // ***UT***, if taking ***ST*** as the transversal, we can say that $\angle S$ and $\angle UTW$ are the same corresponding angles, that is, we get this: $\angle S = \angle UTW$.

Next, using the basics on alternate angles, we can get this, too: $\angle U = \angle UTW$.

Fig. 1



- Corresponding angles with parallel lines are equal.
- Alternate angles with parallel lines are equal.

So now, using the second in the basics above, we can say that since UV is parallel to ST, that is, we have this: $UV \mathbin{//} ST$, if taking UT as the transversal, we can say that $\angle U$ and $\angle UTW$ are the same alternate angles, that is, we get this: $\angle U = \angle UTW$. And we have this, too: $\angle S = \angle UTW$.

Thus, we get this: $\angle S = \angle U$. And by the same token, we can get this, too: $\angle V = \angle T$. As a practice, though, let’s do it now.

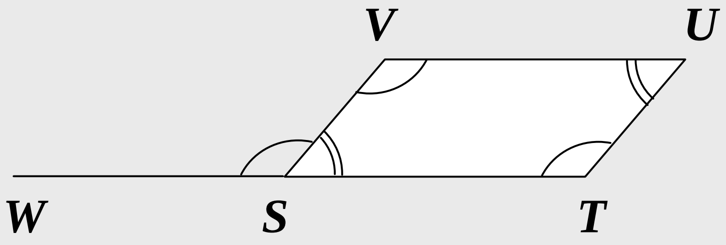
First off, we have these:

- Corresponding angles with parallel lines are equal.
- Alternate angles with parallel lines are equal.

And we are now going to begin with extending the side ***ST***.

This time, though, extend the side ***ST*** the way as follows.

Fig. 2



Then, using the first in the basics above, we can say that

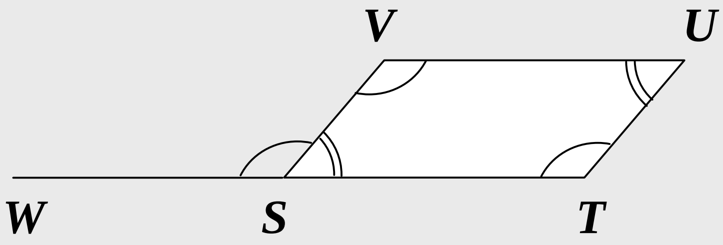
since we have this: ***VS*** // ***UT***, if taking ***ST*** as the transversal,

we can say that ∠***T*** and ∠***VSW*** are the same

corresponding angles, that is, we get this: ∠***T*** = ∠***VSW***.

Next, using the basics on alternate angles stated below, we can get this, too: $\angle V = \angle VSW$.

Fig. 2



- Corresponding angles with parallel lines are equal.
- Alternate angles with parallel lines are equal.

So now, using the second in the basics above, we can say that since we have this: UV // ST, if taking VS as the transversal, we can say that $\angle V$ and $\angle VSW$ are the same alternate angles, that is, we get this: $\angle V = \angle VSW$.

And we have this, too: $\angle T = \angle VSW$.   Thus, we get this: $\angle V = \angle T$, and we’ve got this already: $\angle S = \angle U$.

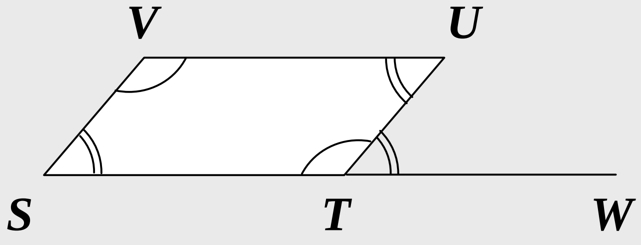
So we can now say that in a parallelogram, two angles facing each other over its center are the same.

So now, referring to the figure below, we have these:

$\angle S = \angle U = \angle UTW$, and $\angle V = \angle STU$.

And we know this: $\angle STU + \angle UTW = 180^{o}$.

Fig. 1



So we get these:

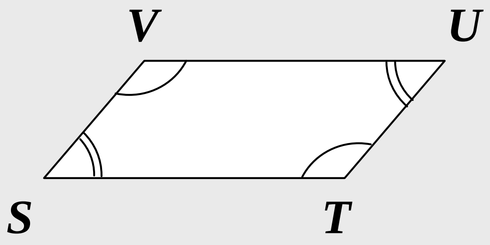
$\angle S + \angle STU = \angle UTW + \angle STU = 180^{o}$.

$\angle V + \angle U = \angle STU + \angle UTW = 180^{o}$.

Thus, we get this: $\angle S + \angle STU = \angle V + \angle U = 180^{o}$.

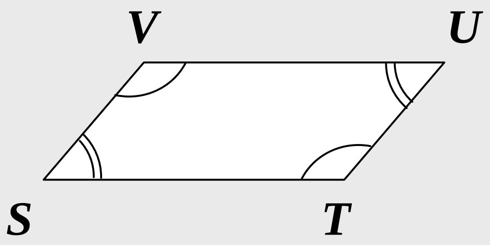
And in the figure below, we have this: $\angle T = \angle STU$.

Fig. 3



So we get this: $\angle \boldsymbol{S} + \angle \boldsymbol{T} = \angle \boldsymbol{V} + \angle \boldsymbol{U} = 180^\circ$ … (1)

Fig. 3



And we have this: $\angle \boldsymbol{V} = \angle \boldsymbol{T}$ … (2)

So applying (2) to (1), we get this, too:

$\angle \boldsymbol{S} + \angle \boldsymbol{V} = \angle \boldsymbol{T} + \angle \boldsymbol{U} = 180^\circ$ … (3)

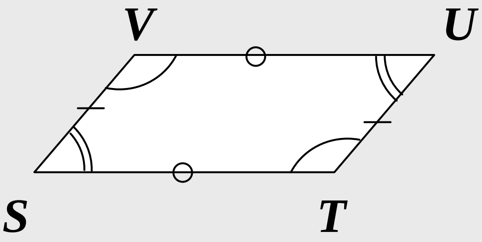
Thus now, putting (1) and (3) together, we get this:

$\angle \boldsymbol{S} + \angle \boldsymbol{T} = \angle \boldsymbol{T} + \angle \boldsymbol{U} = \angle \boldsymbol{U} + \angle \boldsymbol{V} = \angle \boldsymbol{V} + \angle \boldsymbol{S} = 180^\circ$.

So now, we can say that in a parallelogram, every pair of angles next to each other add up to 180°, which is a property of a parallelogram, and also, can be a condition for a parallelogram.

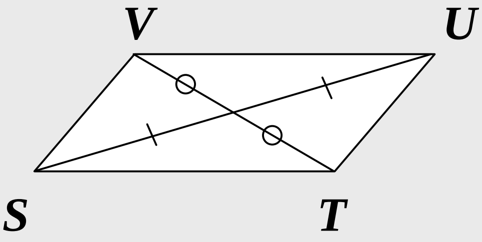
And there are other properties, too.

Fig. 4



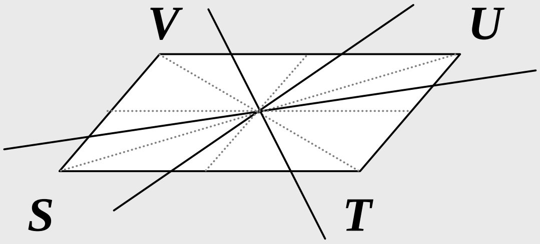
For instance, the two diagonals have the same lengths, and meet at the center which is the midpoint of each diagonal.

Fig. 5



And every line passing through the center bisects the area of a parallelogram.

Fig. 6



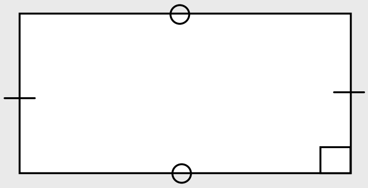
Solving thus, problems with a parallelogram, we can use the properties above.

What then about rectangles?

Among parallelograms, we have rectangles, squares, and rhombuses, called diamonds, too.

If a parallelogram has a right angle, 90^o, it's a rectangle.

Fig. 7



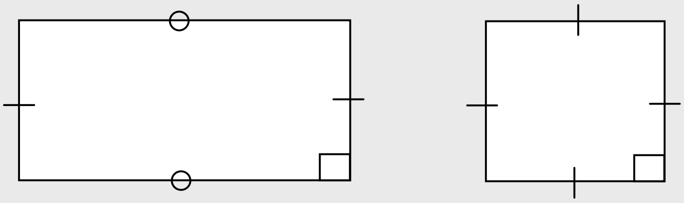
If an angle is 90^o, in a parallelogram, the other angles are right angles, too, so all the four angles are right angles.

What then, about a square?   Is it then, a rectangle, too?

Yes, it is. All squares are rectangles, because a square is a parallelogram where the angles are right angles.

Not vice versa, though. So not all rectangles are squares.

Fig. 8

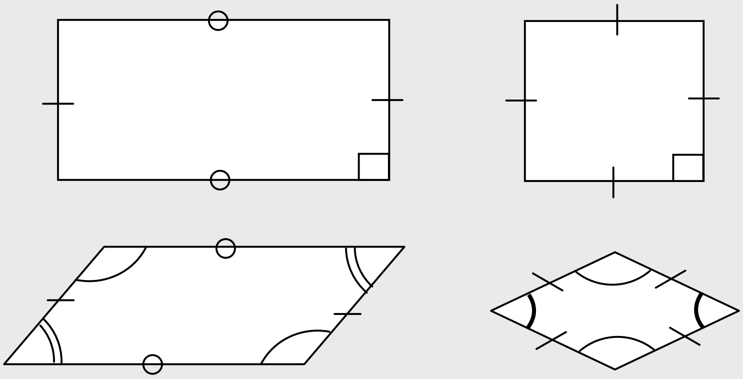


Not all parallelograms have two different pairs of equal sides. In some parallelograms, the four sides are all equal.

So if all the sides are equal, and an angle is 90°, it's a square, and can be called an equilateral rectangle, too.

And of course, there are many parallelograms where the four sides are equal but no angle is a right angle.

Fig. 9



How then do we call them? What parallelograms are they?

They are called rhombuses, and are parallelogram where the four sides are equal, and that’s it. So it doesn’t matter if they have right angles or not.

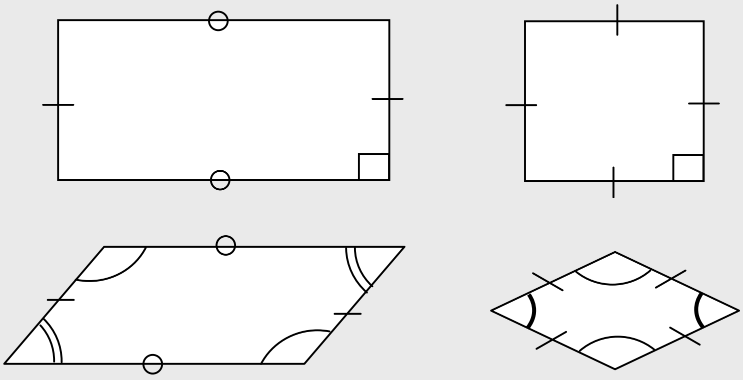


Fig. 9

Mathematicians don’t seem to love many words; they say this: A rhombus is an equilateral parallelogram. That’s it.

So, a rhombus is a parallelogram where all the sides are equal and that’s it. ☺ Is a square then, a rhombus?

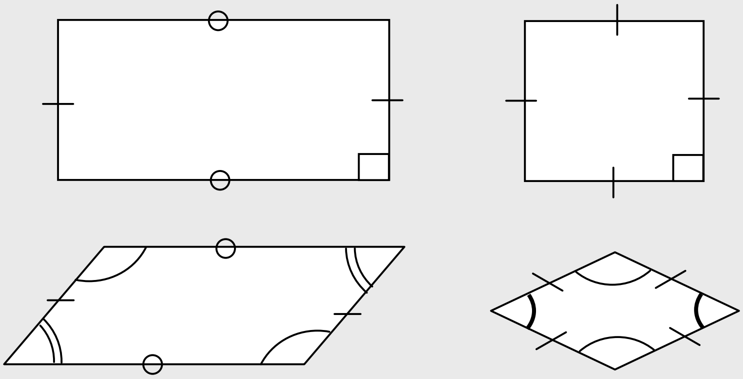
All squares are rhombuses, because all squares are equilateral rectangles, and all rectangles are parallelograms, so squares are equilateral parallelograms, too, and hence rhombuses. In short, squares are rhombuses.

Not vice versa, though. So not all rhombuses are squares. Some are.

And summing up, for now, we can put the ideas this way:

In some parallelograms, every side is equal, in the others, sides next to each other have different lengths, and some have right angles, whereas the others have no right angle.

Fig. 9

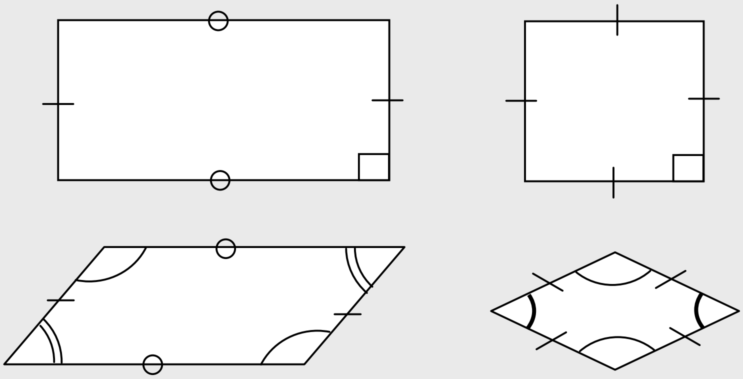


A rectangle is a parallelogram, but not vice versa, because a rectangle has a right angle, but a parallelogram doesn't have to, so it can have no right angle, and some parallelograms are not rectangles.

A square is a rectangle, but not vice versa, because a square is equilateral, all the sides are equal, but a rectangle doesn't have to be, so its four sides may not be all equal, and some rectangles are not squares.

A rhombus is a parallelogram, but not vice versa, because a rhombus is equilateral, but a parallelogram doesn’t have to be, so its four sides may not be all equal, and some parallelograms are not rhombuses.

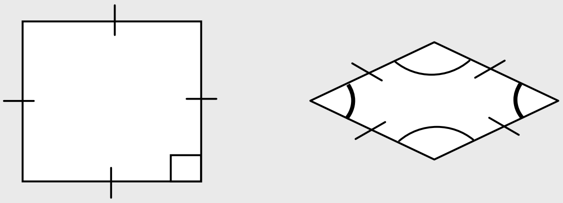
Fig. 9



And since a square is an equilateral parallelogram, it is a rhombus, too, but not vice versa, because a square has a right angle, but a rhombus doesn’t have to, so it can have no right angle, and some rhombuses are not squares.

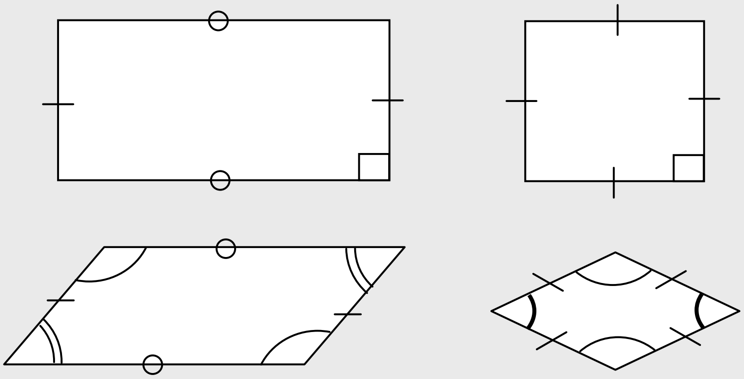
A rhombus can have right angles, so if it has them, it’s a square. So a square is a rhombus as well as a rectangle.

Fig. 10



So among tetragons called parallelograms, we have four kinds, and they are rectangles, squares, rhombuses, and just parallelograms neither rectangles nor rhombuses.

Fig. 9

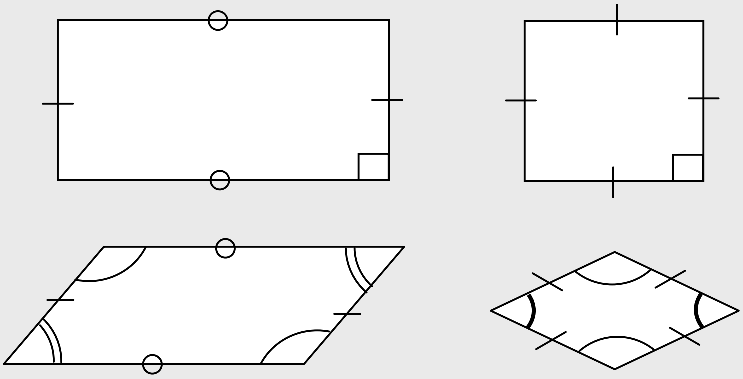


And so much for this lesson.
So we’ll continue this in the next lesson.

Triangle Basics 8

Among tetragons called parallelograms, we have four kinds, and they are rectangles, squares, rhombuses, and just parallelograms neither rectangles nor rhombuses.

Fig. 1



What if in a tetragon, only one pair of sides are parallel?

There are, of course, many tetragons where two sides are parallel, but the other two are not.

Such a tetragon is called a trapezoid. And it doesn’t matter if the other two are parallel or not. If in a tetragon, thus, two sides are parallel, it is a trapezoid.

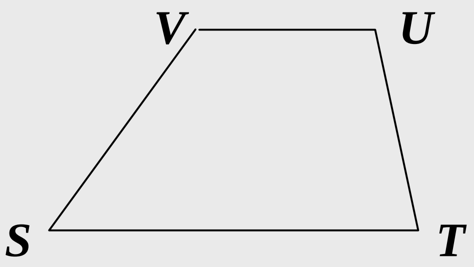


Fig. 2

So in the tetragon ***STUV*** in the figure above, if we get this:
ST // ***UV***, the tetragon is a trapezoid.

A trapezoid is a tetragon where two sides are parallel.
And it doesn’t matter if the other two are parallel or not.

Is a parallelogram then, a trapezoid, too?

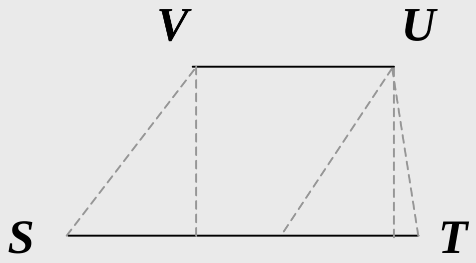
Yes, it is. So all parallelograms are trapezoids, too. It's simply because in a parallelogram, two sides are parallel anyway, since in a parallelogram, each pair of opposite sides are parallel.

So the definition sounds like this: Whatever the tetragon may be, if it has two parallel sides, it's a trapezoid.

So what the definition means is this: If a tetragon needs to be a trapezoid, it has only to have two parallel sides.

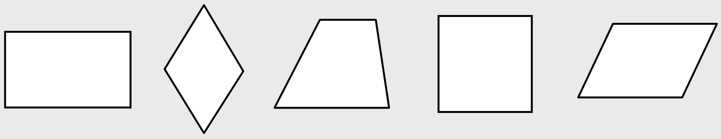
And it doesn't say anything about the other two sides.

Fig. 3



So rectangles are trapezoids, and so are squares, rhombuses, and any other parallelograms.

Fig. 4



And a parallelogram is a trapezoid, but not vice versa, because a trapezoid has only to have two parallel sides, but a parallelogram has to have two pairs of parallel sides.

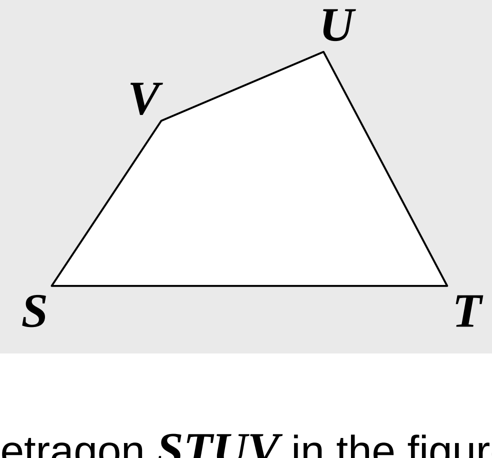
There are, of course, trapezoids that are not parallelograms.

What if in a tetragon, no sides are parallel?

There are, of course, many tetragons where no sides are parallel, and all the sides are not equal.

Such a tetragon is called a trapezium. If in a tetragon, thus, no sides are parallel, it is a trapezium. And if in a tetragon, no sides are parallel, all the four sides are not equal.

Fig. 5



So in the tetragon ***STUV*** in the figure above, if no sides are parallel, the tetragon is a trapezium.

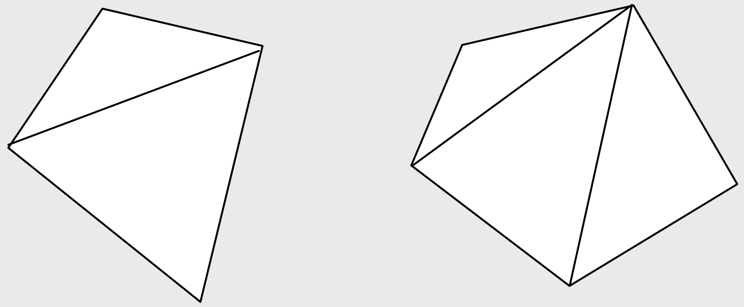
A trapezium is a tetragon where no sides are parallel.
Note that a trapezoid can have four different sides, too, but has to have a pair of sides parallel to each other.

And no matter what tetragon it may be, it can be taken as a sum of two triangles. That is to say that it can be partitioned into two triangles. Every tetragon is made of some triangles. And the same is true of any polygon, too.

And thus, in the world of mathematics, we can say this:

A polygon can be said to be made of triangles. It’s because every polygon can be partitioned into triangles.

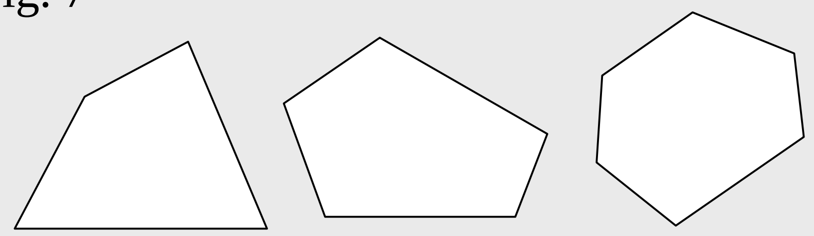
Fig. 6



And the smallest number of triangles in such partitions is two less than the number of the sides in the polygon.

Among those polygons, we have tetragons, pentagons, hexagons, and so forth.

Fig. 7



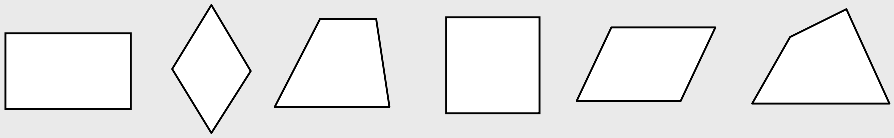
And of all polygons, the most often used are triangles and tetragons. Triangles are the most basic and most often used. And tetragons are the next.

Tetragons can be called quadrangles or quadrilaterals, too. Among tetragons, we have six kinds we assign names to.

They are not, though, separate from one another.

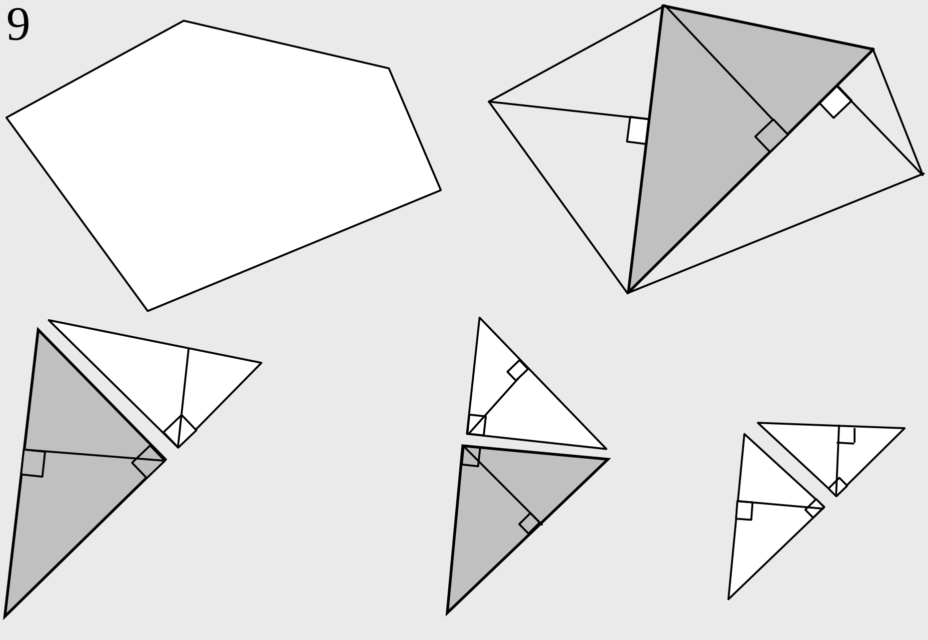
Tetragons include two kinds: trapeziums and trapezoids. And trapezoids include parallelograms, which include rhombuses and rectangles, both of which include squares, each of which is therefore, a rhombus as well as a rectangle.

Fig. 8



They can be said to be made of triangles, the most basic.

Fig. 9



So understanding and getting used to those polygons as trapezoids and parallelograms, we may want to understand and get used to the concept of the most basic ones, triangles.

What is a polygon, though?

It’s a mathematical shape or figure, and is closed in a plane. What then is a plane in math?

It’s a flat and level or even surface.

So a polygon in basic math is a ***closed*** plane figure, and is made of ***straight edges connected end-to-end***.

And those edges are often called ***sides***, and in math, we call them ***line segments***, too.

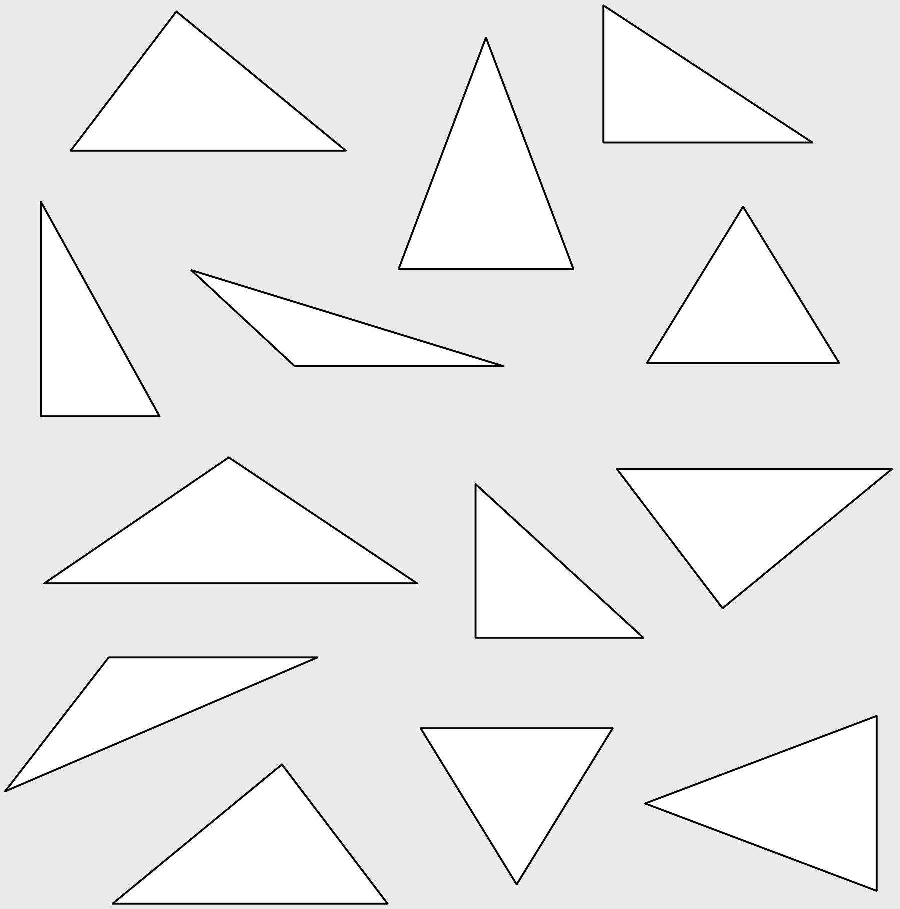
So a polygon in basic math is a closed plane figure made of line segments called sides and connected end-to-end.

In short, it’s a multi-sided closed plane figure.

So a triangle can be called a three-sided closed plane figure.

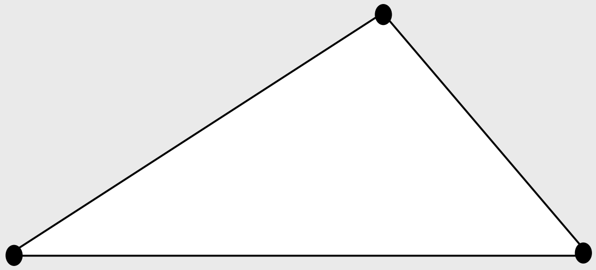
And it is the ***simplest*** polygon, and is ***most often used*** when we do math not only at school, but in many areas of work and jobs in our daily lives.

Fig. 10



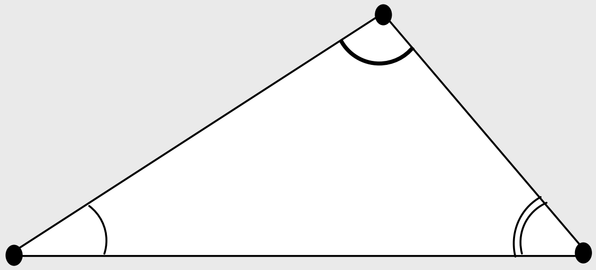
In a triangle, two sides meet at their endpoints, so they meet at a point, which is one of the endpoints each side has.

Fig. 11



And we call such a point a vertex. So a triangle has three vertices. Also, at each vertex, two sides make an angle. So a triangle has three angles, as well as three sides.

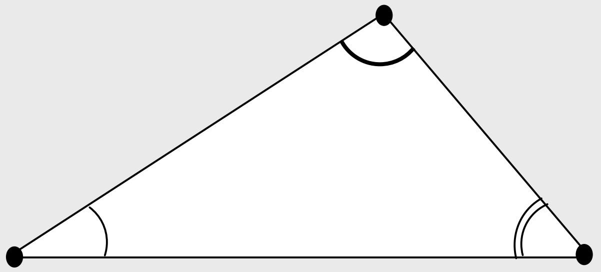
Fig. 12



Doing math, therefore, we may want to define triangles the way as follows.

A triangle is a closed plane figure made of ***three angles*** and ***three sides*** connected ***end-to-end*** at the vertices.

Fig. 12



What polygons do we usually work with doing basic math?

The polygons basic and often used are said to be simple and convex. The polygons shown below are not simple and convex.

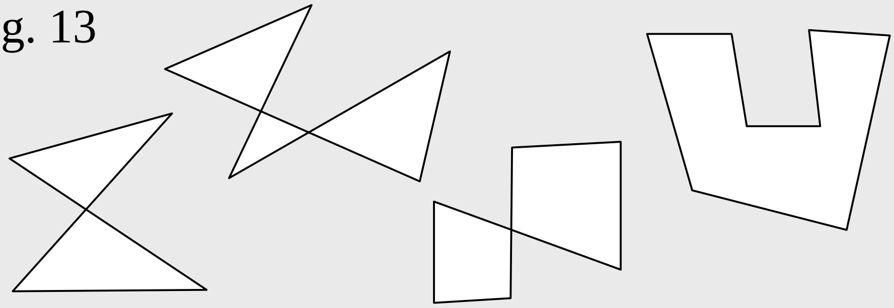


Fig. 13

And a line passing through a polygon simple and convex can cross up to two sides of the polygon, so it cannot cross three or more sides.

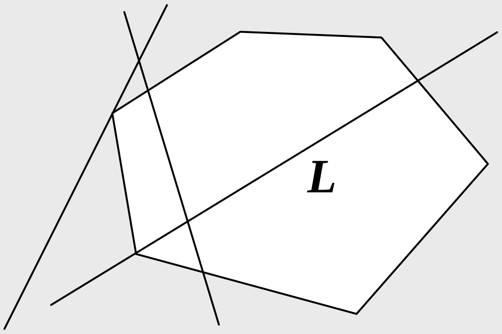


Fig. 14

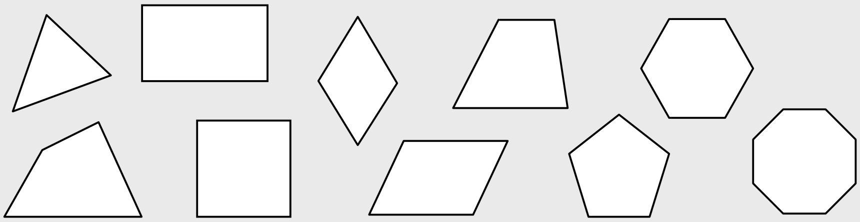
The line ***L*** passes through a point and a side only.

And among those basic polygons, the ones most often used are triangles and tetragons often called quadrangles or quadrilaterals, too, which are four sided, of course.

Other than those two kinds, we don't use a lot. So we may want to know the two very well, their basics. Once we've known the two very well, we can apply the ideas to others.

And they are pentagons (5 - sided), hexagons (6 - sided), heptagons (7 - sided), octagons (8 - sided), nonagons (9 - sided), decagons (10 - sided), hendecagons (11 - sided), dodecagons (12 - sided), etc.

Fig. 15



And all those are made of triangles. So knowing triangles very well, the basics on triangles, we could see better what we can do with many other polygons. Solving a problem in math, we break it into pieces, and put them together in some ways so that we can build the solution.