Examples L in Arithmetic 2

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Examples L

0. Suppose we make integers using numbers from a set ***A* = {7, 0, 9, 8}**, and making each integer, we have to use three different numbers in the set.
What are then all the integers less than 700?

1. Suppose we make integers using numbers from a set ***B* = {5, 0, 9, 4, 6}**, and making each integer, we have to use four different numbers in the set.
Find the number of all the integers < 5000.

2. Doing examples below, use numbers in a set ***C* = {3, 5, 0, 7}**. Note however, each number in the set can be used once only. That is, no number can be used more than once.

2.0. What is the smallest four-digit integer?

2.1. What is the second smallest four-digit integer?

2.2. What is the largest four-digit integer?

2.3. What is the second largest four-digit integer?

3. Find all the integers described as follows.

A. 1’s digit is smaller than 1000’s digit.
B. 10’s digit is lager than 1000’s digit.
C. 1000’s digit is twice 100’s digit.
D. Each integer is between 6000 and 7000.
E. Adding together all the digits in each integer, we get 19.

Suggestions or Solutions
To the Problems in the Examples L

0. Suppose we make integers using numbers from a set *A* = {7, 0, 9, 8}, and making each integer, we have to use three different numbers in the set.
What are then all the integers less than 700?

They are 98, 97, 89, 87, 79, and 78.

If not sure of how to get it, follow the steps below.

If a number is less than 700, the 100's digit is less than or equal to 6.
What number then from the set, can we use for the 100's digit in an integer we make?

We do not have to make a 3-digit integer, because the problem is not asking 3-digit integers. What do we mean by though, a 3-digit integer?

A 3-digit integer is between 1000 and 99. And in math, assuming such an integer is N, we can put it this way: $\mathbf{99 < N < 1000}$.

More precisely though, it is greater than or equal to 100 and less than or equal to 999. And in math, we can put it this way: $\mathbf{100 \leq N \leq 999}$.

And more specifically, since N is an integer, we can put it the way below, too.
$\mathbf{N = 100, 101, 102, \ldots 998, 999}$.

So in this problem, making an integer, we have only to use three numbers from the set. Thus, we can use 0 for the 100's digit in an integer we make.

So we can make 098, 097, 089, 087, 079, and 078, which are all the integers less than 700, and of course, we usually put them this way: 98, 97, 89, 87, 79, and 78.

1. Suppose we make integers using numbers from a set *B* = {5, 0, 9, 4, 6}, and making each integer, we have to use four different numbers in the set.
Find the number of all the integers < 5000.

There are 48 integers.

If not sure of how to get it, follow the steps below.

If a number is less than 5000, what can be the 1000's digit in the number?

It can be less than or equal to 4.
So assuming ***N*** is the integer we want to make, we can put it the way below.
N* = 4000 + 100*a* + 10*b* + *c, where ***a***, ***b***, and ***c*** are numbers that belong to ***C* = {5, 0, 9, 6}**.

So we now have to choose a number for each of ***a***, ***b***, and ***c***.
Then, to begin with, how many choices do we have for ***a***?

We have four. That's because any of 5, 0, 9, and 6 can be used as ***a***. What then about ***b***?

Assuming we have chosen a number for ***a***, we have three choices for ***b***. That's because, of the four numbers in the set ***C***, one has been used as ***a***. What then about ***c***?

Assuming we have chosen two numbers for ***a*** and ***b***, we have two choices for ***c***. That's because, of the four numbers in the set ***C***, one has been used as ***a***, and another has been used as ***b***.

So we have 4 choices for ***a***. And for each choice for ***a***, we have three choices for ***b***. And for each choice for ***b***, we have two choices for ***c***.

Thus, the number of all the integers **< 5000** is 4 x 3 x 2 = 24.

And the integers less than 5000 are as follows.
4965, 4960, 4956, 4950, 4906, 4905
4695, 4690, 4659, 4650, 4609, 4605
4596, 4590, 4569, 4560, 4509, 4506
4096, 4095, 4069, 4065, 4059, 4056

Is that it though?

We do not have to make a 4-digit number, and making a number, we have only to use four different numbers from the set.

That is to say that we can begin with putting 0 in the 1000's digit when making an integer < **5000**.

So assuming ***M*** is such an integer, we can put it the way below.

M* = 1000** x **0 + 100*a* + 10*b* + *c, where ***a***, ***b***, and ***c*** belong to ***D* = {5, 9, 4, 6}**.

That is, ***a*** indicates 100's digit, ***b*** indicates 10's digit, and ***c*** indicates 1's digit.

So we have 4 choices for ***a***. And for each choice for ***a***, we have three choices for ***b***. And for each choice for ***b***, we have two choices for ***c***.

Thus, in this case, too, the number of all the integers < **5000** is 4 x 3 x 2 = 24.
And those integers are as follows.

0965, 0964, 0956, 0954, 0946, 0945
0695, 0694, 0659, 0654, 0649, 0645
0596, 0594, 0569, 0564, 0549, 0546
0496, 0495, 0469, 0465, 0459, 0456

Thus, the number of all the integers < **5000** is 48.

2. Doing examples below, use numbers in a set $E = \{3, 5, 0, 7\}$. Note however, each number in the set can be used once only. So no number can be used more than once.

2.0. What is the smallest four-digit integer?

Making a 4-digit integer, what digit cannot be 0?

It is the highest digit, which is the 1000's digit. So we cannot use 0 in the 1000's digit since we need to make a four-digit integer.

We can however, put 0 in the second highest digit, which is in this case, the 100's digit.
And the smaller the number is in a particular digit, the smaller the integer is.
So the smallest integer is 3057.

2.1. What is the second smallest four-digit integer?

The second smallest is larger than the smallest, and is the smallest of all the others.
So the second smallest is 3075.

2.2. What is the largest four-digit integer?

The larger the number is in a particular digit, the larger the integer is.
So the largest integer is 7530.

2.3. What is the second largest four-digit integer?

The second largest is smaller than the largest, and is the largest of all the others.
So the second largest is 7503.

3. **Find all the integers described as follows:**

A. **1’s digit is smaller than 1000’s digit.**
B. **10’s digit is lager than 1000’s digit.**
C. **1000’s digit is twice 100’s digit.**
D. **Each integer is between 6000 and 7000.**
E. **Adding together all the digits in each integer, we get 19.**

They are 6373, 6382, and 6391.

If not sure of how to get it, follow the steps below.

To begin with, each integer we make is between 6000 and 7000.
So assuming N is the integer, we get $\mathbf{6000 < N < 7000}$.

Thus next, we can see that N is a 4-digit integer positive, of course.
So we can set $N = abcd$, where a is the number in the 1000’s digit, b is the one in the 100’s digit, c is the one in the 10’s digit, and d is the one in the 1’s digit.

Thus, note that in this case,

N is $a \times 1000 + b \times 100 + c \times 10 + d$, and thus, is not $a \times b \times c \times d$.

Normally however, just saying $N = abcd$, we mean $N = a \times b \times c \times d$. So for instance, if we are given just $K = uv$ with no more specification, it means $K = u \times v$.
Then first, in $N = abcd$, what can be the number a?

We know $\mathbf{6000 < N < 7000}$, and N is an integer.
So we get $\mathbf{6001 \leq N \leq 6999}$, that is, N can be any of the integers as follows.
6001, **6002**, …, **6998**, and **6999**. And we have $N = abcd$. So we get $a = 6$.

Next, the statement **C** says that 1000's digit is twice 100's digit.
And we know that $\boldsymbol{a}$ is in the 1000's digit, and $\boldsymbol{b}$ is in the 100's digit.
So we get $\boldsymbol{a} = \mathbf{2} \times \boldsymbol{b}$.
Thus, we get $\boldsymbol{b} = \mathbf{3}$, since $\boldsymbol{a} = \mathbf{6}$.
So we now have $\boldsymbol{N} = \mathbf{63}\boldsymbol{cd}$.

Next, the statement **B** says that 10's digit is lager than 1000's digit.
So we get $\boldsymbol{c} > \mathbf{6} \Rightarrow \boldsymbol{c} = \mathbf{7}, \mathbf{8}$, or $\mathbf{9}$.
Next, the statement **A** says that 1's digit is smaller than 1000's digit.
So we get $\mathbf{0} \le \boldsymbol{d} < \boldsymbol{a} = \mathbf{6} \Rightarrow \boldsymbol{d} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$, or $\mathbf{5}$.

And next, from the statement **E**, we get $\mathbf{6} + \mathbf{3} + \boldsymbol{c} + \boldsymbol{d} = \mathbf{19} \Rightarrow \boldsymbol{c} + \boldsymbol{d} = \mathbf{10}$.
So what can be the next?

We have these:
$\boldsymbol{c} = \mathbf{7}, \mathbf{8}$, or $\mathbf{9}$.
$\boldsymbol{d} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$, or $\mathbf{5}$.
And $\boldsymbol{c} + \boldsymbol{d} = \mathbf{10}$.

So we can pair up the numbers for $\boldsymbol{c}$ and $\boldsymbol{d}$ the way below.

$(\boldsymbol{c}, \boldsymbol{d}) = (\mathbf{7}, \mathbf{3})$, $(\mathbf{8}, \mathbf{2})$, and $(\mathbf{9}, \mathbf{1})$. What do we mean by though, $(\boldsymbol{c}, \boldsymbol{d}) = (\mathbf{7}, \mathbf{3})$?

If $\boldsymbol{c} = \mathbf{7}$, then $\boldsymbol{d} = \mathbf{3}$.

And we have $\boldsymbol{N} = \mathbf{63}\boldsymbol{cd}$.

So the solutions are 6373, 6382, and 6391.