What is the base 10?

Click this to start.

1. What is the base 10?

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, …, 19, 20, 21, …

It is a number system where the radix is 10. So it is the decimal system where we use 10 number alphabets when making numbers. Let's see now what's inside the decimal system and how the system works.

Indicating a value, we can use a number. So for instance, we can use 5 or 17 to indicate five or seventeen. How then do we make the numbers 5 and 17?

We don't just pick a digit 5, and we don't just put together 1 and 7 to make 17. 17 doesn't mean one and seven, does it?

Of course not. We use 17 to mean ten and seven, so we mean seventeen, and not one and seven, which means eight. Thus, when making 17, we don't just pick 1 and 7, and just put them together. What then do we do?

When making numbers, we do actually, build them.
It’s because a number has its structure, as well as a value.
What then is the building material?

Among the material, we use units and places, as well as digits, which are number alphabets.
So we build a number in a number system using digits, together with units and places. And each digit has its place in a number. So a number is a structured object.

We build a structure looking like a string of digits. It’s not just a string of digits though. In a number, digits are not actual amounts or values. What then do we mean by digits?

Digits in a number are just representatives, and each digit shows a number of units. What do we mean by a unit then?

A unit in a number system is a set of equal pieces. And a unit depends on the place in a number. The unit in each place in a number is different. And of course, all the units in a place are equal, and all the pieces in a unit are equal, too.

Understanding how a number is structured, we can see how a number system works, and how digits and their places, together with units and their pieces are related to each other. Let’s now thus, get into the construction.

Constructing a number, we define a number system first.
And defining it, we need to define a radix.
Defining it, we set it to be the number of the alphabets we use when making numbers in the number system.

The alphabets are used as digits, so they are 0, 1, 2, …, and up to the radix. So for instance, if the radix is 10, we make numbers in the decimal system, and the alphabets are 0, 1, …, and 9, so we use up to 10 alphabets when making numbers in the decimal system.

That’s not all though, what the radix is about. There is one more to it, which is the unit. The maximum number of the units a digit can represent is 1 less than the radix. So for instance, if the radix is 10, each digit in a number can represent up to 9 units, and thus, can be from 0 up to 9.
Why not just say 1, but say a unit?

We’ll see the reason when constructing numbers here.
So let’s now construct a number in the decimal system.
A number looks like a string of digits as 10357. For instance, 37 is a two-digit number, and is a string of two digits, for which, we use two alphabets, 3 and 7 from 0, 1, 2, …, and 9.

So now, we need to get each digit constructed.
One digit at a time.
Each digit represents the units in its place, so it has its own place in a number. And each place has its value.

Before a number is constructed, it looks like this:

Each of the empty slots is called ‘a place in a number’.
Though a number seems limited, an actual number is unlimited, so the length is infinite.
Once alphabets as 0, 2, 3, and 7 have been assigned to digits, we can get a particular number as follows.

																			2	0	3	7

The number above is called 2037, read as two thousand thirty seven in the decimal system. Then, the number 2037 is structured the way as follows.

In 2037, the digit 7 is said to be in the 1’s place, and is said to be the 1’s digit. So we say that the 1’s digit is 7.
Why 1’s place or 1’s digit, though?

It’s because of the unit used there. In 2037, the digit 7 represents 7 units, each of which is 1. So 1 is a unit.

> Thus, we use the value of the ***unit*** used in a place as the value of the ***place*** in a number.

So 1 is the value of the place the digit 7 has in 2037, and thus, the digit 7 is in the 1’s place in 2037, and the place value is 1.

Let’s now move on to the digit 5 in another number 59.
What then does the digit 5 mean? Does it mean 5 of 1s?

No, it doesn’t. 5 of 1s are 5, but the 5 in 59 means fifty and not just five. So the 5 in 59 represents 5 units, each of which is a set of ten 1s.

In a place, we put an amount of units, and putting it, we use an alphabet to show the amount. So if putting the alphabet 5 in the 10’s place in the number, we put 5 of the units that are for the 10’s place.
And the value of the place is the value of a unit used or put in the place.
So if we assign an alphabet to a slot, that is, a place, the amount of the units used or put is the alphabet. Thus, if we assign 5 to the 10’s place, the 10’s place gets 5 of its units.
What slot then is the 10’s place in a number?

The second slot from the right end of the number.
What slot then is the 1’s place in a number?

The first slot from the right end of the number. What then is the value of a unit used in the place for the 5 in 59?

It is 10. So the 5 in 59 represents 5 units, each of which is 10. Thus, the 5 in 59 means fifty.

So the 5 in 59 is in the 10’s place. And the place value is 10.

Thus, in 59, the unit used for 9 is 1, and 9 is in the 1’s place. And in 59, the unit used for 5 is 10, and 5 is in the 10’s place, and is the 10’s digit.

So 59 is made of 5 of 10s and 9 of 1s. And for another instance, in 237, what is the unit used for the digit 2?

The unit is 100. So the unit used for the 2 in 237 is 100. And 2 is in the 100’s place, and is the 100’s digit.
Thus, 237 is made of 2 of 100s, 3 of 10s, and 7 of 1s.
So for another instance, how is 5237 is made in the decimal system?

The unit used for the digit 5 in 5237 is 1000. Thus, 5237 is made of 5 of 1000s, 2 of 100s, 3 of 10s, and 7 of 1s.

What if now, we add 3 to 247?

We get 250, of course. Why though?

It's because 247 and 3 are in the decimal system.
How do we get 250 though?

Let's see now what happens to digits in a number if a digit in the number gets 10 of its units in the decimal system.

Then to begin with, we have this:
247 + 3 = 200 + 40 + **7 + 3** = 200 + <u>40</u> + **10**
= 200 + <u>50</u> = 250

Next, in 247, 4 is in the 10's digit, and 7 is in the 1's digit.
So in 247, the 10's digit has 4 units, and the 1's digit has 7 units. Thus, if we add 3 to 247, the 1's digit will get 10 units, but cannot get them. Why not?

It's because 237 and 3 are in the decimal system.
In the decimal system, each digit or place in a number can get from 0 up to 9 units.

So it can get one of these alphabets only: 0, 1, 2, …, and 9.

Thus, in the number 247, what will happen to the 1’s digit if the 1’s digit will get 10 of its units?

The 1’s digit will get 0 unit, but the 10’s digit will get 1 more unit. Thus, we get 247 + 3 = 250, where the 1’s digit is set to 0, and the 10’s digit is set to 4 + 1, which is 5.
So the place to be added to gets 0 unit, but the next higher place gets 1 more unit.

In the decimal system, if a place gets 10 of its units, that place is set to 0, and the next higher place gets 1 more unit.

So for instance, in a number 823, the 8 is in the 100’s place which has 8 units, each of which is worth 100, and if 2 units are added to the 100’s place where 8 is put, the 100’s place gets 8 + 2 = 10 units, so the 100’s place is set to 0, and the 1000’s place gets 1 unit, which is worth 1000.

Before the addition, the 1000’s place in the number 823 was empty, so the number 823 looks like this:

																				8	2	3

And after the addition, we get this:

																			1	0	2	3

Why do 10 units cause the place to be set to 0 though?

It’s because the radix is 10. If the radix is 10, each and every place can get from 0 up to 9 units.

So the radix defines a number system. If the radix is 10, all the numbers we use are made in the decimal system, which means literally 10-alphabet system.

In the decimal system therefore, every time a place gets 10 of its units, that place is set to 0, and the next higher place gets 1 more unit.

A number looks like a string of slots called places, each of which can hold a digit, each of which is from the set of alphabets defined by the radix. So for instance, it can be as follows.

																	3	5	0	8	2	3

Each place has its own value, called the place value.
What value then is the place value?

The value of the unit the place can get. If the place value is 100, it is called the 100's place, and the unit used in the 100's place is a set of a hundred 1s, and thus, is worth 100. In short, the unit in the 100's place is 100.

For instance, in 237, the 3 is the 10's digit, is in the 10's place, and says 3 units, each of which is worth 10, so the place value is 10.
Also in 237, the 2 is in the 100's place, and says 2 units, each of which is worth 100, so the place value is 100.

And if the radix is 10, each place gets from 0 up to 9 units.

What if the radix is 16?

Then, each place gets from 0 up to 15 units. If a place gets 16 units, the place is set to 0, and the next higher place gets one more of its units. For instance, $1F_{[16]} + 1 = 20_{[16]}$, since F in hexadecimal is 15 in decimal.
Note that 0 is 0, and 1 is 1 in any number system.

If 1 is added to $1F_{[16]}$, the 1's digit will get 16 units, since F represents 15 units, so the 1's digit is set to 0, and the 16's digit gets one more unit. The 1 in 1F is the 16's digit, because the radix is 16. So the 16's digit is set to 1 + 1 = 2. Thus, we get 1F + 1 = 20 in hexadecimal, which is equal to 16 x 2, which is 32 in decimal, because the 2 in $20_{[16]}$ represents 2 units, each of which is worth 16. And the 2 in $20_{[16]}$ is in the 16's place.
You'll get familiar with the concept doing example sets.

And if the radix is 10, the base is 10, too. If the radix is 16, so is the base. Why the base then?

We can use a base as a subscript to specify a number to be in a particular number system. For instance, $12_{[10]} = 14_{[8]}$, which means 12 in the decimal system equals 14 in the octal system. In short, 12 is 14 in octal. By the way, $14_{[8]}$ is read as ‘one four in octal’ or ‘one four on the base 8’.

Also, we often use powers when expressing very large or small numbers. Expressing 100000 using power notation, we can put it this way: 10^5, where 10 is called the base, and 5 is called the exponent. Note however, the base in power notation does not mean a number system.

Now, the base comes into play.
The unit for the 100’s digit has 10 of **10 of 1s**. So the unit has 10 equal pieces, and each piece has 10 of 1s, and is from the next lower place that is the 10’s place.

So the next higher digit than the 10’s digit is (10 of 10)’s digit, and thus, is ($10 \times 10 = 100 = 10^2$)’s digit.
And the next higher digit is ($10 \times 10 \times 10 = 10^3$)’s digit.
So on the base 10, each digit can be put the way as follows.

$(1 = 10^0)$'s digit, $(1 \times 10 = 10^1)$'s digit, $(1 \times 10 \times 10 = 10^2)$'s digit, $(1 \times 10 \times 10 \times 10 = 10^3)$'s digit, which is the digit of 10^3, so the next higher is the digit of 10^4, the next higher is the digit of 10^5, the next higher is the digit of 10^6, the next is the digit of 10^7, …   Why $10^0 = 1$ though?

Multiplying 1 by 10 zero times, what do we get?

1. So leaving 1 alone, we just have 1.

Thus, if a number is in the decimal system, and is a positive integer, the digits (the place values) proceed the way as follows.

$10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, \ldots$

And building a number with the digit names, we can get this:

……………….	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

Without using the power notation, we put them the way as follows.

1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, …

They look quite bulky.

How about the digits lower than the 1's digit?

In the decimal system, every unit is a set of 10 equal pieces. The unit for the 1’s digit is 1. What then is the unit for the next lower digit than the 1’s digit?

It is 0.1, because 1 = 10 x 0.1. So 0.1 is the unit for the 0.1’s digit, which is the next lower digit than the 1’s digit. What then is the unit in the next lower digit than the 0.1’s digit?

It is 0.01, because 0.1 = 10 x 0.01. So 0.01 is the unit for the 0.01’s digit, which is the next lower digit than the 0.1’s digit. And we have

$0.1 = 1/10 = 10^{-1}$

$0.01 = 1/100 = 1/(10 \times 10) = 1/10^2 = 10^{-2}$

$0.001 = 1/1000 = 1/(10 \times 10 \times 10) = 1/10^3 = 10^{-3}$

So the digits lower than the 1’s digit proceeds the way as follows. 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8} …

Thus for instance, in a number 257.389,
the 3 is the 0.1’s digit, that is, the digit of 10^{-1},
the 8 is the 0.01’s digit, that is, the digit of 10^{-2},
and the 9 is the 0.001’s digit, that is, the digit of 10^{-3}.

So now, putting together all the digit names in a number with a fractional part, we can get this:

…, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, …

And building a number with the digit names, we can get this:

…	10^5	10^4	10^3	10^2	10^1	10^0	**.**	10^{-1}	10^{-2}	10^{-3}	10^{-4}	…

What then is the dot **.** in the number above?

We use a radix point to separate the fractional part from the whole number part, and the radix point is a dot in some countries, and is a comma in other countries.

For instance, in a number 367.025, the dot is the radix point, and in the decimal system, the radix point is called a decimal point.

What then about other bases as base 8, base 2, base 16, and even base N?
So to begin with what is the base 8?

It is a number system where the radix is 8. So it is the octal system where we use 8 number alphabets when making numbers. And the octal numbers can proceed the way as follows.
0, 1, 2, 3, 4, 5, 6, 7, 10, 11, …, 17, 20, 21, …

Suppose now, when making numbers, we use the octal system where the radix is 8. Then, we use 8 alphabets, which are 0, 1, 2, 3, 4, 5, 6, and 7. What then is the base?

Since the radix is 8, the base is 8, too.
In every number system, numbers have the same structure, and the only difference is the unit and its pieces.

In a number system, a unit is a set of equal pieces, and the number of the equal pieces is the radix.

So if the radix is 8, a unit is a set of 8 equal pieces, and of course, each place can get from 0 up to 7 units.
What then about the place values?

They are similar to those in the decimal system.

In the decimal system, the sequence of place values proceeds the way as follows.
…, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, …

So in the octal system, the sequence of place values proceeds the way as follows.
…, 8^6, 8^5, 8^4, 8^3, 8^2, 8^1, 8^0, 8^{-1}, 8^{-2}, 8^{-3}, 8^{-4}, …

What if the radix, that is, the base is 2?

In the binary system, the sequence of place values proceeds the way as follows.
…, 2^6, 2^5, 2^4, 2^3, 2^2, 2^1, 2^0, 2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, …

If the radix is 2, a unit is a set of 2 equal pieces, and each place can get from 0 up to 1 unit.
And in the binary system, we use 2 number alphabets when making numbers. And the binary numbers can proceed the way as follows.
0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, …

And if the radix is 16, the numbers are in the hexadecimal system, a unit is a set of 16 equal pieces, and each place can get from 0 up to 15 units.

In the hexadecimal system, the sequence of place values proceeds the way as follows.
…, 16^6, 16^5, 16^4, 16^3, 16^2, 16^1, 16^0, 16^{-1}, 16^{-2}, 16^{-3}, 16^{-4}, …

And if the radix is N, where N is an integer ≥ 2, the numbers can be said to be in the N system, a unit is a set of N equal pieces, and each place can get from 0 up to N –1 units.

In the N system, the sequence of place values proceeds the way as follows.
…, N^6, N^5, N^4, N^3, N^2, N^1, N^0, N^{-1}, N^{-2}, N^{-3}, N^{-4}, …