Examples Q in Arithmetic 2

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Examples Q

0. Using each of 3, 0, and 8 once only, how many 2-digit integers can we make?

1. Using each of 5, 1, 4, and 7 once only, how many 3-digit integers can we make?

2. Using each of 4, 6, 0, and 9 once only, how many 3-digit integers can we make?

3. Using once only each of 9 numbers 1, 2, … 8, and 9, how many 9-digit integers can we make?

4. Using once only each of 10 numbers 0, 1, 2, … 8, and 9, how many 10-digit integers can we make?

5. Using once only each of 9 numbers 1, 2, … 8, and 9, how many 6-digit integers can we make?

6. Using once only each of 10 numbers 0, 1, 2, … 8, and 9, how many 8-digit integers can we make?

7. Using once only each of 3 numbers 1, 1, and 2, how many 3-digit integers can we make?

8. Using once only each of 4 numbers 1, 1, 4, and 4, how many 4-digit integers can we make?

9. Using once only each of 5 numbers 1, 1, 4, 4, and 4, how many 5-digit integers can we make?

A. Using once only each of 7 numbers 1, 1, 3, 3, 4, 4, and 4, how many 7-digit integers can we make?

Suggestions or Solutions
To the Problems in the Examples Q

0. Using each of 3, 0, and 8 once only, how many 2-digit integers can we make?

We can make 4 different 2-digit integers.

If not sure of how to get it, follow the steps below.

In a 2-digit integer, the highest digit is 10’s digit.
And each number given has to be used only once.
And we have 0 in the list of the numbers to be used, but cannot use it for the 10’s digit.

So we have 2 choices for the 10’s digit.

We can use however, 0 for the other digit.

So we get 2 choices again, for the 1’s digit.

So we get 2 x 2 = 4 choices, and thus, can make 4 different 2-digit integers.

1. Using each of 5, 1, 4, and 7 once only, how many 3-digit integers can we make?

We can make 24 different 3-digit integers.

If not sure of how to get it, follow the steps below.

In a 3-digit integer, the highest digit is 100’s digit.
And each number given has to be used once only.

So first, we have 4 choices for the 100’s digit.

Then, we get 3 choices for the 10’s digit.

Then, we get 2 choices for the 1’s digit.

So we get 4 x 3 x 2 = 24 choices, and thus, can make 24 different 3-digit integers.

2. Using each of 4, 6, 0, and 9 once only, how many 3-digit integers can we make?

We can make 18 different 3-digit integers.

If not sure of how to get it, follow the steps below.

In a 3-digit integer, the highest digit is 100's digit.
Each number has to be used only once.
And we have 0 in the list of the numbers to be used, but cannot use it for the 100's digit.

So we have 3 choices for the 100's digit.

We can use though, 0 for any of the other lower digits.

So we get 3 choices again, for the 10's digit.

Then, we get 2 choice for 1's digit.

So we get 3 x 3 x 2 = 18 choices, and thus, can make 18 different 3-digit integers.

3. Using once only each of 9 numbers 1, 2, … 8, and 9, how many 9-digit integers can we make?

9!, that is, 362880 different 9-digit integers.

If not sure of how to get it, follow the steps below.

In a 9-digit integer, the highest digit is 10^8's digit, i.e., 100000000's digit.
And each number given has to be used once only.

So first, we have 9 choices for the 10^8's digit.
Then, we get 8 choices for the 10^7's digit.
Then, we get 7 choices for the 10^6's digit.
And so forth. So we get 9 x 8 x 7 x … x 2 x 1 choices.
And 9 x 8 x 7 x … x 2 x 1 is called 9! read as 9 factorial. And 9! = 362880.

And thus, we can make 9! different 9-digit integers, that is, 362880 9-digit integers.

4. Using once only each of 10 numbers 0, 1, 2, … 8, and 9, how many 10-digit integers can we make?

9 x 9! different 10-digit integers. *If not sure of how to get it, follow the steps below.*

In a 10-digit integer, the highest digit is 10^9's digit, i.e., 1000000000's digit.
And each number given has to be used once only.
We have 0 in the list of the numbers to be used, but cannot use it for the 10^9's digit.

So first, we have 9 choices for the 10^9's digit.

We can use though, 0 for any of the other lower digits.
So we get 9 choices again, for the 10^8's digit.

Then, we get 8 choices for the 10^7's digit.

Then, we get 7 choices for the 10^6's digit.
So we get 9 x 9 x 8 x 7 x … x 2 x 1 = 9 x 9! choices.

And thus, we can make 9 x 9! different 10-digit integers.

5. Using once only each of 9 numbers 1, 2, … 8, and 9, how many 6-digit integers can we make?

9!/3!, that is, 60480 different 6-digit integers.

If not sure of how to get it, follow the steps below.

In a 6-digit integer, the highest digit is 10^5's digit, i.e., 100000's digit.

And each number given has to be used once only.

So first, we have 9 choices for the 10^5's digit.
Then, we get 8 choices for the 10^4's digit.
Then, we get 7 choices for the 10^3's digit. And so forth.
So we get 9 x 8 x 7 x 6 x 5 x 4 = 9!/3! choices. And 9!/3! = 362880/6 = 60480.

And thus, we can make 9!/3!, that is, 60480 different 6-digit integers.

6. Using once only each of 10 numbers 0, 1, 2, … 8, and 9, how many 8-digit integers can we make?

9 x 9!/2, that is, 1632960 different 8-digit integers.

If not sure of how to get it, follow the steps below.

In an 8-digit integer, the highest digit is 10^7's digit, i.e., 10000000's digit.
And each number given has to be used once only.
We have 0 in the list of the numbers to be used, but cannot use it for the 10^7's digit.

So first, we have 9 choices for the 10^7's digit.

We can use though, 0 for any of the other lower digits.
So we get 9 choices again, for the 10^6's digit.

Then, we get 8 choices for the 10^5's digit.

Then, we get 7 choices for the 10^4's digit.

So we get 9 x 9 x 8 x 7 x 6 x 5 x 4 x 3 = 9 x 9!/2! = 9 x 9!/2 = 9 x 362880/2

= 9 x 181440 = 1632960 choices.

And thus, we can make 9 x 9!/2 different 8-digit integers, that is, 1632960 different 8-digit integers.

7. Using once only each of 3 numbers 1, 1, and 2, how many 3-digit integers can we make?

We can make 3 different 3-digit integers.

If not sure of how to get it, follow the steps below.

Assuming for instance, we use once only each of three different numbers 1, 2, and 3 to make a 3-digit integer, we get 3 x 2 x 1 = 3! = 6 choices, and thus, can make 6 different 3-digit integers. And all the six integers are as follows. 123, 132, 213, 231, 312, 321.

Now, replacing all the 3s above with 1s, we get 121, 112, 211, 211, 112, 121.

Then, we can see that there are only three different 3-digit integers. That is to say that each different integer appears twice.

So using once only each of 3 numbers 1, 1, and 2, we can make 3 different 3-digit integers, and the integers are 112, 121, 211.

And thus, assuming we get ***x*** different 3-digit integers if we use once only each of 3 numbers, two of which are the same, we get ***x*** x **2! = *y***, where ***y*** is the number of all the 3-digit integers we can make using once only each of 3 different nonzero numbers.

And we know ***y* = 3!**.
So we get ***x*** x **2! = 3!**.

Thus, we get ***x* = 3!/2!**, which is 3.

8. Using once only each of 4 numbers 1, 1, 4, and 4, how many 4-digit integers can we make?

We can make 6 different 4-digit integers.

If not sure of how to get it, follow the steps below.

Assuming for instance, we use once only each of 4 different numbers 1, 2, 3, and 4 to make a 4-digit integer, we get 4 x 3 x 2 x 1 = 4! = 24 choices, and thus, can make 24 different 4-digit integers. And all the 24 integers are as follows.

1234, 1243, 1324, 1342, 1423, 1432.
2134, 2143, 2314, 2341, 2413, 2431.
3124, 3142, 3214, 3241, 3412, 3421.
4123, 4132, 4213, 4231, 4312, 4321.

Now, replacing all the 2s above with 1s, we get

1134, **1143**, **1314**, **1341**, **1413**, **1431**.
1134, 1143, 1314, 1341, 1413, 1431.
3114, **3141**, 3114, 3141, **3411**, 3411.
4113, **4131**, 4113, 4131, **4311**, 4311.

Then, we can see that there are only 12 different 4-digit integers. That is to say that each different integer appears twice.
So using once only each of 4 numbers 1, 1, 3, and 4, we can make 12 different 4-digit integers, and the integers are as follows.

1134, 1143, 1314, 1341, 1413, 1431.
3114, 3141, 3411, 4113, 4131, 4311.

Now, replacing all the 3s above with 4s, we get

1144, 1144, **1414**, **1441**, 1414, 1441.
4114, **4141**, **4411**, 4114, 4141, 4411.

Then, we can see that there are only 6 different 4-digit integers. That is to say that each different integer appears twice.

So using once only each of 4 numbers 1, 1, 4 and 4, we can make 6 different 4-digit integers, and the integers are as follows. 1144, 1414, 1441, 4411, 4141, 4114,

So what pattern can you see?

Suppose we get $\boldsymbol{x}$ different 4-digit integers if we use once only each of 4 nonzero numbers, and two of the four are the same as 1, 1, 3, and 4.

Then, we get $\boldsymbol{x}$ x **2! =** $\boldsymbol{z}$, where $\boldsymbol{z}$ is the number of all 4-digit integers we can make using once only each of 4 different nonzero numbers.

And we know $\boldsymbol{z}$ **= 4!**. So we get $\boldsymbol{x}$ x **2! = 4!**. Thus, we get $\boldsymbol{x}$ **= 4!/2!**, which is 12.

Suppose this time, we get $\boldsymbol{y}$ different 4-digit integers if we use once only each of 4 nonzero numbers, two of which are the same as each other, and the other two are the same as each other, too.

Then, we get $\boldsymbol{y}$ x **2! =** $\boldsymbol{x}$, where $\boldsymbol{x}$ is the number of all 4-digit integers we can make using once only each of 4 nonzero numbers, two of which are the same.

And we know $\boldsymbol{x}$ **= 4!/2!**. So we get $\boldsymbol{y}$ x **2! = 4!/2!**.

Thus, we get $\boldsymbol{y}$ **= (4!/2!)/2! = 4!/2!/2! = 4!/(2!** x **2!)**, which is 6.

9. Using once only each of 5 numbers 1, 1, 4, 4, and 4, how many 5-digit integers can we make?

We can make 10 different 5-digit integers.

If not sure of how to get it, follow the steps below.

Suppose we get $\boldsymbol{x}$ different 5-digit integers if we use once only each of 5 nonzero numbers, two of which are the same. Then, we get $\boldsymbol{x}$ x $\mathbf{2! = \boldsymbol{z}}$, where $\boldsymbol{z}$ is the number of all 5-digit integers we can make using once only each of 5 different nonzero numbers.

And we know $\boldsymbol{z} = \mathbf{5!}$. So we get $\boldsymbol{x}$ x $\mathbf{2! = 5!}$. Thus, we get $\boldsymbol{x} = \mathbf{5!/2!}$, which is 60.

Suppose next, we get $\boldsymbol{y}$ different 5-digit integers if we use once only each of 5 nonzero numbers, two of which are the same as each other, and the other three are the same as each other, too. Then, we get $\boldsymbol{y}$ x $\mathbf{3! = \boldsymbol{x}}$, where $\boldsymbol{x}$ is the number of all 5-digit integers we can make using once only each of 5 nonzero numbers, two of which are the same.

And we know $\boldsymbol{x} = \mathbf{5!/2!}$. So we get $\boldsymbol{y}$ x $\mathbf{3! = 5!/2!}$.

Thus, we get $\boldsymbol{y} = \mathbf{(5!/2!)/3! = 5!/2!/3! = 5!/(2!}$ x $\mathbf{3!)}$, which is 10.

A. Using once only each of 7 numbers 1, 1, 3, 3, 4, 4, and 4, how many 7-digit integers can we make?

We can make 7!/(2! x 2! x 3!), that is, 210 different 7-digit integers.

If not sure of how to get it, follow the steps below.

Suppose we get $\boldsymbol{x}$ different 7-digit integers if we use once only each of 7 nonzero numbers, two of which are the same.

Then, we get $\boldsymbol{x}$ x $\mathbf{2! = \boldsymbol{z}}$, where $\boldsymbol{z}$ is the number of all 7-digit integers we can make using once only each of 7 different nonzero numbers.

And we know $\boldsymbol{z} = \mathbf{7!}$. So we get $\boldsymbol{x}$ x $\mathbf{2! = 7!}$. Thus, we get $\boldsymbol{x} = \mathbf{7!/2!}$, which is 2520.

Suppose next, we get y different 7-digit integers if we use once only each of 7 nonzero numbers, two of which are the same as each other, and two of the other five are the same as each other, too.

Then, we get $y \times 2! = x$, where x is the number of all 7-digit integers we can make using once only each of 7 nonzero numbers, two of which are the same.

And we know $x = 7!/2!$. So we get $y \times 2! = 7!/2!$.

Thus, we get $y = (7!/2!)/2! = 7!/2!/2! = 7!/(2! \times 2!)$, which is 1260.

Suppose this time, we get w different 7-digit integers if we use once only each of 7 nonzero numbers, two of which are the same as each other, two of the other five are the same as each other, and the other three are the same as each other, too.

Then, we get $w \times 3! = y$, where y is the number of all 7-digit integers we can make using once only each of 7 nonzero numbers, two of which are the same, and two of the other five are the same as each other, too.

And we know $y = 7!/(2! \times 2!)$. So we get $w \times 3! = 7!/(2! \times 2!)$.

Thus, we get $w = \{7!/(2! \times 2!)\}/3! = 7!/(2! \times 2! \times 3!)$, which is 210.

What if we use n numbers, u of which are the same, v of which are the same, w of which are the same, x of which are the same, and y of which are the same?

If we make n-digit integers, we can make $n!/(u! \times v! \times w! \times x! \times y!)$ different n-digit integers.