Examples M in Arithmetic 2

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Examples M

0.0. Suppose $\boldsymbol{A}\mathbf{78} + \boldsymbol{B}\mathbf{5}\boldsymbol{C} = \boldsymbol{DEE}\mathbf{4}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, and $\boldsymbol{E}$ indicate digits.
So for instance, $\boldsymbol{A}\mathbf{78} = \boldsymbol{A} \times \mathbf{100} + \mathbf{7} \times \mathbf{10} + \mathbf{8}$. What then is the sum, $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D} + \boldsymbol{E}$?

0.1. Assuming $\mathbf{38}\boldsymbol{A}\mathbf{7} + \mathbf{297}\boldsymbol{B} + \boldsymbol{C}\mathbf{789} = \mathbf{8}\boldsymbol{D}\mathbf{52}$, find the sum, $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D}$.

1. Using the four numbers below, make two different 2-digit integers so that the product of the two is the largest in each of the two cases below.

1.0. Using at least once each of the four numbers
1.1. Using once only each of the four numbers

2, 5, 7, and 8

2. Clark made two 2-digit integers using numbers as follows. 1, 6, 8, and 9.
Then, he flipped both integers. For instance, flipping 91, he gets 19. And then, he took the sum of the two integers, and got 87. What integers then, did he flip? That is, find the two 2-digit integers he made in the first place.

3. Suppose $\boldsymbol{A}$ and $\boldsymbol{B}$ are two different 1-digit integers. What then are $\boldsymbol{AB}$ and $\boldsymbol{A}$ if we get $\boldsymbol{AB} \times \boldsymbol{A} = \mathbf{280}$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ indicate digits? For example, $\boldsymbol{AB} = \boldsymbol{A} \times \mathbf{10} + \boldsymbol{B}$.

4. Romans in the past used numbers the way below, and some of them are still used now. They are often used for decorative purposes.

I = 1, II = 2, III = 3, IV = 4, V = 5, VI = 6, VII = 7, VIII = 8, IX = 9,

X = 10, XL = 40, L = 50, XC = 90,

C = 100, CD = 400, D = 500, CM = 900, M = 1000

For instance,

XXXIV = X + X + X + IV = 10 + 10 + 10 + 4 = 34

MCMXCII = M + CM + XC + II = 1000 + 900 + 90 + 2 = 1992

4.0 Put the Roman numbers below in integers.

XLVI, CMLV, MCMLXXXIX, and MMMDCLXXVIII

4.1. Express the following numbers in the Roman numbers.

94, 952, 1697, and 2069.

Suggestions or Solutions
To the Problems in the Examples M

0.0. Suppose $A78 + B5C = DEE4$, where A, B, C, D, and E indicate digits. So for instance, $A78 = A \times 100 + 7 \times 10 + 8$. What then is the sum, $A + B + C + D + E$?

It is 11 or 22.

If not sure of how to get it, follow the steps below.

To begin with, we can put the three numbers the way below.

$A78 = A \times 100 + 7 \times 10 + 8 \times 1$

$B5C = B \times 100 + 5 \times 10 + C \times 1$

$DEE4 = D \times 1000 + E \times 100 + E \times 10 + 4 \times 1$

So we get $A78 + B5C = (A + B) \times 100 + (7 + 5) \times 10 + (8 + C) \times 1$

$= (A + B) \times 100 + (10 + 2) \times 10 + (8 + C) \times 1$

$= (A + B + 1) \times 100 + 2 \times 10 + (8 + C) \times 1$.

Thus, we can set

$(A + B + 1) \times 100 + 2 \times 10 + (8 + C) \times 1 = D \times 1000 + E \times 100 + E \times 10 + 4 \times 1$

So first, comparing the 1's digits in both sides, we get $8 + C = 4$, which is however, not possible, because we have $0 \leq C \leq 9$, and C is an integer.

Thus, we need to get $8 + C = 14$, so we get $C = 6$. So we need to set

$(A + B + 1) \times 100 + (2 + 1) \times 10 + 4 \times 1 = D \times 1000 + E \times 100 + E \times 10 + 4 \times 1$.

So next, comparing the 10's digits in both sides, we get $E = 3$. Thus, we can set

$(A + B + 1) \times 100 + 3 \times 10 + 4 \times 1 = D \times 1000 + 3 \times 100 + 3 \times 10 + 4 \times 1$.

So next, comparing the 100's digits in both sides, we get $\boldsymbol{A + B + 1 = 3}$.

And we know $\mathbf{0} \le \boldsymbol{A} \le \mathbf{9}$, $\mathbf{0} \le \boldsymbol{B} \le \mathbf{9}$, and $\boldsymbol{A}$ and $\boldsymbol{B}$ are integers. So we can have three cases as follows. $\boldsymbol{(A, B) = (0, 2)}$, $\mathbf{(1, 1)}$, or $\mathbf{(2, 0)}$. Anyway, we get $\boldsymbol{A + B = 2}$.

Isn't it the case though, where $\boldsymbol{A}\mathbf{78}$ and $\boldsymbol{B}\mathbf{5}\boldsymbol{C}$ are 3-digit integers?
That is, why not $\mathbf{1} \le \boldsymbol{A} \le \mathbf{9}$, and $\mathbf{1} \le \boldsymbol{B} \le \mathbf{9}$, but $\mathbf{0} \le \boldsymbol{A} \le \mathbf{9}$, and $\mathbf{0} \le \boldsymbol{B} \le \mathbf{9}$?

It is not said that $\boldsymbol{A}\mathbf{78}$ and $\boldsymbol{B}\mathbf{5}\boldsymbol{C}$ are 3-digit integers. So $\boldsymbol{A}$ and $\boldsymbol{B}$ can be 0.
And it is not said either that $\boldsymbol{DEE}\mathbf{4}$ is a 4-digit integer.
So it can be the case where we get $\boldsymbol{D = 0}$.

Now, we have $\boldsymbol{A + B = 2}$.
So we can now set $\mathbf{3} \text{ x } \mathbf{100 + 3} \text{ x } \mathbf{10 + 4} \text{ x } \mathbf{1} = \boldsymbol{D} \text{ x } \mathbf{1000 + 3} \text{ x } \mathbf{100 + 3} \text{ x } \mathbf{10 + 4} \text{ x } \mathbf{1}$.
Thus, next, comparing the 1000's digits in both sides, we get $\boldsymbol{D = 0}$.

What about a case where $\boldsymbol{A + B + 1 = 13}$ though?

We can get, of course, $\boldsymbol{A + B = 12}$, too. For instance, we can get $\boldsymbol{A = 3}$, and $\boldsymbol{B = 9}$.
And in that case, we need to set

$$\mathbf{1} \text{ x } \mathbf{1000 + 3} \text{ x } \mathbf{100 + 3} \text{ x } \mathbf{10 + 4} \text{ x } \mathbf{1} = \boldsymbol{D} \text{ x } \mathbf{1000 + 3} \text{ x } \mathbf{100 + 3} \text{ x } \mathbf{10 + 4} \text{ x } \mathbf{1}.$$

So we get $\boldsymbol{D = 1}$. Thus, we get these:

$$\boldsymbol{(A + B) + C + D + E = 2 + 6 + 0 + 3 = 11}.$$

$$\boldsymbol{(A + B) + C + D + E = 12 + 6 + 1 + 3 = 22}.$$

0.1. Assuming $38A7 + 297B + C789 = 8D52$, find the sum, $A + B + C + D$.

To begin with, we can put the four numbers the way below.

$\mathbf{38A7 = 3 \times 1000 + 8 \times 100 + A \times 10 + 7}$

$\mathbf{297B = 2 \times 1000 + 9 \times 100 + 7 \times 10 + B}$

$\mathbf{C789 = C \times 1000 + 7 \times 100 + 8 \times 10 + 9}$

$\mathbf{8D52 = 8 \times 1000 + D \times 100 + 5 \times 10 + 2}$

So we get $\mathbf{38A7 + 297B + C789}$

$\mathbf{= (3 + 2 + C) \times 1000 + (8 + 9 + 7) \times 100 + (A + 7 + 8) \times 10 + (7 + B + 9)}$

$\mathbf{= (C + 5) \times 1000 + (20 + 4) \times 100 + (A + 10 + 5) \times 10 + (B + 10 + 6)}$

$\mathbf{= (C + 5 + 2) \times 1000 + (4 + 1) \times 100 + (A + 5 + 1) \times 10 + (B + 6)}$

$\mathbf{= (C + 7) \times 1000 + 5 \times 100 + (A + 6) \times 10 + (B + 6)}$

Thus, we get $\mathbf{38A7 + 297B + C789 = 8D52} \Rightarrow$
$\mathbf{(C + 7) \times 1000 + 5 \times 100 + (A + 6) \times 10 + (B + 6) = 8 \times 1000 + D \times 100 + 5 \times 10 + 2}$.

So first, comparing the 1's digits in both sides, we get $\mathbf{B + 6 = 2}$, which is however, not possible, because we have $\mathbf{0 \le B \le 9}$, and $\mathbf{B}$ is an integer.

Thus, we need to get $\mathbf{B + 6 = 12}$, so we get $\mathbf{B = 6}$. Thus, we need to set
$\mathbf{(C + 7) \times 1000 + 5 \times 100 + (A + 6 + 1) \times 10 + 2 = 8 \times 1000 + D \times 100 + 5 \times 10 + 2}$.

So next, comparing the 10's digits in both sides, we get $\mathbf{A + 6 + 1 = A + 7 = 5}$, which is however, not possible, because we have $\mathbf{0 \le A \le 9}$, and $\mathbf{A}$ is an integer.

And thus, we need to get: $\mathbf{A + 7 = 15}$, so we get: $\mathbf{A = 8}$. So we need to set:

$\mathbf{(C + 7) \times 1000 + (5 + 1) \times 100 + 5 \times 10 + 2 = 8 \times 1000 + D \times 100 + 5 \times 10 + 2}$.

So next, comparing the 100's digits in both sides, we get: $\mathbf{D = 6}$. And thus, we can set:

$\mathbf{(C + 7) \times 1000 + 6 \times 100 + 5 \times 10 + 2 = 8 \times 1000 + 6 \times 100 + 5 \times 10 + 2}$.

And next, comparing the 1000's digits in both sides, we get $\mathbf{C + 7 = 8} \Rightarrow \mathbf{C = 1}$.

Thus, we get $\mathbf{A + B + C + D = 8 + 6 + 1 + 6 = 21}$.

1. Using the four numbers below, make two different 2-digit integers so that the product of the two is the largest in each of the two cases below.

1.0. Each number can be used more than once.
1.1. Each number can be used once only.

2, 5, 7, and 8

The larger the numbers are, the larger their product gets.
Basically, multiplying two numbers, we do it digit by digit.
And the larger the digit, the larger the number.

1.0. Making first, all possible 2-digit integers, we can get

22, 25, 27, 28, 55, 52, 57, 58, 77, 72, 75, 78, 88, 82, 85, and 87.

And next, taking from the list above, the largest and the second largest, and taking the product of the two, we get the largest product. And the two are 88 and 87.

1.1. Making first, all possible pairs of two different 2-digit integers, we can get

(25 and 78), (25 and 87), (52 and 78), (52 and 87), (27 and 58), (27 and 85), (72 and 58), (72 and 85), (28 and 57), (28 and 75), (82 and 57), and (82 and 75)

And among all the pairs above, either of (72 and 85) and (82 and 75) makes the product the largest. And taking the product of the two in each pair, we get
85 x 72 = 6120, and 82 x 75 = 6150.

So the two integers we have to make are 82 and 75.

2. Clark made two 2-digit integers using numbers as follows. 1, 6, 8, and 9. Then, he flipped both integers. For instance, flipping 91, he gets 19. And then, he took the sum of the two integers, and got 87. What integers then, did he flip? That is, find the two 2-digit integers he made in the first place.

The two integers made in the first place can be **(81, 69)** or **(86, 91)**.

If not sure of how to get it, follow the steps below.

Suppose first, ***AB*** and ***CD*** are the two integers made in the first place.
Note that ***AB*** is not ***A* x *B*** but ***A* x 10 + *B***. And the same is true for ***CD***, too.

Then, ***A*** and ***C*** are numbers in the 10's digits, and ***B*** and ***D*** are numbers in the 1's digits. And of course, ***A*** is one of 1, 6, 8, and 9. And the same is true for ***B***, ***C***, and ***D***, too.

Next, flipping the two integers, he gets: ***BA*** and ***DC***.
Then, ***B*** and ***D*** are the numbers in the 10's digits, and ***A*** and ***C*** are the numbers in the 1's digits. And taking the sum of the two integers, he gets ***BA* + *DC* = 87**.

What then is ***A* + *C***?

It can be the case where ***A* + *C* = 7**, or ***A* + *C* = 17**.
And we know that ***A*** and ***C*** can be any of the numbers in the list: 1, 6, 8, and 9.
So assuming first, ***A* + *C* = 7**, we get **(*A*, *C*) = (1, 6)** or **(6, 1)**.

And next, we have ***B* + *D* = 8**, which is however, not possible, because ***B*** and ***D*** have to be from the list, but the list does not have the numbers for ***B*** and ***D***.
So it cannot be the case where ***A* + *C* = 7**.

Thus next, moving on to the case where ***A* + *C* = 17**, we get **(*A*, *C*) = (8, 9)** or **(9, 8)**.
And we can put ***BA*** and ***DC*** this way: ***BA* = *B* x 10 + *A***, and ***DC* = *D* x 10 + *C***.

Thus, we get $BA + DC = (B + D) \times 10 + A + C$.
And we know $A + C = 17$, and $BA + DC = 87$.
So we get $BA + DC = (B + D) \times 10 + 17 = (B + D + 1) \times 10 + 7 = 87$.

Thus next, comparing the 10's digits, we get $B + D + 1 = 8 \Rightarrow B + D = 7$.

And we know that B and D can be any of the numbers in the list: 1, 6, 8, and 9.
So we get $(B, D) = (1, 6)$ or $(6, 1)$.

Thus, putting threads together, we have

$(A, B, C, D) = (8, 1, 9, 6)$, $(8, 6, 9, 1)$, $(9, 1, 8, 6)$, or $(9, 6, 8, 1)$.

So assuming first, $(A, B, C, D) = (8, 1, 9, 6)$, we get $BA + DC = 18 + 69 = 87$.

Assuming next, $(A, B, C, D) = (8, 6, 9, 1)$, we get $BA + DC = 68 + 19 = 87$, too.

Assuming next, $(A, B, C, D) = (9, 1, 8, 6)$, we get $BA + DC = 19 + 68 = 87$, too.

And assuming next, $(A, B, C, D) = (9, 6, 8, 1)$, we get: $BA + DC = 69 + 18 = 87$, also.

So the two integers made in the first place seem to make either of the four pairs below:

$(AB, CD) = (81, 96)$, $(86, 91)$, $(91, 86)$, and $(96, 81)$.

However, **(81, 96)** is no other than **(96, 81)**, and **(86, 91)** is no other than **(91, 86)**.

Thus, the two integers made in the first place can be **(81, 69)** or **(86, 91)**.

3. **Suppose *A* and *B* are two different 1-digit integers. What then, are *AB* and *A* if we get: *AB* x *A* = 280, where *A* and *B* indicate digits? For example, *AB* = *A* x 10 + *B*.**

***AB* = 56**, and ***A* = 5**.

If not sure of how to get it, follow the steps below.

To begin with, we can set ***AB* x *A* = (*A* x 10 + *B*) x *A* = *A* x *A* x 10 + *A* x *B***.
So we get $\mathbf{10}\boldsymbol{A}^{\mathbf{2}} + \boldsymbol{A} \times \boldsymbol{B} = \mathbf{280}$, where $\mathbf{10}\boldsymbol{A}^{\mathbf{2}}$ is 10 times $\boldsymbol{A}^{\mathbf{2}}$, that is, $\mathbf{10} \times \boldsymbol{A}^{\mathbf{2}}$.
And we know $\mathbf{10}\boldsymbol{A}^{\mathbf{2}}$ is a multiple of 10. What then, about ***A* x *B*** in $\mathbf{10}\boldsymbol{A}^{\mathbf{2}} + \boldsymbol{A} \times \boldsymbol{B} = \mathbf{280}$?

We can say ***A* x *B*** is a multiple of 10, too, because 280 is a multiple of 10.
What then, ***A*** and ***B*** can be?

First, each of ***A*** and ***B*** can be one of 0, 1, 2, 3, … 8, and 9, since ***A*** and ***B*** are numbers used for digits. So next, we can put **(*A*, *B*)** the way below.

(*A*, *B*) = (2, 5), (5, 2), (4, 5), (5, 4), (6, 5), (5, 6), (8, 5), and (5, 8).

Thus, taking the product ***AB* x *A*** using each pair above, we get these:

(*A*, *B*) = (2, 5) ⇒ **(*AB*, *A*)** = (25, 2) ⇒ ***AB* x *A*** = 25 x 2 ≠ 280.

(*A*, *B*) = (5, 2) ⇒ **(*AB*, *A*)** = (52, 5) ⇒ ***AB* x *A*** = 52 x 5 ≠ 280.

(*A*, *B*) = (4, 5) ⇒ **(*AB*, *A*)** = (45, 4) ⇒ ***AB* x *A*** = 45 x 4 ≠ 280.

(*A*, *B*) = (5, 4) ⇒ **(*AB*, *A*)** = (54, 5) ⇒ ***AB* x *A*** = 54 x 5 ≠ 280.

(*A*, *B*) = (6, 5) ⇒ **(*AB*, *A*)** = (65, 6) ⇒ ***AB* x *A*** = 65 x 6 ≠ 280.

• **(*A*, *B*) = (5, 6) ⇒ (*AB*, *A*) = (56, 5) ⇒ *AB* x *A* = 56 x 5 = 280**.

(*A*, *B*) = (8, 5) ⇒ **(*AB*, *A*)** = (85, 8) ⇒ ***AB* x *A*** = 85 x 8 ≠ 280.

(*A*, *B*) = (5, 8) ⇒ **(*AB*, *A*)** = (58, 5) ⇒ ***AB* x *A*** = 58 x 5 ≠ 280.

Thus, we get ***AB* = 56**, and ***A* = 5**.

4. Romans in the past used numbers the way below, and some of them are still used now. They are often used for decorative purposes.

I = 1, II = 2, III = 3, IV = 4, V = 5, VI = 6, VII = 7, VIII = 8, IX = 9,
X = 10, XL = 40, L = 50, XC = 90,
C = 100, CD = 400, D = 500, CM = 900, M = 1000

For instance, XXXIV = X + X + X + IV = 10 + 10 + 10 + 4 = 34
MCMXCII = M + CM + XC + II = 1000 + 900 + 90 + 2 = 1992

4.0 Put in integers the Roman numbers below.

XLVI, **CMLV**, **MCMLXXXIX**, and **MMMDCLXXVIII**

XLVI = XL + VI = 40 + 6 = 46

CMLV = CM + L + V = 900 + 50 + 5 = 955

MCMLXXXIX
= M + CM + L + X + X + X + IX
= 1000 + 900 + 50 + 10 + 10 + 10 + 9 = 1989

MMMDCLXXVIII
= M + M + M + D + C + L + X + X + V + III
= 1000 + 1000 + 1000 + 500 + 100 + 50 + 10 + 10 + 5 + 3 = 3678

4.1. Express in the Roman numbers theses numbers: 94, 952, 1697, and 2069.

94 = 90 + 4 = XC + IV = XCIV
952 = 900 + 50 + 2 = CM + L + II = CMLII
1697 = 1000 + 900 + 50 + 40 + 7 = M + CM + L + XL + VII = MCMLXLVII
2069 = 2000 + 40 + 9 = 1000 + 1000 + 50 + 10 + 9 = M + M + L + X + IX = MMLXIX.