Examples J in Arithmetic 2

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Examples J

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A. What is the smallest positive number that has five digits above the point and seven digits below the point?

B. What is the smallest negative number that has 10 digits above the point and two digits below the point?

C. What is the largest 10-digit number where all digits are different, and three of the digits are above the point?

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E. What is the smallest 12-digit number where five digits are above the point, and every digit above the point is different from any digit below the point?

F. Find the smallest 12-digit positive number where 7 digits are above the point, and every digit above the point is different from any of the digits below the point.

G. Find the smallest 12-digit number where six digits are above the point, and are different, and every digit above the point is different from any digit below the point.

Suggestions or Solutions
To the Problems in the Examples J

0. What is the largest number?

There is no largest number. Why not, though?

A number can be infinitely large.
So no matter how large number we may come up with, we can still make one the larger.

-9, < -2 < 0 < 1 < 3 < 9 < 9999 < 99999 < 9999999 < 999999999, ….

Thus, there is no largest number.

1. What is the largest positive number?

There is no largest positive number. Why not?

A positive number can be infinitely large. So no matter how large positive number we may come up with, we can still make the larger one.

0 < 0.0001 < 0.01 < 1 < 3 < 9 < 99 < 9999 < 9999999 < 9999999999999 < … … …

Therefore, there is no largest positive number.

2. What is the largest negative number?

There is no largest number negative. Why not, though?

A number is negative if it is less than 0.
And the smaller the magnitude, the larger the number if the number is negative.
So a negative number can be indefinitely large, since it is less than 0. How?

The magnitude of a negative number can be infinitely small. And the smaller the magnitude, the larger the number if the number is negative.

-9 < -2 < -1 < -0.1 < -0.001 < -0.000001 < -0.0000000000001 < … … … < 0.

And we know 0 is not negative, of course.
So no matter how large negative number we may come up with, we can still make one the larger, which is still negative though. Thus, there is no largest number negative.

3. What is the smallest number?

There is no smallest number. Why not?

A negative number can be infinitely small. And the larger the magnitude, the smaller the number if the number is negative.

99 > 9 > 5 > 1 > 0 > -0.0001 > -0.01 > -1 > -3 > -99 > -999 > -999999999 > … … …

So no matter how small negative number we may come up with, we can still make one the smaller. Thus, there is no smallest number.

4. What is the smallest positive number?

There is no smallest positive number. Why not, though?

A number is positive if it is greater than 0.
And the smaller the magnitude, the smaller the number if the number is positive.
And the magnitude of the positive number can be infinitely small.
So a positive number can be infinitely small although it is greater than 0.

9 > 5 > 1 > 0.1 > 0.0001 > 0.0000001 > 0.000000000001 > … … … > 0

And we know 0 is not positive. So no matter how small positive number we may come up with, we can still make one the smaller. So there is no smallest positive number.

5. What is the smallest negative number?

There is no smallest negative number. Why not?

A number is negative if it is less than 0.
And the larger the magnitude, the smaller the number if the number is negative.
And the magnitude of the negative number can be infinitely large.

So a negative number can be infinitely small.

0 > -0.000001 > -0.001 > -0.1 > -1 > -5 > -99 > -9999 > -99999999 > … … …

Thus, no matter how small negative number we may come up with, we can still make one the smaller. Therefore, there is no smallest negative number.

6. What is the largest number where there are ten digits and two of the digits are below (to the right of) the point?

The number is 99,999,999.99. How?

To begin with, the number we want is positive. And the larger the digits, the larger the number if the number is positive.
And we know each digit is one of the integers from 0 to 9. So the largest digit can be 9.
And next, in the number, 8 digits are above the point, and 2 digits are below the point.

So the number we want is 99,999,999.99.

7. What is the largest positive number where there are ten digits, and seven of the digits are above (to the left of) the point?

The number is 9,999,999.999. How?

First, the number we want is positive. And the larger the digits, the larger the number if the number is positive. And we know each digit is one of the integers from 0 to 9. So the largest digit can be 9.
And next, the number has 7 digits above the point, and 3 digits below the point.
So the number we want is 9,999,999.999.

8. What is the largest negative number where there are ten digits, and four of the digits are below the point?

The number is –100,000.0001. How?

First, the number we want is negative.
And the smaller the magnitude, the larger the negative number.

So the smaller the digit, the smaller the magnitude, so the larger the negative number.

And we know that the smallest number we can use as a digit in a number is 0.

And we know that the number we want is made of 10 digits, of which seven are above the point, and three are below the point. To what then can we set the digits in the number we want?

We cannot set all the digits to 0s, because setting all to 0s, we just get 0.

The second smallest number we can use as a digit is 1.

So we want to set the highest digit to 1, and put the negative sign in front.

Then, we get –1000000.0000, which is however, the same as –1000000, which does not have four digits below the point. So it is not the number we want.

What do we mean by a number that has 4 digits below the point?

At least, the lowest digit, that is, the fourth digit below the point is not 0.
So for instance, a number with 4 digits below the point can be 0.0128 or 12.0105.

What then is the number we want?

In the number, the highest digit is 1, the lowest digit is 1, too, and the rest of all the digits are 0s. So the number we want is –100,000.0001.

And let’s now put all the ideas together in one page.

8. What is the largest negative number where there are ten digits, and four of the digits are below the point?

First, the number we want is negative.
And the smaller the magnitude, the larger the negative number.
So the smaller the digit, the smaller the magnitude, so the larger the negative number.
And we know that the smallest number we can use as a digit is 0, and that the number is made of 10 digits, of which seven are above the point, and three are below the point. To what then can we set the digits in the number we want?

We cannot set all the digits to 0s, because setting all to 0s, we just get 0.
And the second smallest number we can use as a digit is 1.
So we want to set the highest digit to 1, and put the negative sign in front.
Then, we get –1000000.0000, which is however, the same as –1000000, which does not have four digits below the point. So it is not the number we want.
What do we mean by a number that has 4 digits below the point?

At least, the lowest digit, that is, the fourth digit below the point is not 0.
So for instance, a number with 4 digits below the point can be 0.0128 or 12.0105.
What then is the number we want?

In the number, the highest digit is 1, the lowest digit is 1, too, and the rest of all the digits are 0s. So the number we want is –100,000.0001.

9. What is the smallest number where there are twelve digits, and three of those are below the point?

The number is –999,999,999.999. How?

First, the number we want is negative. And the larger the digits, the smaller the number if the number is negative. And we know each digit is an integer from 0 to 9. So the largest digit can be 9.

And next, the number has 9 digits above the point, and 3 digits below the point.
So the number we want is –999,999,999.999.

A. What is the smallest positive number that has five digits above the point and seven digits below the point?

The number is 10,000.0000001. How?

First, the number we want is positive.
And the smaller the magnitude, the smaller the positive number.
So the smaller the digit, the smaller the magnitude, so the smaller the positive number.

And we know that the smallest number we can use as a digit is 0, and that the number we want is made of 12 digits, of which 5 are above the point, and 7 are below the point.
To what then, can we set the digits in the number we want?

We cannot set all the digits to 0s, because setting all to 0s, we just get 0.
And the second smallest number we can use a digit is 1.
So we want to set the highest digit to 1.

Then, we get 10000.0000000, which is however, no other than 10000, which does not have 7 digits below the point. So it is not the number we want.
What do we mean by a number that has 7 digits below the point?

At least, the lowest digit, that is, the seventh digit below the point is not 0.
So for instance, a number with 7 digits below the point can be 101.0117008.
What then is the number we want?

In the number, the highest digit is 1, the lowest digit is 1, too, and the rest of all the digits are 0s. So the number we want is 10,000.0000001.
And let's now put all the ideas together in one page.

A. What is the smallest positive number that has five digits above the point and seven digits below the point?

First, the number we want is positive.
And the smaller the magnitude, the smaller the positive number.
So the smaller the digit, the smaller the magnitude, so the smaller the positive number.
And we know that the smallest number we can use as a digit is 0, and that the number we want is made of 12 digits, of which 5 are above the point, and 7 are below the point.
To what then, can we set the digits in the number we want?

We cannot set all the digits to 0s, because setting all to 0s, we just get 0.
And the second smallest number we can use a digit is 1.
So we want to set the highest digit to 1.

Then, we get 10000.0000000, which is however, no other than 10000, which does not have 7 digits below the point. So it is not the number we want.
What do we mean by a number that has 7 digits below the point?

At least, the lowest digit, that is, the seventh digit below the point is not 0.
So for instance, a number with 7 digits below the point can be 101.0117008.
What then is the number we want?

In the number, the highest digit is 1, the lowest digit is 1, too, and the rest of all the digits are 0s. So the number we want is 10,000.0000001.

B. What is the smallest negative number that has 10 digits above the point and two digits below the point?

The number is –9,999,999,999.99. How?

First, the larger the digits, the smaller the number if the number is negative. And we know each digit is an integer from 0 to 9. So the largest digit can be 9.

And next, the number has 10 digits above the point, and 3 digits below the point.

So the number we want is –9,999,999,999.99.

C. What is the largest 10-digit number where all digits are different, and three of the digits are above the point?

The number is 987.6543201. How?

First, the number we want is positive.
And the larger the magnitude, the larger the positive number.
So the larger the digit, the larger the magnitude, so the larger the positive number.

We know however, all the digits in the number we want are different.
To what then can we set the digits in the number we want?

First, assuming ***A*** is the number we want, and ignoring the point, that is, the dot in ***A***, we can set ***A* = 9876543210**.
And next, we know that three digits are above the point.
So can we just set ***A* = 987.6543210**?

We know ***A* = 987.6543210 = 987.654321**, which does not have 10 digits.
So we want to set ***A* = 987.6543201**.

D. What is the largest 10-digit negative number where all digits are different, and five of the digits are below the point?

The number is –10234.56789. How?

The smaller the magnitude, the larger the negative number.
So the smaller the digit, the smaller the magnitude, so the larger the negative number.

We know however, all the digits in the number we want are different.
To what then can we set the digits in the number we want?

First, assuming ***A*** is the number we want, and ignoring the point, that is, the dot in ***A***, we can try setting ***A* = –0123456789**, which is however, no other than **–123456789**.
Can we then set ***A* = –1234567890**?

The smaller the digit, the smaller the magnitude, so the larger the negative number.
So we want to set ***A* = –1023456789**.

And next, we want ***A*** to be the smallest of all the negative numbers that have 5 digits above the point and 5 digits below the point.

So the number we want is **–10234.56789**.

E. What is the smallest 12-digit number where five digits are above the point, and every digit above the point is different from any of the digits below the point?

The number is –99999.8888888. How?

First, the number we want is negative, since a number negative is smaller than a number positive. And the larger the magnitude, the smaller the negative number.

So the larger the digit, the larger the magnitude, so the smaller the negative number.
Thus first, assuming ***A*** is the smallest 12-digit number with no digit below the dot, we can set **A = –999999999999**.

Next, we know in the number we want, 5 digits are above the point and every digit above the point is different from any of the digits below the point.
And the second largest number we can use as a digit is 8.
So the number we want is **–99999.8888888**.

F. Find the smallest 12-digit positive number where 7 digits are above the point, and every digit above the point is different from any of the digits below the point.

The number is 1,000,000.22222. How?

The smaller the magnitude, the smaller the positive number.
The smaller the digit, the smaller the magnitude, so the smaller the positive number.

Thus first, assuming ***A*** is the smallest 12-digit positive number with no digit below the dot, we can set ***A* = 100000000000**.

Next, we know in the number we want, 7 digits are above the point and every digit above the point is different from any of the digits below the point.

So we cannot use 1 as any digit below the dot.
And the second smallest number we can use as a digit is 2.

So the number we want is 1,000,000.22222.

G. Find the smallest 12-digit number where six digits are above the point, and are different, and every digit above the point is different from any digit below the point.

The number is –987654.333333. How?

First, the number we want is negative.
And the larger the magnitude, the smaller the negative number.
So the larger the digit, the larger the magnitude, so the smaller the negative number.
Thus next, assuming ***A*** is the smallest 12-digit number with no digit below the dot, to what number can we set ***A***?

We can set ***A* = –999999999999**.
Next, we know in the number we want, 6 digits are above the point, and are different.
So assuming ***B*** is the smallest 12-digit number where 6 digits are above the point, and are different, to what number can we set ***B***?

We can set ***B* = –987654.999999**.
And next, we know in the number we want, every digit above the point is different from any of the digits below the point.

So we cannot use any digit above the dot as any digit below the dot.
What then are the numbers can we use as digits below the dot?

They are 0, 1, 2, and 3.
And we know this: the larger the magnitude, the smaller the negative number.
So the larger the digit, the larger the magnitude, so the smaller the negative number.

And 3 is the largest of 3, 2, 1, and 0.
So the number we want is –987654.333333.