Examples R in Arithmetic 2

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Examples R

0. Find all the integers that satisfy all the statements below.

   A. They are 2-digit integers positive.
   B. 4 and 6 can divide the integers.
   C. In each of the integers, the one’s digit is twice the ten’s digit.

1. Find all the integers that satisfy all below.

   A. They are 3-digit integers positive.
   B. 3, 4, 5 and 6 can divide the integers.
   C. In each of the integers, the ten’s digit is twice the hundred’s digit.

2. Find the integer described as follows.

   A. It is a 2-digit integer positive.
   B. Adding together the two numbers in both digits, we get an 8.
   C. It is an odd integer, and is closest to 40.

3. Find the date of birth described as follows.

   A. The date is a 2-digit odd integer, and the difference between the 2 digits is 7.
   B. The 100's digit of the year is the same as one of the two digits in the date.
   C. The 1's digit in the month is 0.
   D. In the year, the sum of the numbers in the 1000's and the 100's digits is equal to the sum of the numbers in the other digits.
   E. And in the year, the 1's digit is less than the 10's digit, and the year is made of odd integers.

Suggestions or Solutions
To the Problems in the Examples R

0. Find all the integers that satisfy all the statements below.

A. They are 2-digit integers positive.
B. 4 and 6 can divide the integers.
C. In each of the integers, the one's digit is twice the ten's digit.

12, 24, 36, or 48 *If not sure of how to get it, follow the steps below.*

• From the statement **A**, we can say this:
Assuming $\boldsymbol{N}$ is such an integer, we can set $\boldsymbol{N = 10a + b}$, where $\boldsymbol{1 \le a \le 9}$, $\boldsymbol{0 \le b \le 9}$, and $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers. Why not $\boldsymbol{0 \le a \le 9}$ but $\boldsymbol{1 \le a \le 9}$ though?

That's because $\boldsymbol{N}$ is a 2-digit integer positive. If $\boldsymbol{a = 0}$, $\boldsymbol{N}$ cannot be a 2-digit integer.

• From the statement **B**, we can say this:
$\boldsymbol{N}$ is a multiple of a number. What then, is the number?

We have $4 = 2 \times 2 = 2^2$, and $6 = 2 \times 3$.
So the number is $2^2 \times 3 = 12$, which is the least common multiple of 4 and 6.
And the least common multiple is often called the **LCM**, which is the acronym.
And thus, we can set $\boldsymbol{N = 12n}$ where $\boldsymbol{n}$ is an integer $\boldsymbol{\ge 1}$.

• From the statement **C**, we can say this:
We know $\boldsymbol{a}$ is the 10's digit, and $\boldsymbol{b}$ is the 1's digit. So we get $\boldsymbol{b = 2a}$.

So putting threads together,

To begin with, in the statement **B**, we have $\boldsymbol{N = 12n}$ for $\boldsymbol{n}$ = 1, 2, 3, 4 …

From **A** though, we get $\boldsymbol{10 \le N \le 99}$.

So we get $\boldsymbol{N}$ = 12, 24, 36, 48, 60, 72, 84, 96.

And in the **C**, we have $\boldsymbol{b = 2a}$ where $\boldsymbol{a}$ is the 10's digit, and $\boldsymbol{b}$ is the 1's digit.

And thus, we get $\boldsymbol{N}$ = 12, 24, 36, or 48.

1. Find all the integers that satisfy all below.

- **A. They are 3-digit integers positive.**
- **B. 3, 4, 5 and 6 can divide the integers.**
- **C. In each of the integers, the ten's digit is twice the hundred's digit.**

120, 240, 360, or 480	*If not sure of how to get it, follow the steps below.*

• From the statement **A**, we can say this:

Assuming $\boldsymbol{N}$ is such an integer, we can set $\boldsymbol{N = 100a + 10b + c}$, where $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are integers, and $\boldsymbol{1 \le a \le 9}$, $\boldsymbol{0 \le b \le 9}$, and $\boldsymbol{0 \le c \le 9}$. Why $\boldsymbol{1 \le a \le 9}$, though?

That's because $\boldsymbol{N}$ is a 3-digit integer positive.

• From the statement **B**, we can say this:

$\boldsymbol{N}$ is a multiple of a number. What then is the number?

It is the least common multiple of all the divisors, 3, 4, 5 and 6.

That is, it is the **LCM** of 3, 4, 5 and 6. What then is the **LCM**?

We have $4 = 2 \times 2 = 2^2$, and $6 = 2 \times 3$. So the **LCM** is $2^2 \times 3 \times 5 = 60$.

And thus, we can set $\boldsymbol{N = 60n}$ where $\boldsymbol{n}$ is an integer $\geq$ **2**.

Why not $\boldsymbol{n}$ is an integer $\geq$ **1** but $\boldsymbol{n}$ is an integer $\geq$ **2**, though?

It's because $\boldsymbol{N}$ is a 3-digit integer.

• From the statement **C**, we can say this:

We know $\boldsymbol{a}$ is the 100's digit, and $\boldsymbol{b}$ is the 10's digit. So we get $\boldsymbol{b = 2a}$.

So putting threads together,

To begin with, in the statement **B**, we have $\boldsymbol{N = 60n}$ for $\boldsymbol{n} = 2, 3, 4 \ldots$

From **A** though, we get $\boldsymbol{100 \leq N \leq 999}$.
So we get $\boldsymbol{N}$ = 120, 180, 240, 300, 360, … 900, 960.

And in the **C**, we have $\boldsymbol{b = 2a}$ where $\boldsymbol{a}$ is the 100's digit, and $\boldsymbol{b}$ is the 10's digit.

And thus, we get $\boldsymbol{N}$ = 120, 240, 360, or 480.

2. Find the integer described as follows.

A. It is a 2-digit integer positive.
B. Adding together the two numbers in both digits, we get an 8.
C. It is an odd integer, and is closest to 40.

It is 35. *If not sure of how to get it, follow the steps below.*

• From the statement **A**, we can say this:
Assuming $\boldsymbol{N}$ is the integer, we can set $\boldsymbol{N = 10a + b}$, where $\mathbf{1} \leq \boldsymbol{a} \leq \mathbf{9}$, $\mathbf{0} \leq \boldsymbol{b} \leq \mathbf{9}$, and $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers.

• From the statement **B**, we can say this:
The two numbers in both digits are $\boldsymbol{a}$ and $\boldsymbol{b}$. So we get $\boldsymbol{a + b = 8}$.

And we have $\mathbf{1} \leq \boldsymbol{a} \leq \mathbf{9}$, and $\mathbf{0} \leq \boldsymbol{b} \leq \mathbf{9}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers.

So we can set $\boldsymbol{(a, b)}$ = (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1).

• And the statement **C** says $\boldsymbol{N}$ is an odd integer.

And in **A**, we have $\boldsymbol{N = 10a + b}$. So $\boldsymbol{b}$ is an odd integer.

And we have $\mathbf{0} \leq \boldsymbol{b} \leq \mathbf{9}$. So we can set $\boldsymbol{b}$ = 1, 3, 5, 7, 9.

And in **B**, we have $\boldsymbol{(a, b)}$ = (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1).

So we can reduce it this way: $\boldsymbol{(a, b)}$ = (1, 7), (3, 5), (5, 3), (7, 1).

And thus, $\boldsymbol{N}$ is one of the integers as follows. 17, 35, 53, 71.

• And also, the statement **C** says $\boldsymbol{N}$ is closest to 40.

So we can see this: $\boldsymbol{N = 35}$.

3. **Find the date of birth described as follows.**

A. **The date is a 2-digit odd integer, and the difference between the 2 digits is 7.**
B. **The 100's digit of the year is the same as one of the two digits in the date.**
C. **The 1's digit in the month is 0.**
D. **In the year, the sum of the numbers in the 1000's and the 100's digits is equal to the sum of the numbers in the other digits.**
E. **And in the year, the 1's digit is less than the 10's digit, and the year is made of odd integers.**

Oct 29, 1991 or 1973. *If not sure of how to get it, follow the steps below.*

To begin with, a date of birth is composed of the date, the month and the year.

A. The date is a 2-digit odd integer, and the difference between the 2 digits is 7.

First, assuming the date is $\boldsymbol{d}$, we can set $\mathbf{1 \leq \boldsymbol{d} \leq 31}$.
Next, the date is a 2-digit odd integer. So we can set $\boldsymbol{d}$ = 11, 13, 15, … or 31.
And also, we can set it this way: $\boldsymbol{d} \mathbf{= 2\boldsymbol{k} + 11}$, where $\boldsymbol{k}$ = 0, 1, 2, … 10.
And we can put it this way, too: $\boldsymbol{d} \mathbf{= 2\boldsymbol{k} + 1}$, where $\boldsymbol{k}$ = 5, 6, 7, … 15.

Next, the difference between the 2 digits is 7. So we get $\boldsymbol{d} \mathbf{= 29}$.
And therefore, the date must be 29.

B. The 100's digit of the year is the same as one of the two digits in the date.

So assuming the year is $\boldsymbol{wxyz}$, where $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ are digits, we get $\boldsymbol{x}$ = 2 or 9.

C. The 1's digit in the month is 0.

First, assuming the month is $\boldsymbol{m}$, we can set $\mathbf{1 \leq \boldsymbol{m} \leq 12}$.
And the 1’s digit in the month is 0. So we get $\boldsymbol{m} \mathbf{= 10}$, which is the only choice.
And therefore, the month must be October.

D. In the year, the sum of the numbers in the 1000's and the 100's digits is equal to the sum of the numbers in the other digits.

In $\boldsymbol{B}$, we assume that the year is $\boldsymbol{wxyz}$, where $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ are digits.
So $\boldsymbol{w}$ is the 1000’s digit, and $\boldsymbol{x}$ is the 100’s digit.
And thus, we get $\boldsymbol{w + x = y + z}$. And in $\boldsymbol{B}$, we have $\boldsymbol{x} = 2$ or 9.
So we get $\boldsymbol{w + 2 = y + z}$, or $\boldsymbol{w + 9 = y + z}$.

E. And in the year, the 1's digit is less than the 10's digit, and the year is made of odd integers.

Now, the year is made of odd integers.
So of the two cases where $\boldsymbol{w + 2 = y + z}$, and $\boldsymbol{w + 9 = y + z}$, we want to use $\boldsymbol{w + 9 = y + z}$.

Next, in the year, the 1's digit is less than the 10's digit.
We know $\boldsymbol{y}$ is the 10’s digit, $\boldsymbol{z}$ is the 1’s digit. So we get $\boldsymbol{y} > \boldsymbol{z}$.

And we know the year is made of odd integers. So we get

$\boldsymbol{y = 9 \Rightarrow z = 1, 3, 5, 7}$
$\boldsymbol{y = 7 \Rightarrow z = 1, 3, 5}$
$\boldsymbol{y = 5 \Rightarrow z = 1, 3}$
$\boldsymbol{y = 3 \Rightarrow z = 1}$
$\boldsymbol{y \neq 1}$

And we know $\boldsymbol{w}$ is the 1000’s digit, and is an odd integer.
And using common sense, we get $\boldsymbol{w = 1}$. And we have $\boldsymbol{w + 9 = y + z}$.
So we get $\boldsymbol{w = 1 \Rightarrow w + 9 = 10 \Rightarrow y + z = 10}$.
And we know that $\boldsymbol{y = 9 \Rightarrow z = 1}$, $\boldsymbol{3}$, $\boldsymbol{5}$, $\boldsymbol{7}$, and that $\boldsymbol{y = 7 \Rightarrow z = 1}$, $\boldsymbol{3}$, $\boldsymbol{5}$.
So we get $\boldsymbol{(y, z)} = (9, 1)$ or $(7, 3)$.

And thus, the date of birth is Oct 29, 1991 or 1973.