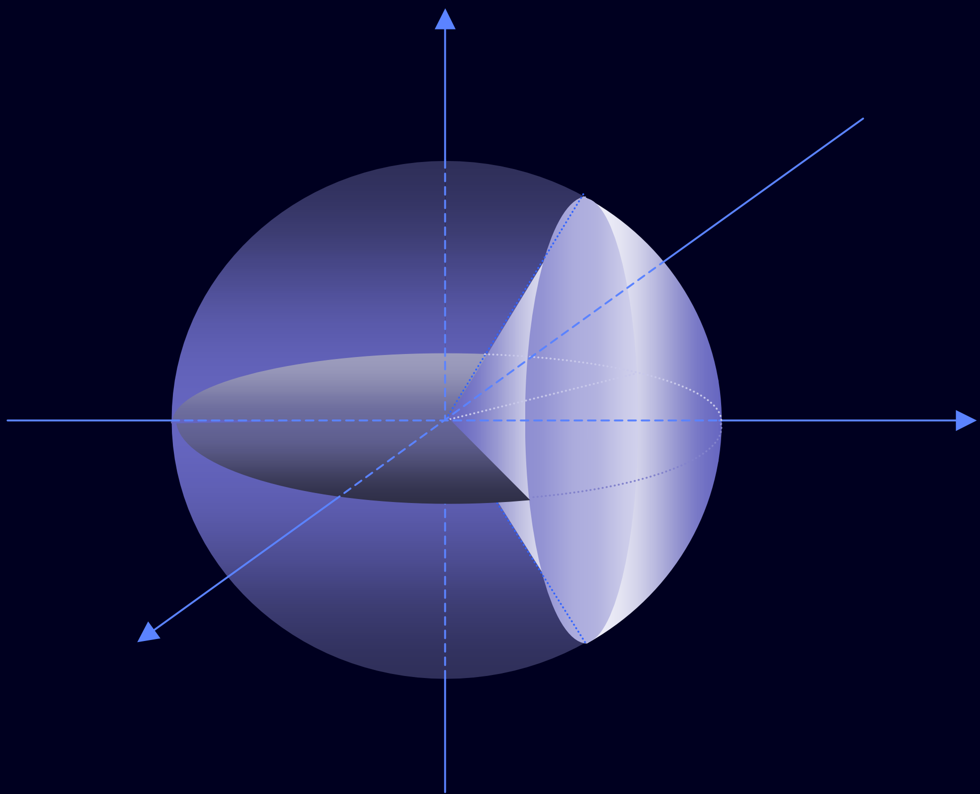
삼각함수

Trigonometry



김성렬

Seong R. Kim

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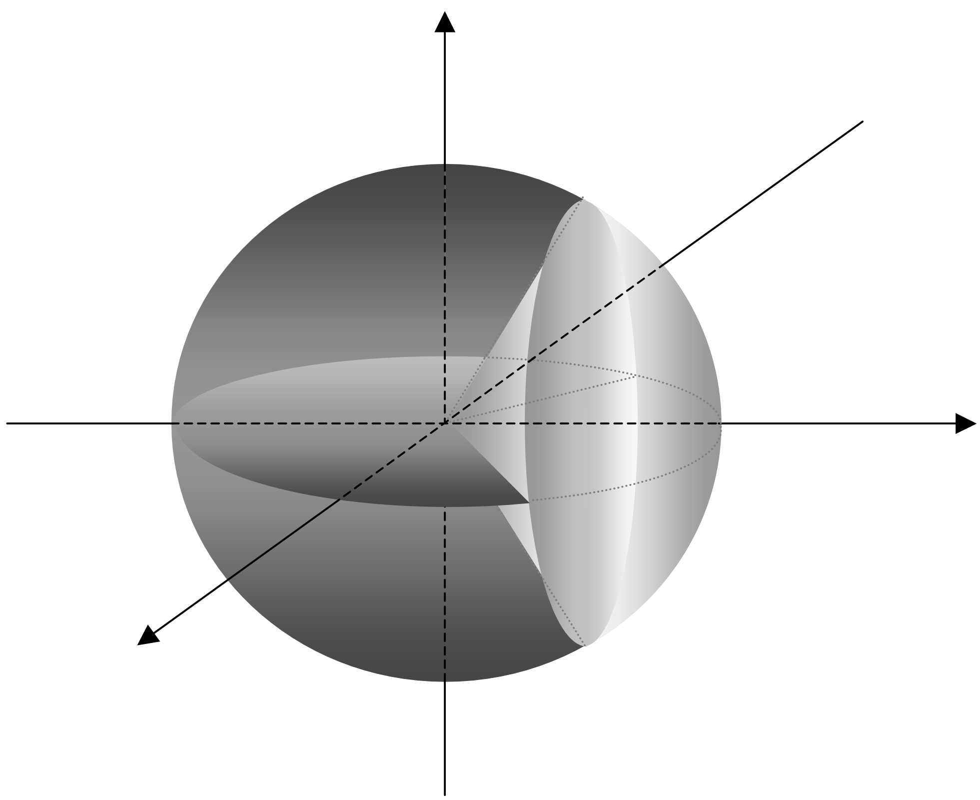


Seong R. KIM

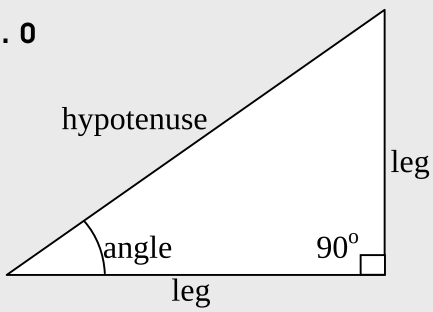
Contents

0.0. The Basics

To begin with, what is trigonometry, and what is it about?

It's about ***ratio***s. And we have three basic ratios in trigonometry. Each is between two sides in a triangle where an angle is 90^o, called a right angle. So such a triangle is called a right triangle.

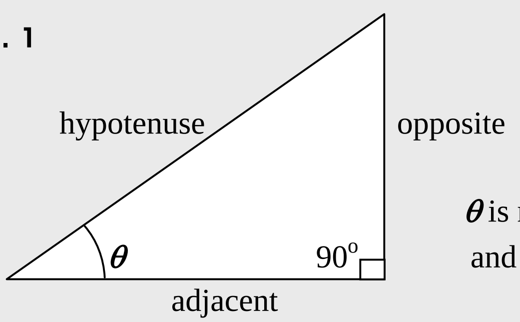
Fig. 0



In a right triangle, an angle is 90^o, called a right angle, and is between the two sides called legs. And showing a right angle, we use a small rectangle

And doing trigonometry, we name the sides in a right triangle the way as follows.

Fig. 1



θ is read as theta, and indicates an angle.

As in Fig. 1, the side slanted and connecting the two legs is called the ***hypotenuse***,

the side adjacent to the angle θ is called the ***adjacent*** for short,
so the angle θ is between the hypotenuse and the adjacent,

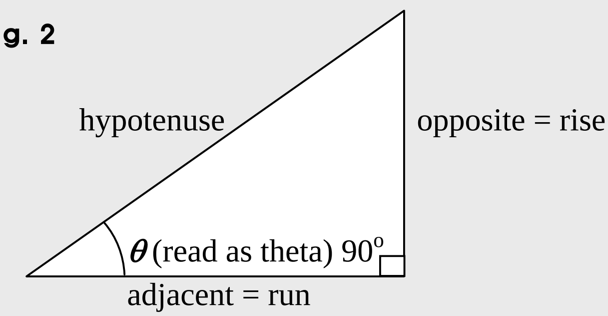
and the side opposite to the angle is just called the ***opposite***.

Thus, a right triangle is made of the hypotenuse, the adjacent, and the opposite.
What then are the three basic ratios?

They are the sine, the cosine, and the tangent.   What are they about?

To get straight to the point, the ***sine*** is about the side called the ***opposite***, and the ***cosine*** is about the side called the ***adjacent.***   What about the tangent?

Fig. 2



The ***tangent*** is the ***slope*** of the hypotenuse, is often called ***rise over run***, and is in fact, ***the sine over the cosine***, and thus, is the opposite over the adjacent.

$$\text{the tangent} = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}} = \frac{\text{the sine}}{\text{the cosine}} = \frac{\textbf{\textit{rise}}}{\textbf{\textit{run}}}$$

And if the hypotenuse is 1, the sine is the opposite, and the cosine is the adjacent.
Opposite to what and adjacent to what?

Opposite to an angle, adjacent to the angle,
and the angle is between the adjacent and the hypotenuse.

So the angle matters, since the ratios depend on the angle.

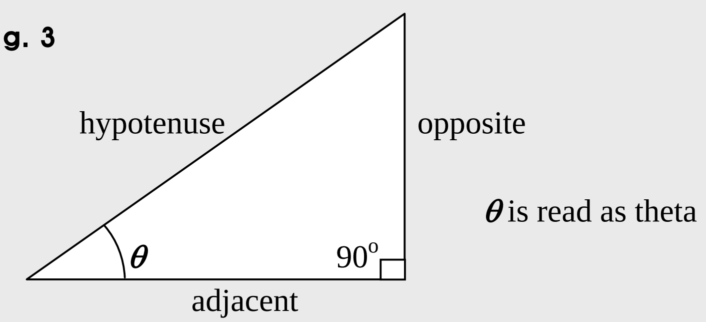
In short,
the ***sine*** is about the ***opposite***,
the ***cosine*** is about the ***adjacent***,
the ***tangent*** is the ***slope***,
and the adjacent and the hypotenuse make the angle.

In fact, if the hypotenuse is 1, the ***sine*** is just the ***opposite***, and the ***cosine*** is simply the ***adjacent***. And anyway, the ***tangent*** is the ***slope***. What slope?

The slope of the hypotenuse. Let's now cover a bit more of the detail.
So first, trigonometry is about ratios. What ratios then?

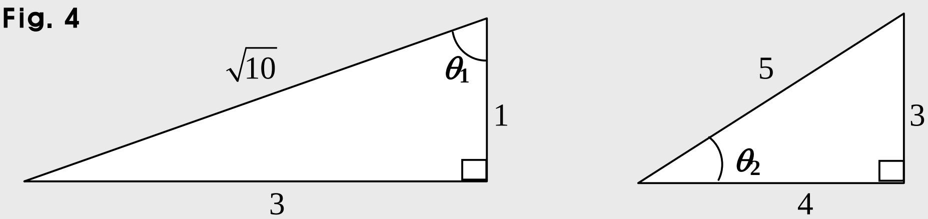
Trigonometric ratios. Each ratio is between two sides in a right triangle.

Fig. 3



So it's a *ratio* of one side to the other. In a right triangle, we can make a ratio using two from the three sides, and make three ratios. Each is called the ***sine***, the ***cosine***, or the ***tangent***. In fact, more ratios exist, but are derived ones. So basically, trigonometry is about trigonometric ratios, often called ***trig ratios***. That's not it, though. One more to it.

Fig. 4



In a right triangle, two sides make an angle, three angles are made, and one of the three matters, since trig ratios depend on that angle. Which angle is it, and how does it matter?

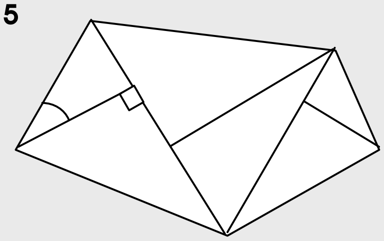
Finding a trig ratio, we need to know ***which angle***. The angle is between the two sides that we use finding the cosine. So the hypotenuse makes that angle with the adjacent. Wrong angle, wrong ratio. (By the way, in math, a right angle means 90^{o}.) If the angle changes, all ratios change. So the angle matters, and in this book, the angle is called the ***governing*** angle, since the angle governs the ratios.

Thus, trigonometry has a lot to do with angles. And we can say that the choice of the adjacent matters, too, so we need to make sure which side the adjacent is, because the governing angle depends on the choice of the adjacent, since the adjacent makes the governing angle with the hypotenuse. So right adjacent, right ratios.

(Please note that governing angle is not an official designation. I just call it that way for convenience, so it's effective in this book only.) What then is the use of those ratios?

Using trig ratios, we can find the length, force or weight, speed or velocity, etc. in structural settings involving angles or triangles. How to use the ratios then?

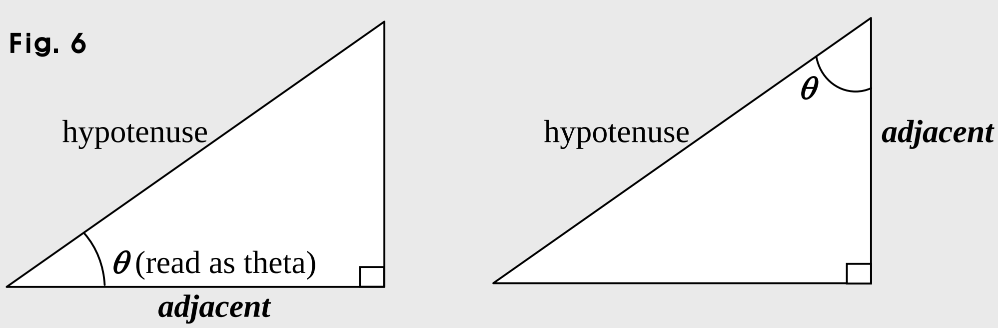
Fig. 5



In a structure made of right triangles as shown in Fig. 5 above, multiplying known components by proper trig ratios, we can get unknown components. The proper ratios depend on how each unknown component is related to its governing angle. We'll cover such relations and how the ratios work later.

Let's now briefly, cover the basics of three ratios. Their names are the sine, the cosine, and the tangent. So to begin with, what is the cosine, and what is it about?

Fig. 6



It's about the ***adjacent***. Adjacent to what?

Adjacent to the angle. What angle then?

As shown in Fig. 7 below, θ is the angle, and is called the governing angle in this book. So the governing angle is between the hypotenuse and the adjacent. That angle matters. Specifying a trig ratio, we use the governing angle, along with the name. So in the case of Fig. 7, we can say ***the cosine of*** θ, often just say cosine θ, and θ is read as theta.

For instance, if $\theta = 30^o$, we can say the cosine of 30^o, or just cosine 30^o. Usually though, it's written as **cos 30^o**, read as cosine thirty degrees.
By the way, we can express angles in two different ways. One is in degrees, and the other is in radians, covered in a later section. Note that the length of the side adjacent to the governing angle is often just called ***the adjacent***.

Fig. 7

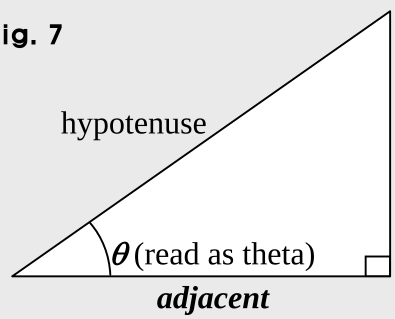
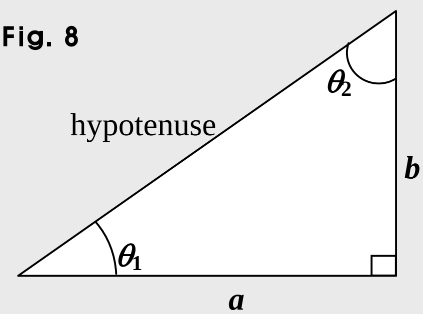


Fig. 8



In Fig. 8, if θ_2 is the governing angle, ***b*** is the side adjacent to the governing angle, and if θ_1 is the governing angle, ***a*** is the side adjacent to the governing angle. So we need to make sure where the governing angle is, or which side the adjacent is.

> Now, the ***cosine*** is the ***ratio*** between the ***two sides forming the governing angle***, one of the two is the ***adjacent***, and the other is the ***hypotenuse***.

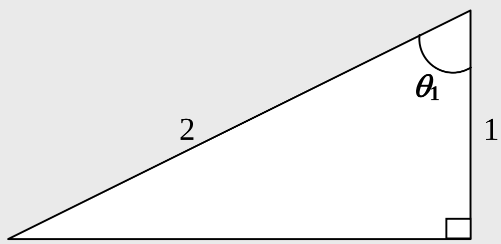
We can put the cosine in several different ways as follows.

The cosine is ***the adjacent*** *compared* to the hypotenuse. In other words, the cosine is *the ratio of* ***the adjacent*** to the hypotenuse, that is, ***the adjacent*** *over* the hypotenuse.
The same story in different words, of course.

For instance, if in Fig. 9, the cosine is $\frac{1}{2}$, the adjacent is half the hypotenuse.

Fig. 9

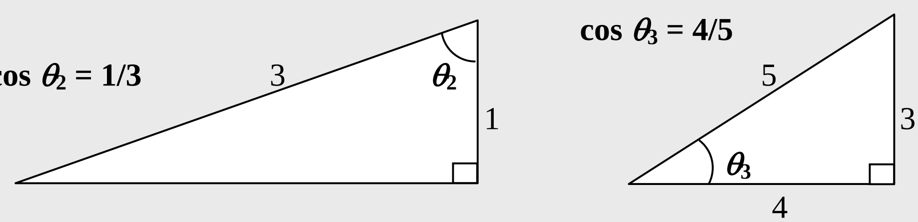
$\cos \theta_1 = 1/2$



If the cosine is $\frac{1}{3}$ as shown in Fig. 10, the hypotenuse is 3 times the adjacent.

Fig. 10

$\cos \theta_2 = 1/3$



As in Fig. 10 above, if the adjacent is 4, and the hypotenuse is 5, the cosine is $\frac{4}{5}$, so we can say that the ***cosine*** is the ***adjacent over*** the ***hypotenuse***.

$$\cos\theta = \frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}}$$

How then can we use the cosine?

Suppose in a right triangle, we know an angle between the hypotenuse and a leg, and the leg is the adjacent. Then, if multiplying the hypotenuse by the cosine of the angle, what do we get?

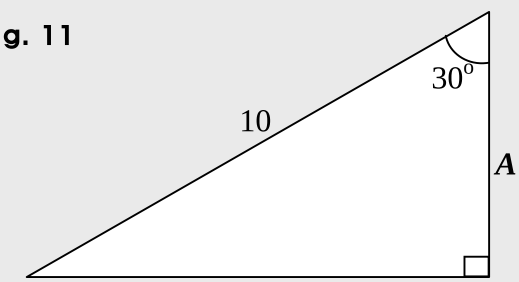
We get the adjacent.

$$\cos\theta = \frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}}, \text{ so } \textbf{\textit{hypotenuse}} \cdot \cos\theta = \textbf{\textit{hypotenuse}} \cdot \frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}} = \textbf{\textit{adjacent}}$$

Thus, doing a problem where we know the hypotenuse, but the adjacent is unknown, we can use the cosine to get the adjacent. And using the cosine in this case, we multiply the hypotenuse by the cosine to get the adjacent.

For instance, suppose in a right triangle, the hypotenuse is 10, the governing angle is 30^o, and the adjacent is ***A***, which is unknown.

Fig. 11



Multiplying the hypotenuse 10 by the cosine 30^o, we get $10 \cdot \cos 30^o = 10 \cdot \frac{\boldsymbol{A}}{10} = \boldsymbol{A}$.

And we have cos $30^o = 0.5 = \frac{1}{2}$. So we get $10 \cdot \cos 30^o = 10 \cdot \frac{1}{2} = 5$.

Thus, we get ***A* = 5**. In sum, ***A* =** $10 \cdot \cos 30^o = 10 \cdot \frac{1}{2} = 5$, where 10 is the hypotenuse, and ***A*** is the adjacent. So we can use the cosine to find the adjacent.

So what is the ***cosine*** about?

About the side adjacent to the governing angle.
The cosine is about ***the adjacent***.

Using the ***cosine***,
we multiply the hypotenuse by the cosine
to get the ***adjacent***.

Note that when the hypotenuse is 1, the cosine is actually, the adjacent itself, because the cosine is the adjacent over the hypotenuse. We'll see that it is convenient when we work with trigonometric functions. For the cosine function $\boldsymbol{y = f(x) = \cos x}$, where $\boldsymbol{x}$ is an angle, the graph shows how the adjacent changes as the angle $\boldsymbol{x}$ changes.
Let's now, move on to the sine, and talk about what the sine is about.

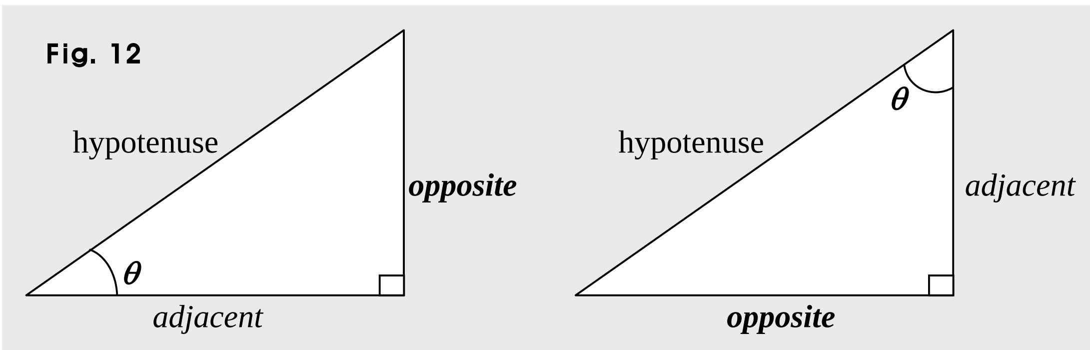


Fig. 12

The ***sine*** is about the side ***opposite***. What opposite?

Opposite to the angle. What angle then?

The governing angle. The side ***opposite to*** an angle is the side ***facing*** the angle.
As in Fig. 13 below, $\boldsymbol{\theta}$ is the governing angle, since the side labeled ***opposite*** is *facing* the angle $\boldsymbol{\theta}$.
Specifying a trig ratio, we need to use the governing angle, also. So in Fig. 11, we can say ***the sine of θ***, or just say ***sine θ***, written as **sin** $\boldsymbol{\theta}$, read as sine theta.
For instance, if $\theta = 60^o$, we say the sine of 60^o, or say sine 60^o, written as **sin** 60^o.

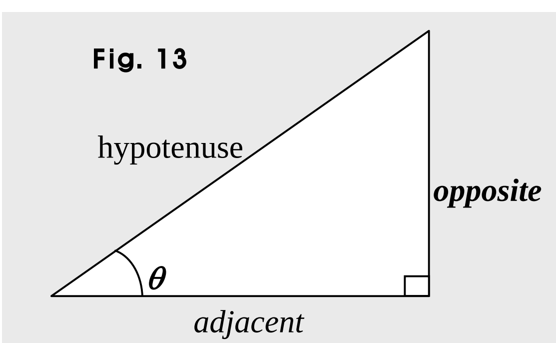


Fig. 13

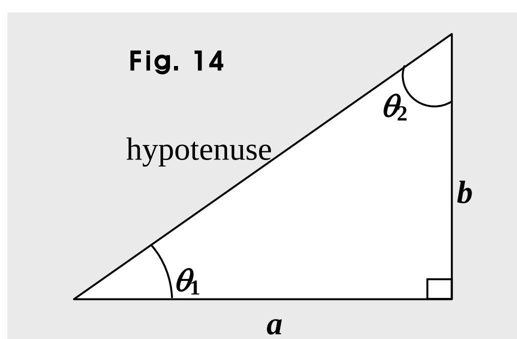


Fig. 14

In Fig. 14 above, if θ_2 is the governing angle, then $\boldsymbol{a}$ is the side ***opposite to*** the angle θ_2, and if θ_1 is the governing angle, $\boldsymbol{b}$ is the side ***opposite to*** the angle θ_1. So we need to make sure which angle the governing angle is.

The length of the side opposite to the angle is often just called ***the opposite***.

Now, the ***sine*** is the ***ratio*** between two sides,
one is the ***opposite***, and the other is the ***hypotenuse***.

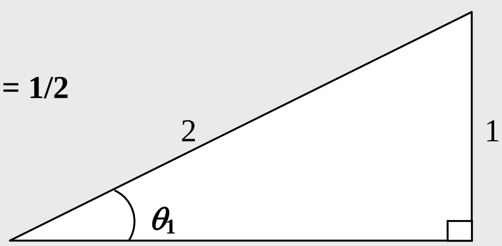
We can put the same idea of the sine in several different ways as follows.

The sine is ***the opposite*** *compared* to the hypotenuse. In other words, the sine is *the ratio of **the opposite*** to the hypotenuse, that is, ***the opposite*** *over* the hypotenuse.

For instance, as shown in Fig. 15, if the sine is 0.5, the opposite is half the hypotenuse.

Fig. 15

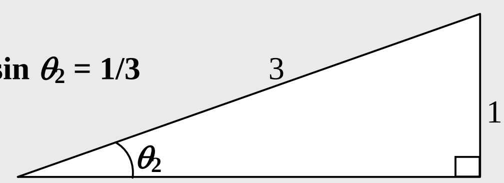
$\sin \theta_1 = 0.5 = 1/2$



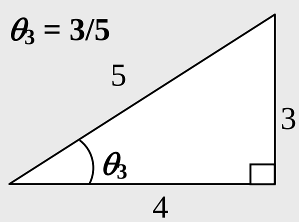
If as shown in Fig. 16, the sine is $\frac{1}{3}$, the hypotenuse is 3 times the opposite.

Fig. 16

$\sin \theta_2 = 1/3$



$\sin \theta_3 = 3/5$



As shown in Fig. 16 above, if the opposite is 3 and the hypotenuse is 5, the sine is $\frac{3}{5}$, and thus, the sine is the ***opposite over*** the ***hypotenuse***.

$$\sin\theta = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{hypotenuse}}}$$

How then can we use the sine?

Suppose in a right triangle, we know an angle opposite to a leg. Then, the leg is the opposite, so if multiplying the hypotenuse by the sine of the angle, what do we get?

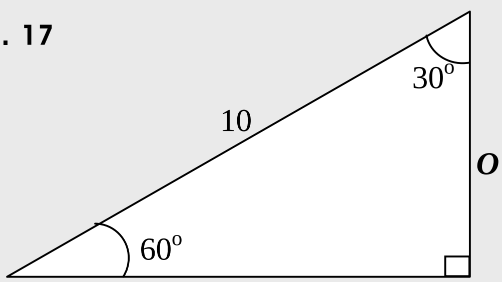
.

We get the opposite.

$$\sin\theta = \frac{\boldsymbol{opposite}}{\boldsymbol{hypotenuse}}, \text{ so } \boldsymbol{hypotenuse}\cdot\sin\theta = \boldsymbol{hypotenuse}\cdot\frac{\boldsymbol{opposite}}{\boldsymbol{hypotenuse}} = \boldsymbol{opposite}$$

For instance, suppose in a right triangle, the hypotenuse is 10, the governing angle is 60^o, and the opposite is ***O***, which is unknown.

Fig. 17



Multiplying the hypotenuse 10 by the sine, we get $10\cdot\sin 60^o = 10\cdot\frac{\boldsymbol{O}}{10} = \boldsymbol{O}$.

And we have sin 60^o = 0.5 = $\frac{1}{2}$. So we get $10\cdot\sin 60^o = 10\cdot\frac{1}{2} = 5$.

Thus, we get ***O* = 5**. In sum, we get $\boldsymbol{O} = 10\cdot\sin 60^o = 10\cdot\frac{1}{2} = 5$.

Using the sine, we multiply by the sine, the known amount, which is the hypotenuse. What then do we get by the multiplication?

We get the opposite, which is the unknown. Therefore, using the sine, we multiply the hypotenuse by the sine to get the opposite.

So what is the ***sine*** about?

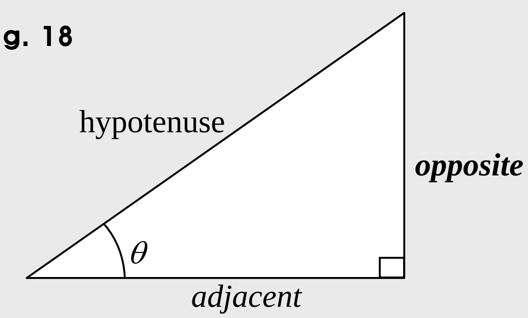
About the side opposite to the governing angle.
The sine is about ***the opposite***.

Using the ***sine***,
we multiply the hypotenuse by the sine
to get the ***opposite***.

Note that when the hypotenuse is 1, the sine is actually, the opposite itself, because the sine is the opposite over the hypotenuse. We'll see that it's convenient when talking about trig functions. For the sine function $\boldsymbol{y = f(x) = \sin x}$, where $\boldsymbol{x}$ is an angle, the graph shows how the opposite changes as the angle $\boldsymbol{x}$ changes.

Let's now move on to the tangent. What is the tangent?

Fig. 18



The ***tangent*** is a ***slope***. It is the slope of the hypotenuse.
The slop of the hypotenuse is the ratio of the opposite to the adjacent.
So the tangent is a slope, and is the ratio of the opposite to the adjacent.

The ***tangent*** is the ***ratio*** between two sides,
one is the ***opposite***, and the other is the ***adjacent***.

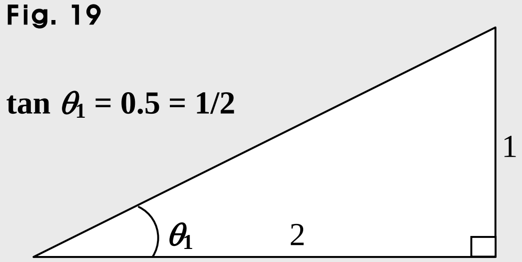
We can put the tangent in several different ways as follows.

The tangent is ***the opposite*** *compared* to ***the adjacent***. In other words, the tangent is *the ratio of* ***the opposite*** to ***the adjacent***, that is, ***the opposite*** *over* ***the adjacent***.

Exactly the same story in different words.

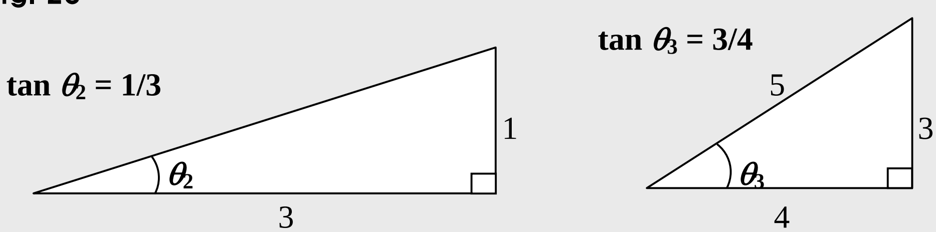
For instance, as shown in Fig. 19, if the tangent is 0.5, the opposite is half the adjacent.

Fig. 19



If the tangent is $\frac{1}{3}$ as shown in Fig. 20, the adjacent is 3 times the opposite.

Fig. 20



If as shown in Fig. 18 above, the opposite is 3 and the adjacent is 4, the tangent is $\frac{3}{4}$,

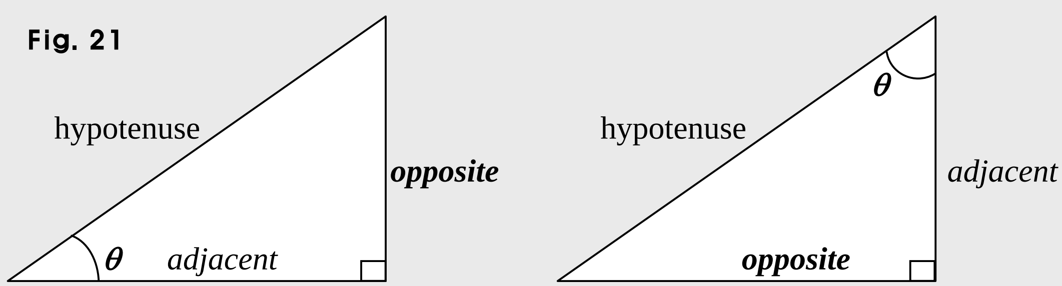
and thus, the tangent is the ***opposite over*** the ***adjacent***.

$$\tan\theta = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}}$$

How then can we use the tangent?

When using the tangent, we can also, multiply the tangent by a side in a right triangle.

Fig. 21



The side is the adjacent. When using the tangent, we want to get the opposite knowing the adjacent. The adjacent is adjacent to the governing angle.

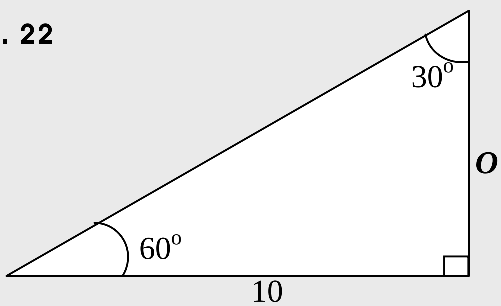
Suppose in a right triangle, we know an angle opposite to a leg. Then, the leg is the opposite, and the other leg is the adjacent, so if multiplying the adjacent by the tangent, what do we get?

We get the opposite.

$$\tan\theta = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}}, \text{ so } \ \textbf{\textit{adjacent}} \cdot \tan\theta = \textbf{\textit{adjacent}} \cdot \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}} = \textbf{\textit{opposite}}$$

For instance, suppose in a right triangle, the adjacent is 10, the governing angle is 60^{o}, and the opposite is ***O***, which is unknown. How then can we get the opposite ***O***?

Fig. 22



Multiplying the adjacent 10 by the tangent, we get $10 \cdot \tan 60^{o} = 10 \cdot \frac{\boldsymbol{O}}{10} = \boldsymbol{O}$.

And we have $\tan 60^{o} = \frac{\sqrt{3}}{3}$. So we get $10 \cdot \tan 60^{o} = 10 \cdot \frac{\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$.

Thus, we get $\boldsymbol{O} = \frac{10\sqrt{3}}{3}$. In sum, we get $\boldsymbol{O} = 10 \cdot \tan 60^{o} = 10 \cdot \frac{\sqrt{3}}{3} = \frac{10\sqrt{3}}{3}$.

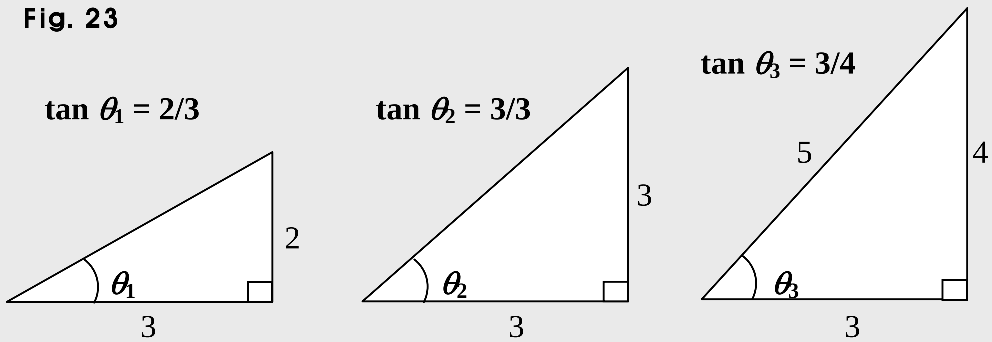
Now, we know 10 is the adjacent, and ***O*** is the opposite.
Using the tangent, we multiply by the tangent the known amount, which is the adjacent.
What then do we get using the tangent?

We get the opposite. So we can use the tangent to find the opposite.
Thus,, though the tangent is not about the opposite, we can easily get the opposite using the tangent.

Using the ***tangent***,
we multiply the adjacent by the tangent
to get the ***opposite***.

Now, if the tangent is not about the opposite, what then is it about?

Fig. 23



As shown in Fig. 23 above, the bigger the opposite, the bigger the tangent, and the hypotenuse gets steeper.
So the slope of the hypotenuse gets bigger as the tangent gets bigger. And in fact, the ***tangent*** is a ***slope***, and is about a rate of change, will be briefly covered shortly.
The tangent is the slope of the hypotenuse.
The slop of the hypotenuse is the ratio of the opposite to the adjacent.

The ratio is the opposite *compared to* the adjacent. For instance, if the opposite is 8 and the adjacent is 4, the opposite compared to the adjacent is 2, because it's $\frac{8}{4}$.

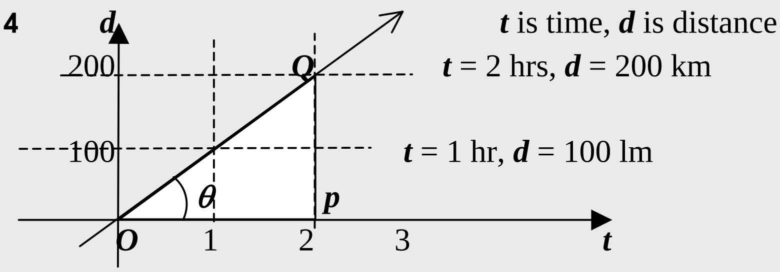
The tangent is ***a slope***.

The tangent is a slope and a slope can be a rate of change.
So the tangent can be a rate of change. If the slope is bigger, so is the tangent, and the change is faster.

A rate of change indicates how fast or slow an amount increases or decreases.

For instance, a time rate of change in distance is a speed as 100 Km/hour.

Fig. 24



The triangle ***OPQ*** is a right triangle, so ***OQ*** is the hypotenuse, and **tan *θ*** is the slope of the hypotenuse ***OQ***, and is a rate of change, which is in this case, a speed, which is a time rate of change in distance.
The longer the opposite, the steeper the hypotenuse, so the slope is bigger.

Fig. 25

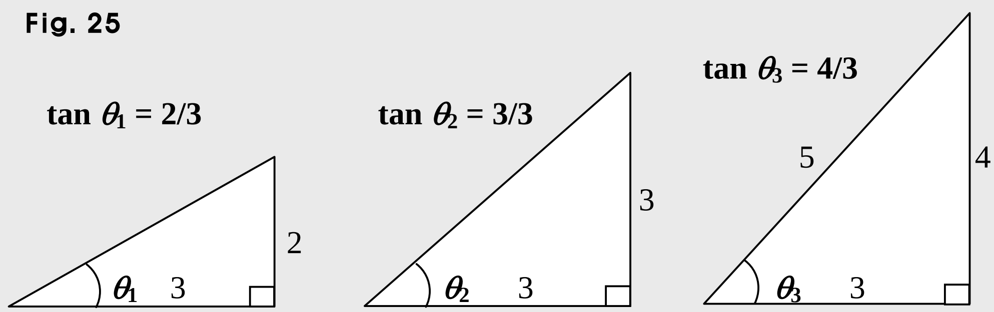
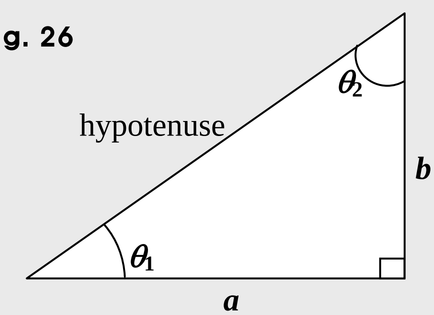


Fig. 26



So in Fig. 26, the ratio is the length of ***b*** *compared to* the length of ***a***.

In short, the ratio is ***b*** compared to ***a***, which means the ratio is $\frac{b}{a}$.

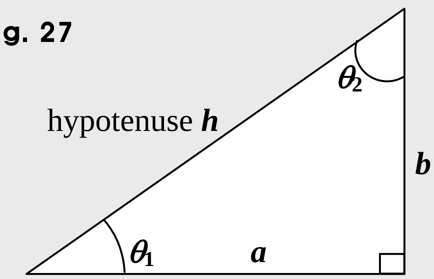
How then do we call the ratio?

It is the tangent, and is the slope, the slope of the hypotenuse, since it shows how steep the hypotenuse is. So the tangent is the slope that shows the steepness of the hypotenuse.

The ***longer*** the ***opposite***, the ***bigger*** the ***slope***, so is the tangent.
The ***longer*** the ***adjacent***, the ***smaller*** the ***slope***, so is the tangent.

Let's go over now, the idea of the tangent.
The tangent is a slope, which is a ratio.   What ratio then?

Fig. 27



It is the ratio of the opposite to the adjacent.
So ***the tangent*** is ***the opposite over the adjacent***. We often call the opposite a rise, and call the adjacent a run. Thus, ***the tangent***, that is, ***a slope*** is often called ***rise over run***.

$$\textbf{tangent = slope} = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\textbf{tangent = slope} = \frac{\textit{rise}}{\textit{run}}$$

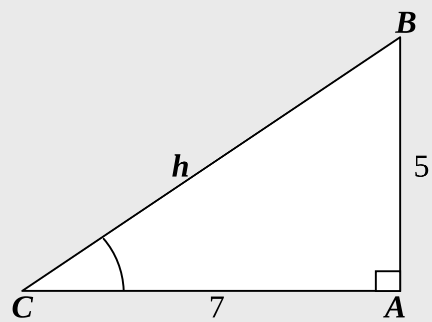
In Fig. 27, if the governing angle is θ_1, the tangent is $\frac{b}{a}$, since b is the opposite, and a is the adjacent. So the tangent of θ_1 is $\frac{b}{a}$, and we put it this way: $\tan \theta_1 = \frac{b}{a}$.

If however, θ_2 is the governing angle, the tangent is $\frac{a}{b}$, since a is the opposite, and b is the adjacent. And we put it this way: $\tan \theta_2 = \frac{a}{b}$.

For instance, if the opposite is 3 and the adjacent is 4, the tangent is $\frac{3}{4}$.

What is the tangent if the opposite is 5 and the adjacent is 7?

Fig. 28



In Fig. 28, the angel ***C*** is the governing angle, so 5 is the opposite, and 7 is the adjacent; thus, the tangent is $\frac{5}{7}$. And we can put it this way: **tan *C*** $= \frac{5}{7}$.

So what is the ***tangent***?

The ***slope*** of the hypotenuse.
The tangent is about ***a rate of change*** as speed.

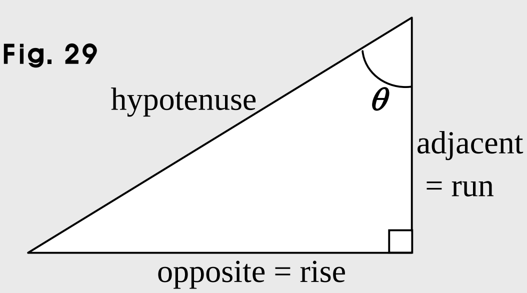
tangent = slope = $\frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}}$

tangent = slope = $\frac{\textbf{\textit{rise}}}{\textbf{\textit{run}}}$

We now have covered, briefly, the three basic trig ratios, which are the sine, the cosine, and the tangent.

The ***sine*** is about the ***opposite***,
the ***cosine*** is about the ***adjacent***,
and the ***tangent*** is a ***slope***.
More specifically,
the sine is ***opposite over hypotenuse***,
the cosine is ***adjacent over hypotenuse***,
and the tangent is a slope, ***opposite over adjacent***,
often put ***rise over run***.
And the governing angle matters. So make sure where the governing angle is.

Fig. 29

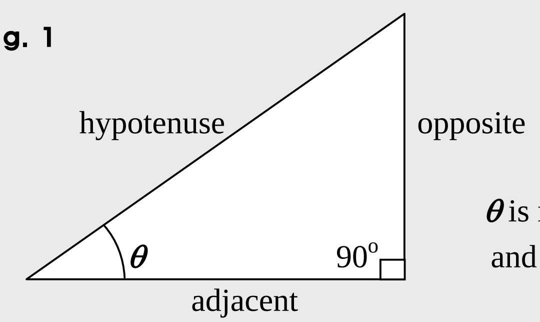


In fact, we don’t name the sides that way doing trigonometry.
Of the two legs, one is called the opposite, and the other is called the adjacent.
Why opposite?

It’s because the leg is opposite to the angle. And we just call it the opposite.
What then about the adjacent?

It’s the leg adjacent to the angle. And we just call it the adjacent.

Fig. 1



θ is read as theta,
and indicates an angle.

Thus, in trigonometry, the side called the hypotenuse connects the two legs called the opposite and the adjacent.   What then about the ratios?

In trigonometry, we work with three basic ratios, the sine, the cosine, and the tangent.
To get straight to the point, just keep in mind that

the sine is about the opposite,
the cosine is about the adjacent,
the tangent is about the slope of the hypotenuse,
and the angle matters.

We can even say that
the sine is the opposite, and the cosine is the adjacent.
And of course, the tangent is the slope.

What about the angle?

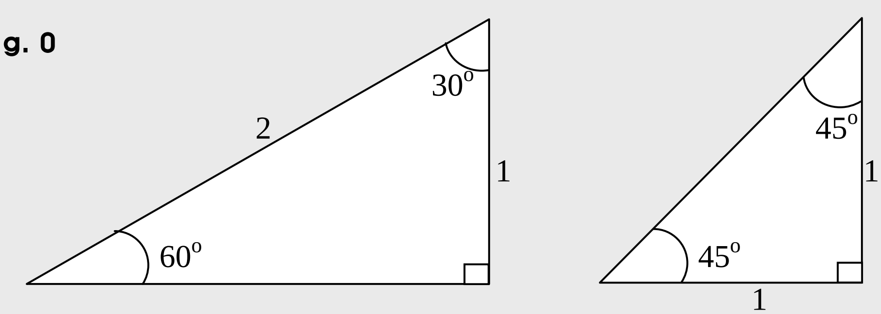
The angle is between the hypotenuse and the adjacent, and the ratios depends on it.

0.1. Review & Special Angles

There are three angles we often use when learning and working with trig ratios. The three are 30^o, 45^o, and 60^o. And we can call them special angles in trigonometry. It's probably because the angles are easy to create and express visually, and we can readily get and express the trig ratios for those angles. So we can get the ratios for the angles without looking the table of trig ratios and angles.

How then to create the angles and how to get the ratios?

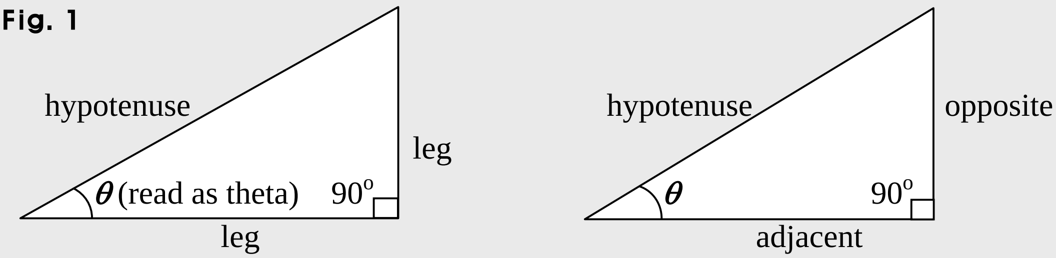
Fig. 0



To begin with, we may want to go over quickly the basic trig ratios.

A trig ratio is from a ***right triangle***. In a right triangle, an angle is 90^o, called a right angle. So a right triangle. One side is the hypotenuse, and the others are called the legs, and make a right angle, 90^o. Indicating a right angle, 90^o, we often use a small rectangle.

Fig. 1



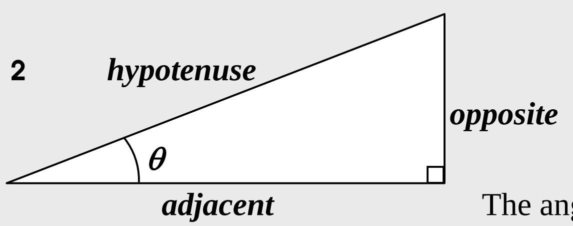
Of the two legs in a right triangle, the leg opposite to the angle θ is called ***the opposite***, and the leg adjacent to the angle θ is called ***the adjacent***. A trig ratio is made of two sides in a right triangle. And we have three basic trig ratios. What then, are the three?

The sine, the cosine, and the tangent. Getting a trig ratio, we pick two sides in a right triangle, and then, take the ratio between the two. We don't just pick two sides though. How then, do we choose the two?

Two sides make an important angle in a right triangle. And in this book, we call the angle the *governing angle*. That's because the angle governs the ratios.
What sides are those two then?

One is the adjacent, and the other is the hypotenuse. So the adjacent makes the governing angle with the hypotenuse.

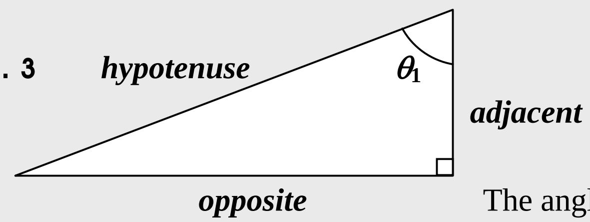
Fig. 2



The angle θ is the governing angle.

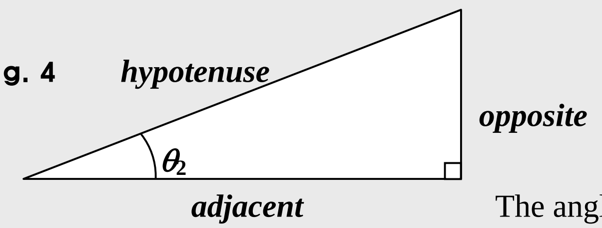
The governing angle is between the adjacent and the hypotenuse.
The adjacent is the leg next to, that is, adjacent to the governing angle.
How then do we get each ratio?

Fig. 3



The angle θ_1 is the governing angle.

Fig. 4



The angle θ_2 is the governing angle.

The sine is the ratio of the *opposite* to the hypotenuse, that is, the opposite over the hypotenuse; thus, $\sin\theta = \dfrac{\textbf{\textit{opposite}}}{\textbf{\textit{hypotenuse}}}$. And the sine is about the opposite.

The cosine is the ratio of the *adjacent* to the hypotenuse, so is the adjacent over the hypotenuse; thus, $\cos\theta = \dfrac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}}$. And the cosine is about the adjacent.

And the tangent is the ratio of the *opposite* to the *adjacent*, so is the opposite over the adjacent; thus, $\tan\theta = \dfrac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}}$. And the tangent is a slope, and is often put $\dfrac{\textbf{\textit{rise}}}{\textbf{\textit{run}}}$, where ***rise*** is a ***vertical*** amount, and ***run*** is a ***horizontal*** amount

How then do we use the ratios?

If getting ***A*** by using the ratio of ***A*** to ***B***, we multiply ***B*** by the ratio.
So if multiplying ***B*** by the ratio of ***A*** to ***B***, we can get ***A***.
Thus, ***A*** is the product of ***B*** and the ratio of ***A*** to ***B***. So?

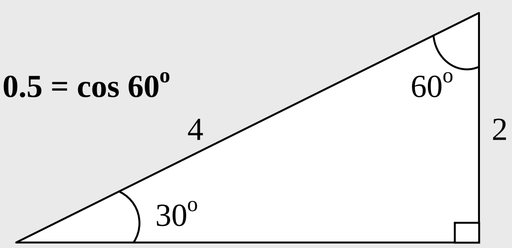
So getting the product of the hypotenuse and the sine, we get the opposite.
Getting the product of the hypotenuse and the cosine, we get the adjacent.
And getting the product of the adjacent and the tangent, we get the opposite.

Let's now take some examples.

So for instance, as shown in Fig. 5 below, in a right triangle where the governing angle is 30°, the hypotenuse is twice the opposite. Thus, the ratio of the opposite to the hypotenuse is $\frac{1}{2}$, which means, if the hypotenuse is 4, the opposite is 2.

Fig. 5

$\sin 30^{\circ} = 2/4 = 0.5 = \cos 60^{\circ}$



Thus, **$\sin 30^{\circ} = \frac{2}{4} = \frac{1}{2} = 0.5$**.

So if multiplying the hypotenuse by **$\sin 30^{\circ}$**, we get the opposite, that is,

$\textbf{\textit{hypotenuse}} \cdot \sin 30^{\circ} = 4 \cdot \frac{1}{2} = \frac{4}{2} = 2$, which is the opposite.

Next, in the right triangle above, if the governing angle is 60°, the adjacent is half the hypotenuse. So the ratio of the adjacent to the hypotenuse is $\frac{1}{2}$, also, that is, if the hypotenuse is 4, the adjacent is 2.

Thus, **$\cos 60^{\circ} = \frac{2}{4} = \frac{1}{2} = 0.5$**. So if multiplying the hypotenuse by **$\cos 60^{\circ}$**, we get the adjacent, that is, $\textbf{\textit{hypotenuse}} \cdot \cos 60^{\circ} = 4 \cdot \frac{1}{2} = \frac{4}{2} = 2$, which is the adjacent.

Notice that **$\sin 30^{\circ} = \cos 60^{\circ}$**, which is an example of a trig identity, since $30^{\circ} + 60^{\circ} = 90^{\circ}$. Thus for instance, we get **$\cos 17^{\circ} = \sin 73^{\circ}$**.

So taking the product of a trig ratio and a relevant side, we can get the side we want. If the ratio is the sine or the cosine, the relevant side is the hypotenuse. And if the ratio is the tangent, the relevant side is the adjacent. Thus:

We can get the opposite multiplying the hypotenuse by the sine, **sin** θ.

$$\textit{\textbf{hypotenuse}} \cdot \sin\theta = \textit{\textbf{hypotenuse}} \cdot \frac{\textit{\textbf{opposite}}}{\textit{\textbf{hypotenuse}}} = \textit{\textbf{opposite}}$$

We can get the adjacent multiplying the hypotenuse by the cosine, **cos** θ.

$$\textit{\textbf{hypotenuse}} \cdot \cos\theta = \textit{\textbf{hypotenuse}} \cdot \frac{\textit{\textbf{adjacent}}}{\textit{\textbf{hypotenuse}}} = \textit{\textbf{adjacent}}$$

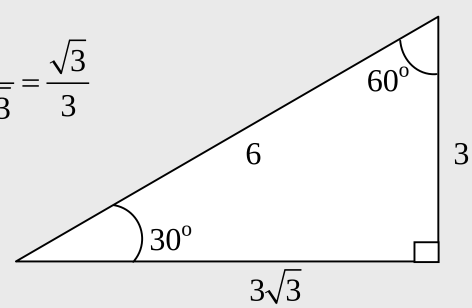
And we can get the opposite multiplying the adjacent by the tangent, tan θ.

$$\textit{\textbf{adjacent}} \cdot \tan\theta = \textit{\textbf{adjacent}} \cdot \frac{\textit{\textbf{opposite}}}{\textit{\textbf{adjacent}}} = \textit{\textbf{opposite}}$$

Let's next, move on to an example of the tangent.
In a right triangle in Fig. 6 below, if the governing angle is 30°, the opposite is 3, and the adjacent is $3\sqrt{3}$. the adjacent is $\sqrt{3}$ times the opposite. So the ratio of the opposite to the adjacent is $\frac{1}{\sqrt{3}}$, which means, if the adjacent is $3\sqrt{3}$, the opposite is 3.

Fig. 6

tan 30° = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

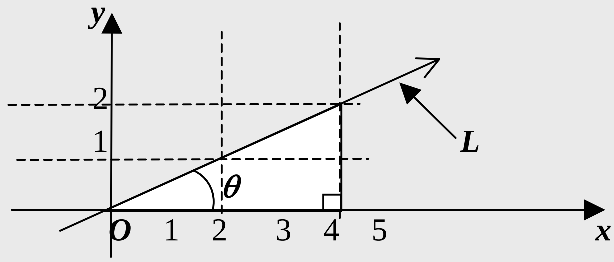


Thus, $\tan 30^{\circ} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$. So if multiplying the adjacent by **tan 30°**, we get the opposite. That is, $\textit{\textbf{adjacent}} \cdot \tan 30^{\circ} = 3\sqrt{3} \cdot \frac{1}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}} = 3$, which is the opposite.

Also, of the thee trig ratios, one has another important use, and is the tangent.
The tangent explains ***how much the hypotenuse is slanted***, and thus, is ***the slope*** of the hypotenuse in a right triangle.

So the tangent is a slope, and we use the tangent when we want to find the slope of a line.

Fig. 7



As shown in Fig. 7 above, the hypotenuse in the right triangle is a part of a line ***L***.

The equation of the line ***L*** is $\boldsymbol{y} = \frac{1}{2}\boldsymbol{x}$, where $\frac{1}{2}$ is the slope.

And $\tan\theta = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}} = \frac{2}{4} = \frac{1}{2}$, which is the slope of the hypotenuse.

So we can put the equation of ***L*** this way, too: $\boldsymbol{y} = \tan\theta \cdot \boldsymbol{x}$, and can say that the tangent can be the slope of a line in the ***x-y*** plane.

There is one more idea, though, we can use when using the tangent.
The tangent, that is, the slope does not only explain how much the hypotenuse is slanted, that is, the steepness of the hypotenuse, but more importantly, specifies

a rate of change as a velocity or speed.

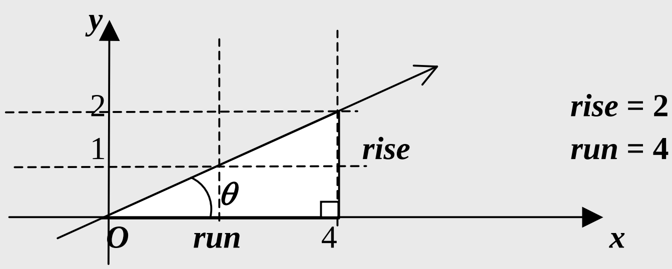
A ***velocity*** is a time ***rate of change*** in distance. And a speed is the magnitude of a velocity, since velocity is a vector, so has a direction, as well as a maginitude.

For instance, if a speed is 100 km/h, the distance traveled for every hour is 100 km. So in two hours, the distance covered is 200 km.

So the tangent, that is, the slope can indicate how fast or slow an amount changes (increases or decreases).

And the slope is often expressed as rise over run, i.e., $\frac{\textbf{\textit{rise}}}{\textbf{\textit{run}}}$, where ***rise*** is the ***opposite***, and ***run*** is the ***adjacent*** in a right triangle as in Fig. 8 below.

Fig. 8

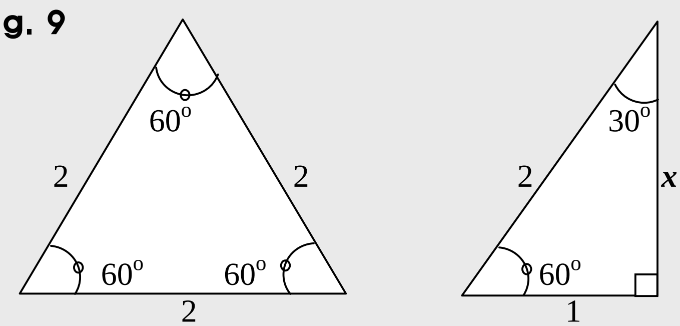


The rise is a vertical amount as a production or distance, so is usually an output, and the run is a horizontal amount as a number of people or time, so is usually an input. Actually in practice, each can be any that changes in amount as altitude, temperature, weight, pressure, volume, area, etc.

And we have three special angles, which are $\mathbf{30^o}$, $\mathbf{45^o}$, and $\mathbf{60^o}$.
We can easily get the trig ratios for those angles.
We can readily get them using a regular triangle and an isosceles right triangle.

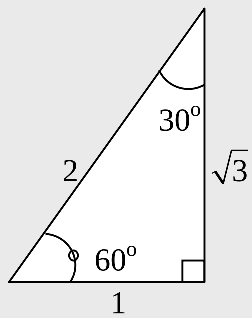
In a regular (equilateral) triangle, every angle is equal, so is 60^o, and in an isosceles right triangle, two angles are equal, and thus, are 45^o each.
So to begin with, cutting in half a regular triangle, we can get a right triangle as in Fig. 9.

Fig. 9



Next, using the Pythagorean Theorem (the distance formula), we can get $\boldsymbol{x^2 + 1^2 = 2^2 = 4 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}}$, since $\boldsymbol{x} > \boldsymbol{0}$. So we get this:

Fig. 10



Thus, taking the trig ratios of 60° and those of 30°, we get

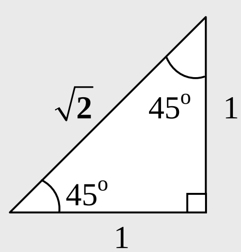
$$\sin 60^o = \frac{\boldsymbol{opposite}}{\boldsymbol{hypotenuse}} = \frac{\sqrt{3}}{2}, \text{ and } \sin 30^o = \frac{\boldsymbol{opposite}}{\boldsymbol{hypotenuse}} = \frac{1}{2}$$

$$\cos 60^o = \frac{\boldsymbol{adjacent}}{\boldsymbol{hypotenuse}} = \frac{1}{2}, \text{ and } \cos 30^o = \frac{\boldsymbol{adjacent}}{\boldsymbol{hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\tan 60^o = \frac{\boldsymbol{opposite}}{\boldsymbol{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}, \text{ and } \tan 30^o = \frac{\boldsymbol{opposite}}{\boldsymbol{adjacent}} = \frac{1}{\sqrt{3}}, \text{ which is } \frac{\sqrt{3}}{3}.$$

And the next is an isosceles right triangle as shown in Fig. 11 as follows.

Fig. 11



Taking the trig ratios of 45^{o}, we get

$$\sin 45^{o} = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{hypotenuse}}} = \frac{1}{\sqrt{2}}\text{, which is } \frac{\sqrt{2}}{2}.$$

$$\cos 45^{o} = \frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

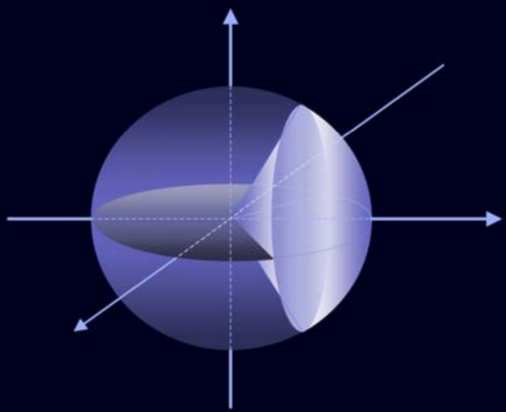
$$\tan 45^{o} = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}} = \frac{1}{1} = 1$$

And putting threads together, we have these:

$$\sin 30^{o} = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{hypotenuse}}} = \frac{1}{2}, \quad \sin 45^{o} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \sin 60^{o} = \frac{\sqrt{3}}{2}$$

$$\cos 30^{o} = \frac{\textbf{\textit{adjacent}}}{\textbf{\textit{hypotenuse}}} = \frac{\sqrt{3}}{2}, \quad \cos 45^{o} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \cos 60^{o} = \frac{1}{2}$$

$$\tan 30^{o} = \frac{\textbf{\textit{opposite}}}{\textbf{\textit{adjacent}}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}, \quad \tan 45^{o} = \frac{1}{1} = 1, \quad \tan 60^{o} = \sqrt{3}$$



삼각함수

Trigonometry