# What is a radix?



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# 0. What is a radix?

We can make and use numbers to show values.

How then do we make a number?

Do we just pick a number we need, and use it? For instance, do we just pick 5, and use it to show five?
And do we make a number putting numbers together, and use it? For instance, do we make 15 putting 1 and 5 together, and use it to show fifteen?

We don’t make a number that way.
We make a number using a number system. Strictly speaking thus, we set up a number system first, make a number using the system, and use it in accordance with the system.

Even doing math though, we don’t usually think about a number system when we make and use numbers.
What number system then do we use?

Normally, it is understood unless specified otherwise, we use the decimal number system, often called the decimal system for short when we make and use numbers. Decimal means ten. And number systems can matter in math.

A number can have or indicate a value. In math though, many different numbers can indicate the same value. Some numbers thus, look different, but share the same value. For instance, 11, 12, 13, 14, 15, 21, 23, 102, and 111 can have the same value in math. And if the same value is eleven, the value of 13 above is not thirteen but eleven. And the same is true for 102 and the others, too. Why though?

It’s because each of 11, 13, and the others has a different number system.
In our daily lives, we use the decimal system, so 11 means eleven, and 13 means thirteen in the decimal system.
Thus, our usual number system is the decimal system.
If however, 13 means eleven, it is in a different system, which is called the octal system. Octal means eight. So 13 in octal is 11 in decimal. And if 15 means eleven, it is in another system called the senary system. Senary means six. Thus, 15 in senary is 11 in decimal.

So on the other hand, the same number can have different values, too. For instance, 11 in binary is 3 in decimal, 11 in senary is 7 in decimal, and 11 in octal is 9 in decimal.

The use of a number to show a particular value depends on the number system where we make and use numbers.
How then a number system is made?

When we make a number system, we define the system.
And defining it, we use a radix, often called a base, too.
What then is a radix, a base?

Straight to the point, it is the maximum number of unique digits we can use when making a number in a number system. Unique digits?

They are the digits different from each other.
33228 is for instance, a 5-digit number, so it's made of 5 digits, but has 3 unique digits only, which are 2, 3, and 8.
Not quite clear?

In a number, each digit used is like an alphabet as K or T. For instance, a word 'nice' is made of n, i, c, and e. And a number '125' can be said to be made of 1, 2, and 5.

Thus, 1, 2, and 5 can be called number alphabets.
In English, the maximum number of alphabets is 26.
In the decimal system, the maximum number of number alphabets is 10, and they are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, each of which is a digit. So a number alphabet is a digit.

Making a number in math, we use digits as alphabets.

A number in math looks like a word as ‘nice’ or ‘good’, but is quite different from a word in nature and structure.

Suppose when making a number, we can use two digits only, which are 0 and 1, and no other digit is allowed. Then, the radix is 2.

We can take a digit as a number alphabet.
Using thus, 0 and 1 only to make numbers in a number system, we make numbers using the two alphabets only, which are 0 and 1. So we can make numbers in the system the way as follows. 0, 1, 10, 11, 100, 101, 110, …

Then, we make and use numbers in the binary number system, called the binary system for short.
So in the binary system, the radix is 2.

And the radix is 10 in the decimal system.
Thus, a radix often called a base is the maximum number of number alphabets we can use when making numbers in a number system.

We don’t just put number alphabets together though. It’s because each digit has a place in a number. For instance, a number 15 has two digits, and is a two-digit number, where 1 and 5 are the two, so 15 has two places. Each and every digit in a number has its own place in the number.
How then do we know its place in a number?

Each place in a number has its value, and we call it a place value. Suppose for instance, 325 is in the decimal system. Then in 325, the digit 3 has a place whose value is a hundred, the digit 2 has a place whose value is ten, and the digit 5 has a place whose value is one.

So in 325, 3 is in the hundred’s place, 2 is in the ten’s place, and 5 is in the one’s place.
How many hundreds then are there in the hundred’s place in a number 325?

Three. So the 3 in 325 means not three but three hundreds.

How many tens are there in the ten’s place in 325?

Two. So the 2 in 325 means not two but twenty.
How many ones does the one’s place have in 125?

Five. So the 5 in 325 means five.
We know 3 of hundreds is three hundreds, 2 of tens is twenty, and 5 of ones is five.
So 325 is normally called three hundred twenty five.

In the decimal system where the radix is 10, we have to use one of ten alphabets in each place in a number we make. And the ten alphabets are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

So ten is the maximum number of the number alphabets that can be used when we make numbers in the decimal system. And ten is the radix or the base in the decimal system.

We can make a number system where the radix is more than ten. If the radix is more than ten, we normally use English alphabets on top of those ten number alphabets listed above.  So what if the radix is 16?

If the radix is 16, we make and use numbers in the hexadecimal system, since hexadecimal means 16, and we use A, B, C, D, E, and F, together with 0, 1, 2, …, and 9. So we use 16 alphabets in the hexadecimal system. What then about the values of A, B, C, D, E, and F?

They are taken as digits. It's because each of those has been assigned a value. Thus, if putting them in ascending order, we put them the way as follows.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.

So A means ten, B means eleven, C means twelve, D means thirteen, E means fourteen, and F means fifteen. So in the hexadecimal system, we use those 16 digits when making numbers.

Thus, a radix is the maximum number of alphabets we use when making numbers in a number system.

Thus, the radix in the decimal system is 10, since decimal means 10, so the maximum number of alphabets is 10. What then is the radix in the nonary system where nonary means 9?

The radix or the base is 9 in the nonary system where each digit in a number has to have only one of 9 alphabets as follows. 0, 1, 2, 3, 4, 5, 6, 7, and 8.
What then is the radix in the octal system where octal means 8?

The radix or the base is 8 in the octal system where each digit in a number has to have only one of 8 alphabets as follows. 0, 1, 2, …, 7.
What then is the radix in the binary system where binary means 2?

It is 2, so the radix or the base is 2 in the binary system where each digit in a number has to have only one of 2 alphabets as follows. 0 and 1.
Why do we call a radix a base, too, though?

When specifying the number system a number belongs to, we often add the radix to the number as a subscript the way as follows. $12_{[8]}$, which means 12 is in the octal system.

‘12 in octal’ means 12 is in the octal system.
’10 in decimal’ means 10 is in the decimal system.
So $12_{[8]}$ means 12 in octal, and $10_{[10]}$ means 10 in decimal.

And we have $12_{[8]} = 10_{[10]}$. So 12 in octal is equivalent to 10 in decimal. $12_{[8]}$ is an octal number, and $10_{[10]}$ is a decimal number. Normally though, we omit the underscore [10] for a number in decimal if not ambiguous or confusing.

So "$12_{[8]} = 10$" means 12 in octal is equal to 10 in decimal. Usually though, we just say that 12 in octal is 10.

We will cover how the conversion is made between systems, and cover the arithmetic operations in each system in other sections or in example sets.

Some examples of number systems are as follows.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, … decimal

0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, … nonary

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, … octal

0, 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22,, 23, … septenary

0, 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, … senary

0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, … quinary

0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, … quaternary

0, 1, 2, 10, 11, 12, 20, 21, 22, 30, 31, 32, 40, … ternary

0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, … binary

0, 1, 2, …, 9, A, B, C, D, E, F, 10, 11, 12, …, 19, 1A, 1B, …,

99, 9A, 9B, …, 9F, A0, A1, …, AF, B0, B1, … hexadecimal

What is the number of alphabets in English?

26. What then is the number of alphabets in the decimal system?

10. So we don’t need to say the maximum if not ambiguous.

> Thus, a radix is the number of alphabets we use to make numbers in a number system.