Examples I in Arithmetic 2

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Examples I

0. Find the number of all positive integers that are less than 784, and have 7 in the 100's digit, and 3 in the 1's digit.

1. Find the number of all positive integers that are less than 1004, and have 8 in the 100's digit, and 5 in the 1's digit.

2. Put the numbers in the list below in ascending order, that is, put the smallest number first and the largest number last: 7852, 7694, 7926, 7799, 7901, and 7890.

3. Assuming ***d*** is a single digit number, find ***d*** that makes 59***d***4 greater than 5***d***99.

4. Assuming ***d*** is a single digit number, find ***d*** that makes 8***d***09 greater than 89***d***5.

5.0. Find all positive integers that can satisfy all below at the same time:

A. The 100's digit is 3 larger than the 10's digit.
B. The 10's digit is 1 lager than the 1's digit.
C. The 1's digit is lager than 2.
D. The integers are between 3000 and 4000.

5.1. Find all positive integers that can satisfy all below at the same time:

A. The 100's digit is 1 larger than the 10's digit.
B. The 10's digit is 2 larger than the 1's digit.
C. The 1's digit is lager than 1.
D. The integers are between 3000 and 5000.

5.2. Find all positive integers that can satisfy all below at the same time:

A. The 100's digit is 3 smaller than the 10's digit.
B. The 10's digit is 2 larger than the 1's digit.
C. The 1's digit is smaller than 7.
D. The integers are between 3000 and 5000.

Suggestions or Solutions
To the Problem in the Example 0

Find the number of all positive integers that are less than 784, and have 7 in the 100's digit, and 3 in the 1's digit.

There are 9 integers.

If not sure of how to get it, follow the steps below.

Assuming first, ***A*** is a positive integer less than 784, we can say that ***A*** can be one of all the integers from 1 to 783.

So assuming next, ***B*** is a positive integer that is less than 784, and has 7 in the 100's digit, and 3 in the 1's digit, we can set ***B* = 7*D*3** where ***D*** is the 10's digit.

What number then can be ***D***?

As each digit in a number, we can use one of ten single digit numbers, which are 0, 1, 2, 3, …, 9.

And we know that ***B*** is an integer positive and less than 784.
And we set ***B* = 7*D*3** where ***D*** is the number in the 10's digit.

So we can use as ***D*** one of 9 numbers that are 0, 1, 2, 3, …, 8.

How many integers then, can we use as ***B***?

We can use 9 numbers as ***D***. And thus, there are 9 integers that can be ***B***.

They are 783, 773, 763, 753, 743, 733, 723, 713, and 703.

Suggestions or Solutions
To the Problem in the Example 1

Find the number of all positive integers that are less than 1004, and have 8 in the 100's digit, and 5 in the 1's digit.

There are 10 integers.

If not sure of how to get it, follow the steps below.

Assuming first, ***A*** is a positive integer less than 1004, we can say that ***A*** can be one of all the integers from 1 to 1004.

So assuming next, ***B*** is a positive integer that is less than 1004, and has 8 in the 100's digit, and 5 in the 1's digit, we can set ***B* = *T*8*D*5** where ***T*** is the 1000's digit, and ***D*** is the 10's digit.

Then first, what can be ***T***?

As each digit in a number, we can use one of ten single digit numbers, which are 0, 1, 2, 3, …, 9.

And we know that ***B*** is an integer positive and less than 1004.
So we can use 0 only as ***T***.

And thus, we can now set ***B* = 8*D*5**.

And we can use as ***D*** one of ten numbers, which are these: 0, 1, 2, 3, …, 0.

So there are 10 integers that can be ***B***.

And they are 895, 885, 875 . . . 815, and 805.

Suggestions or Solutions
To the Problem in the Example 2

Put the numbers in the list below in ascending order, that is, put the smallest number first and the largest number last: 7852, 7694, 7926, 7799, 7901, and 7890.

So putting all those numbers in ascending order, we can put them the way below:

7694, **7799**, **7852**, **7901**, and **7926**.

It won’t probably take too long to get it, because we can look at all the numbers in the list, and the list is not too long. *If want to see how though, follow the steps below.*

If positive, the smaller the digit, the smaller the number.

So what number can be the smallest in a list of positive numbers if all the numbers have the same number of digits?

Of the numbers positive, the smallest can be the number where the highest digit is the smallest if all the numbers have the same number of digits.

So in short, if the highest digit is the smallest, the number can be the smallest.

What if however, all the highest digits in all the numbers are the same?

Then, we check the next highest digit, that is, the second highest digit, which is in this case, the hundred’s digit.

And if positive, the smaller the digit, the smaller the number.
So what is the smallest of all in the list below?

7852, 7694, 7926, 7799, 7901, and **7890**.

We can see that **7694** has 6 in the 100’s digit, and that 6 is the smallest of all the 100’s digit.

So **7694** is the smallest in the list above.

And the second smallest is **7799**.

Thus, eliminating the two from the list, we now have **7852, 7926, 7901,** and **7890**.

Next, we can see that the two numbers 7852 and 7890 have 8 in the 100’s digit.
So in the two, we need to look at the next lower digit, which is the 10’s digit.

Then, 5 is smaller than 9, so **7852** is the smaller of the two, and thus, is the third smallest.

Then, of course, **7890** is the fourth smallest.

Thus, we are now left with **7926** and **7901**, of which 7901 is the smaller.

So putting all the numbers in the list in ascending order, we can put them the way below:

7694, **7799**, **7852**, **7901**, and **7926**.

Suggestions or Solutions
To the Problem in the Example 3

Assuming *d* is a single digit number, find *d* that makes 59*d*4 greater than 5*d*99.

We get: ***d*** = 0, 1, 2 . . . 7, or 8.

If not sure of how to get it, follow the steps below.

We want 59***d***4 to be larger than 5***d***99.

And if positive, the larger the digit, the larger the number.

So we may want to begin with checking the highest digits in the two numbers given.

To begin with, the 1000's digits are the same.

So moving on to the next lower digit, which is the 100's digit, we have 9 and ***d***.

And we want 59***d***4 to be larger than 5***d***99. So for now, what does ***d*** have to be?

We can see for now, that ***d*** has to be less than or equal to 9.
Thus, we get ***d*** = 0, 1, 2 . . . 8, or 9.

And of course, we want to check to see if all the numbers can work for ***d***.

If ***d*** is one of 0, 1, 2 . . . 7, and 8, we get 59***d***4 > 5***d***99.
If however, ***d* = 9**, we cannot get 59***d***4 > 5***d***99. So we get ***d* ≠ 9**.

Thus, we get ***d*** = 0, 1, 2 . . . 7, or 8.

Suggestions or Solutions
To the Problem in the Example 4

Assuming *d* is a single digit number, find *d* that makes 8*d*09 greater than 89*d*5.

There is no value for ***d***.

If want to see why not, follow the steps below.

We want 8***d***09 to be larger than 89***d***5.

And if positive, the larger the digit, the larger the number.
So we may want to begin with checking the highest digits in the two numbers given.

To begin with, the 1000's digits are the same.
So moving on to the next lower digit, which is the 100's digit, we have ***d*** and 9.

And we want 8***d***09 to be larger than 89***d***5. So for now, what ***d*** has to be?

We can see for now, that ***d*** has to be equal to 9, since ***d*** is a digit, which is one of 0, 1, 2, …, 8, or 9.

Thus, assuming ***d* = 9**, we get 8***d***09 = 8909, and 89***d***5 = 8995, which is however, not the case we want, because we want this: 8***d***09 > 89***d***5 for some ***d***.

So there is no value for ***d***.

Suggestions or Solutions
To the Problem 0 in the Example 5

Find all positive integers that can satisfy all below at the same time:

A. **The 100's digit is 3 larger than the 10's digit.**
B. **The 10's digit is 1 lager than the 1's digit.**
C. **The 1's digit is lager than 2.**
D. **The integers are between 3000 and 4000.**

The numbers are as follows. **3743**, **3854**, and **3965**.

If not sure of how to get it, follow the steps below.

Assuming first, that ***n*** is an integer described above, we can say from the fact ***D***, that ***n*** is a 4-digit integer.
So we can set ***n* = *thdu***, where ***t*** is the 1000's digit, ***h*** is the 100's digit, ***d*** is the 10's digit, and ***u*** is the 1's digit. Then first, what does ***t*** have to be?

We can say that ***t* = 3**, since ***n*** is between 3000 and 4000.

Now, we have set ***n* = *thdu***, where ***t*** is the 1000's digit, ***h*** is the 100's digit, ***d*** is the 10's digit, and ***u*** is the 1's digit. And we have the facts below.

A. **The 100's digit is 3 larger than the 10's digit.**
B. **The 10's digit is 1 lager than the 1's digit.**
C. **The 1's digit is lager than 2.**

So next, from the fact **A**, what can we say about ***h*** and ***d***?

The fact **A** is saying that the 100's digit is 3 larger than the 10's digit.
And we have set ***n* = *thdu***, where ***t*** is the 1000's digit, ***h*** is the 100's digit, ***d*** is the 10's digit, and ***u*** is the 1's digit. So from the fact **A**, we can say that ***h* = 3 + *d***.

And we have the other two facts below.

B. The 10's digit is 1 lager than the 1's digit.
C. The 1's digit is lager than 2.

So next, from the fact **B**, what can we say about $\boldsymbol{d}$ and $\boldsymbol{u}$?

The fact **B** is saying that the 10's digit is 1 lager than the 1's digit.

And we have set $\boldsymbol{n = thdu}$, where $\boldsymbol{t}$ is the 1000's digit, $\boldsymbol{h}$ is the 100's digit, $\boldsymbol{d}$ is the 10's digit, and $\boldsymbol{u}$ is the 1's digit. So from the fact **B**, we can say that $\boldsymbol{d = 1 + u}$.

And next, we have the fact **C**, which is saying that the 1's digit is lager than 2.
So next, from the fact **C**, what can we say about $\boldsymbol{u}$?

The fact **C** is saying that the 1's digit is lager than 2.

And we set $\boldsymbol{n = thdu}$, where $\boldsymbol{t}$ is the 1000's digit, $\boldsymbol{h}$ is the 100's digit, $\boldsymbol{d}$ is the 10's digit, and $\boldsymbol{u}$ is the 1's digit.
So from the fact **C**, we can say that $\boldsymbol{u} > \mathbf{2}$. Thus for now, we get $\boldsymbol{u} = 3, 4, 5, 6, 7, 8, \text{or } 9$.

So to begin with, we get $\boldsymbol{u = 3 \Rightarrow d = 1 + u = 4 \Rightarrow h = 3 + d = 7}$.

Next, we get $\boldsymbol{u = 4 \Rightarrow d = 1 + u = 5 \Rightarrow h = 3 + d = 8}$.

Next, we get $\boldsymbol{u = 5 \Rightarrow d = 1 + u = 6 \Rightarrow h = 3 + d = 9}$.

And next, we get $\boldsymbol{u = 6 \Rightarrow d = 1 + u = 7 \Rightarrow h = 3 + d = 10}$, which is however, is not allowed, since $\boldsymbol{h}$ is a digit.

So we get $\boldsymbol{(u, d, h)} = (3, 4, 7), (4, 5, 8), \text{and } (5, 6, 9)$.

And we know $\boldsymbol{n = thdu}$, where $\boldsymbol{t = 3}$. So we get $\boldsymbol{n} = \mathbf{3743}, \mathbf{3854}, \text{or } \mathbf{3965}$.

Let's now, put all the ideas together in one page.

5.0. Find all positive integers that can satisfy all below at the same time:

A. **The 100's digit is 3 larger than the 10's digit.**
B. **The 10's digit is 1 lager than the 1's digit.**
C. **The 1's digit is lager than 2.**
D. **The integers are between 3000 and 4000.**

Assuming first, that $\boldsymbol{n}$ is an integer described above, we can say from the fact **D**, that $\boldsymbol{n}$ is a 4-digit integer.

So we can set $\boldsymbol{n = thdu}$, where $\boldsymbol{t}$ is the 1000's digit, $\boldsymbol{h}$ is the 100's digit, $\boldsymbol{d}$ is the 10's digit, and $\boldsymbol{u}$ is the 1's digit.

Also, we can see that $\boldsymbol{t = 3}$, since $\boldsymbol{n}$ is between 3000 and 4000.

Next, from the fact **A**, we can say that $\boldsymbol{h = 3 + d}$.

Next, from the fact **B**, we can say that $\boldsymbol{d = 1 + u}$.

And next, from the fact **C**, we can say that $\boldsymbol{u} > \mathbf{2}$.
Thus for now, we get $\boldsymbol{u}$ = 3, 4, 5, 6, 7, 8, or 9.

So to begin with, we get $\boldsymbol{u = 3 \Rightarrow d = 1 + u = 4 \Rightarrow h = 3 + d = 7}$.

Next, we get $\boldsymbol{u = 4 \Rightarrow d = 1 + u = 5 \Rightarrow h = 3 + d = 8}$.

Next, we get $\boldsymbol{u = 5 \Rightarrow d = 1 + u = 6 \Rightarrow h = 3 + d = 9}$.

And next, we get $\boldsymbol{u = 6 \Rightarrow d = 1 + u = 7 \Rightarrow h = 3 + d = 10}$, which is however, is not allowed since $\boldsymbol{h}$ is a digit.

So we get $\boldsymbol{(u, d, h)}$ = (3, 4, 7), (4, 5, 8), and (5, 6, 9).

And we know $\boldsymbol{n = thdu}$, where $\boldsymbol{t = 3}$.

So we get $\boldsymbol{n = 3743}$, **3854**, or **3965**.

Suggestions or Solutions
To the Problem 1 in the Example 5

Find all positive integers that can satisfy all below at the same time:

A. **The 100's digit is 1 larger than the 10's digit.**
B. **The 10's digit is 2 larger than the 1's digit.**
C. **The 1's digit is lager than 1.**
D. **The integers are between 3000 and 5000.**

Assuming first, that $\boldsymbol{n}$ is an integer described above, we can say from the fact **D**, that $\boldsymbol{n}$ is a 4-digit integer.
So we can set $\boldsymbol{n = thdu}$, where $\boldsymbol{t}$ is the 1000's digit, $\boldsymbol{h}$ is the 100's digit, $\boldsymbol{d}$ is the 10's digit, and $\boldsymbol{u}$ is the 1's digit.
And also, we can see that $\boldsymbol{t = 3}$ or $\boldsymbol{4}$, since $\boldsymbol{n}$ is between 3000 and 5000.

Next, from the fact **A**, we can say that $\boldsymbol{h = 1 + d}$.
Next, from the fact **B**, we can say that $\boldsymbol{d = 2 + u}$.
And next, from the fact **C**, we can say that $\boldsymbol{u > 1}$. So we get $\boldsymbol{u}$ = 2, 3, 4, 5, 6, 7, 8, or 9.

So to begin with, we get $\boldsymbol{u = 2 \Rightarrow d = 2 + u = 4 \Rightarrow h = 1 + d = 5}$.

Next, we get $\boldsymbol{u = 3 \Rightarrow d = 2 + u = 5 \Rightarrow h = 1 + d = 6}$.

Next, we get $\boldsymbol{u = 4 \Rightarrow d = 2 + u = 6 \Rightarrow h = 1 + d = 7}$.

Next, we get $\boldsymbol{u = 5 \Rightarrow d = 2 + u = 7 \Rightarrow h = 1 + d = 8}$.

Next, we get $\boldsymbol{u = 6 \Rightarrow d = 2 + u = 8 \Rightarrow h = 1 + d = 9}$.

And next, we get $\boldsymbol{u = 7 \Rightarrow d = 2 + u = 9 \Rightarrow h = 1 + d = 10}$, which is however, is not allowed, since $\boldsymbol{h}$ is a digit.

So we get: $\boldsymbol{(u, d, h)}$ = (2, 4, 5), (3, 5, 6), (4, 6, 7), (5, 7, 8), and (6, 8, 9).

And we know $\boldsymbol{n = thdu}$, where $\boldsymbol{t = 3}$ or $\boldsymbol{4}$.

So we get $\boldsymbol{n}$ **= 3542**, **4542**, **3653**, **4653**, **3764**, **4764**, **3875**, **4875**, **3986**, or **4986**.

Suggestions or Solutions
To the Problem 2 in the Example 5

Find all positive integers that can satisfy all below at the same time:

- **A.** **The 100's digit is 3 smaller than the 10's digit.**
- **B.** **The 10's digit is 2 larger than the 1's digit.**
- **C.** **The 1's digit is smaller than 7.**
- **D.** **The integers are between 3000 and 5000.**

Assuming first, that n is an integer described above, we can say from the fact **D**, that n is a 4-digit integer. So we can set $n = thdu$, where t is the 1000's digit, h is the 100's digit, d is the 10's digit, and u is the 1's digit.

Also, we can see that $t = 3$ or 4, since n is between 3000 and 5000.

Next, from the fact **A**, we can say that $h = d - 3$.
Next, from the fact **B**, we can say that $d = 2 + u$.
And next, from the fact **C**, we can say that $u < 7$. So we get $u = 0, 1, 2, 3, 4, 5,$ or 6.

So to begin with, we get $u = 0 \Rightarrow d = 2 + u = 2 \Rightarrow h = d - 3 = -1$, which is however, is not allowed since h is a digit.

Next, we get $u = 1 \Rightarrow d = 2 + u = 3 \Rightarrow h = d - 3 = 0$.

Next, we get $u = 2 \Rightarrow d = 2 + u = 4 \Rightarrow h = d - 3 = 1$.

Next, we get $u = 3 \Rightarrow d = 2 + u = 5 \Rightarrow h = d - 3 = 2$.

Next, we get $u = 4 \Rightarrow d = 2 + u = 6 \Rightarrow h = d - 3 = 3$.

Next, we get $u = 5 \Rightarrow d = 2 + u = 7 \Rightarrow h = d - 3 = 4$.

And next, we get $u = 6 \Rightarrow d = 2 + u = 8 \Rightarrow h = d - 3 = 5$.

So we get $(u, d, h) = (1, 3, 0), (2, 4, 1), (3, 5, 2), (4, 6, 3), (5, 7, 4)$, and $(6, 8, 5)$.

And we know $n = thdu$, where $t = 3$ or 4.

So we get $n =$ 3031, 4031, 3142, 4142, 3253, 4253, 3364, 4364, 3475, 4475, 3586, or 4586.