Examples U in Arithmetic 2

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Examples U

Factorizing (factoring) an integer, we find first, all its divisors that are prime numbers, called primes, for short, and then, take the product of all the primes. And we call all those primes prime factors. So factorizing an integer, we put the integer in the product of all its prime factors. And normally, if a same prime repeats, we put it in a power, that is, the power of the prime. So for instance, 8 is factorized to $2 \cdot 2 \cdot 2$, which is therefore, put in 2^3, which is a power of 2, and is called 2 to the third power, or simply, 2 cubed.

0. Factorize the integers below.

0. 100

1. 2205

2. 126126

3. 42636

4. 525505955

5. 65816751

6. 7807877640

7. 2380749780

8. 980132580713

9. 333028705229

The example below is for those students who took courses on powers and radicals.

1. Put the numbers below in terms of powers of primes.

We have identities (or formulas) on powers, and one of them is as follows.
Assuming $\boldsymbol{n}$, $\boldsymbol{p}$, and $\boldsymbol{q}$ are real numbers, we can get $\boldsymbol{n^{pq} = (n^p)^q = (n^q)^p}$.

For instance, $0.8 = 2^3 10^{-1} = 2^3(2 \cdot 5)^{-1} = 2^{3-1}5^{-1} = 2^2 5^{-1}$.

0. 0.032

1. 12.5

2. $\sqrt{\mathbf{22.05}}$

3. –1.26126

4. $\sqrt[3]{\mathbf{426.36}}$

5. $\sqrt{\mathbf{0.525505955}}$

6. $\sqrt[4]{\mathbf{65.816751}}$

7. $\sqrt[5]{\mathbf{-98013.2580713}}$

2. Put in ascending order, the numbers as follows. 3^{55}, 4^{44}, and 5^{33}.

Suggestions or Solutions
To the Problems in the Examples U

Factorizing (factoring) an integer, we find first, all its divisors that are prime numbers, called primes, for short, and then, take the product of all the primes. And we call all those primes prime factors. So factorizing an integer, we put the integer in the product of all its prime factors. And normally, if a same prime repeats, we put it in a power, that is, the power of the prime. So for instance, 8 is factorized to 2·2·2, which is therefore, put in 2^3, which is a power of 2, and is called 2 to the third power, or simply, 2 cubed.

0. **100 =** $2\cdot 50 = 2\cdot 2\cdot 25 = 2\cdot 2\cdot 5\cdot 5 = 2^2\cdot 5^2 = 2^2 5^2$.

1. **2205 =** $5\cdot 441 = 5\cdot 3\cdot 147 = 5\cdot 3\cdot 3\cdot 49 = 3^2 5\cdot 7^2$

2. **126126 =** $2\cdot 63063 = 2\cdot 3\cdot 21021 = 2\cdot 3^2 7007 = 2\cdot 3^2 7\cdot 1001 = 2\cdot 3^2 7^2 143 = 2\cdot 3^2 7^2 11\cdot 13$

3. **42636 =** $2\cdot 21318 = 2^2 10659 = 2^2 3\cdot 3553 = 2^2 3\cdot 11\cdot 323 = 2^2 3\cdot 11\cdot 17\cdot 19$

4. **525505955 =** $5\cdot 105101191 = 5\cdot 13\cdot 8084707 = 5\cdot 13\cdot 17\cdot 475571 = 5\cdot 13\cdot 17\cdot 23\cdot 20677$

$= 5\cdot 13\cdot 17\cdot 23^2 899 = 5\cdot 13\cdot 17\cdot 23^2 29\cdot 31$.

5. **65816751 =** $3\cdot 21938917 = 3\cdot 7\cdot 3134131 = 3\cdot 7^2 447733 = 3\cdot 7^2 11\cdot 40703$

$= 3\cdot 7^2 11\cdot 13\cdot 3131 = 3\cdot 7^2 11\cdot 13\cdot 31\cdot 101$.

6. **7807877640 =** $2^2 1951969410 = 2^2 10\cdot 195196941 = 2^3 5\cdot 195196941$

$= 2^3 5\cdot 3\cdot 65065647 = 2^3 5\cdot 3^2 21688549 = 2^3 5\cdot 3^2 17\cdot 1275797 = 2^3 5\cdot 3^2 17\cdot 29\cdot 43993$

$= 2^3 5\cdot 3^2 17\cdot 29^2 1517 = 2^3 5\cdot 3^2 17\cdot 29^2 37\cdot 41$

7. **2380749780 =** $10\cdot 238074978 = 10\cdot 2\cdot 119038489 = 2^2 5\cdot 3\cdot 39679163$

$= 2^2 3\cdot 5\cdot 19\cdot 2088377 = 2^2 3\cdot 5\cdot 19\cdot 23\cdot 90799 = 2^2 3\cdot 5\cdot 19\cdot 23\cdot 29\cdot 3131 = 2^2 3\cdot 5\cdot 19\cdot 23\cdot 29\cdot 31\cdot 101$

8. **980132580713 =** $11\cdot 89102961883 = 11\cdot 13\cdot 6854073991 = 11\cdot 13\cdot 17\cdot 403180823$

$= 11\cdot 13\cdot 17^2 23716519 = 11\cdot 13\cdot 17^2 23\cdot 1031153 = 11\cdot 13\cdot 17^2 23\cdot 29\cdot 35557$

$= 11\cdot 13\cdot 17^2 23\cdot 29\cdot 31\cdot 1147 = 11\cdot 13\cdot 17^2 23\cdot 29\cdot 31^2 37$

9. **333028705229 =** $11\cdot 30275336839 = 11^2 2752303349 = 11^2 17\cdot 161900197$

$= 11^2 17^2 9523541 = 11^2 17^2 19\cdot 501239 = 11^2 17^2 19^2 26381 = 11^2 17^2 19^2 23\cdot 1147$

$= 11^2 17^2 19^2 23\cdot 31\cdot 37$

1. Put the numbers below in terms of powers of primes.

Note:

Assuming $\boldsymbol{n}$, $\boldsymbol{p}$, and $\boldsymbol{q}$ are real numbers, we can get $\boldsymbol{n^{pq} = (n^p)^q = (n^q)^p}$.

For instance, $\mathbf{0.8 = 2^3 10^{-1} = 2^3(2 \cdot 5)^{-1} = 2^{3-1}5^{-1} = 2^2 5^{-1}}$.

0. 0.032 = $32 \cdot 10^{-2} = 4 \cdot 8 \cdot 10^{-2} = 2^2 2^3 10^{-2} = 2^5 10^{-2} = 2^5(2 \cdot 5)^{-2} = 2^5 2^{-2} 5^{-2} = 2^{5-2}5^{-2} = 2^3 5^{-2}$

$(= 2^3/5^2)$

1. 12.5

$125 = 5 \cdot 25 = 5^3$, and thus, $12.5 = 5^3 10^{-1} = 5^3(2 \cdot 5)^{-1} = 5^3 2^{-1} 5^{-1} = 5^{3-1}2^{-1} = 5^2 2^{-1}$ $(= 5^2/2)$

2. $\boldsymbol{\sqrt{22.05}}$

$22.05 = 2205 \cdot 10^{-2}$, and $2205 = 3^2 5 \cdot 7^2$.

So $22.05 = 3^2 5 \cdot 7^2 10^{-2} = 3^2 5 \cdot 7^2 (2 \cdot 5)^{-2} = 3^2 5 \cdot 7^2 2^{-2} 5^{-2} = 2^{-2} 3^2 5^{1-2} 7^2 = 2^{-2} 3^2 5^{-1} 7^2$.

And thus, $\boldsymbol{\sqrt{22.05}} =$ $(2^{-2}3^2 5^{-1} 7^2)^{1/2} = 2^{-1} 3 \cdot 5^{-1/2} 7$ $(= 3 \cdot 7/(2 \cdot 5^{1/2}))$

3. –1.26126

$1.26126 = 126126 \cdot 10^{-5}$, and $126126 = 2 \cdot 3^2 7^2 11 \cdot 13$. So we get

$1.26126 = 2 \cdot 3^2 7^2 11 \cdot 13 \cdot 10^{-5} = 2 \cdot 3^2 7^2 11 \cdot 13(2 \cdot 5)^{-5} = 2^{1-5} 3^2 5^{-5} 7^2 11 \cdot 13 = 2^{-4} 3^2 5^{-5} 7^2 11 \cdot 13$

$= 3^2 7^2 11 \cdot 13 / (2^4 5^5)$. And thus, $-1.26126 = -3^2 7^2 11 \cdot 13 / (2^4 5^5)$.

4. $\sqrt[3]{\mathbf{426.36}}$ To begin with, $426.36 = 42636 \cdot 10^{-2}$, and $42636 = 2^2 3 \cdot 11 \cdot 17 \cdot 19$.

So $426.36 = 2^2 3 \cdot 11 \cdot 17 \cdot 19 \cdot 10^{-2} = 2^2 3 \cdot 11 \cdot 17 \cdot 19 (2 \cdot 5)^{-2} = 2^{2-2} 3 \cdot 5^{-2} 11 \cdot 17 \cdot 19$

$= 2^0 3 \cdot 5^{-2} 11 \cdot 17 \cdot 19 = 1 \cdot 3 \cdot 5^{-2} 11 \cdot 17 \cdot 19 = 3 \cdot 5^{-2} 11 \cdot 17 \cdot 19$.

And thus, $\sqrt[3]{\mathbf{426.36}}\ \mathbf{=}\ (3 \cdot 5^{-2} 11 \cdot 17 \cdot 19)^{1/3} = 3^{1/3} 5^{-2/3} 11^{1/3} 17^{1/3} 19^{1/3} = 3^{1/3} 11^{1/3} 17^{1/3} 19^{1/3} / 5^{2/3}$.

5. $\sqrt{\mathbf{0.525505955}}$

$525505955 = 5 \cdot 13 \cdot 17 \cdot 23^2 29 \cdot 31$.

So $0.525505955 = 5 \cdot 13 \cdot 17 \cdot 23^2 29 \cdot 31 \cdot 10^{-1} = 2^{-1} 5^{1-1} 13 \cdot 17 \cdot 23^2 29 \cdot 31 = 2^{-1} \cdot 1 \cdot 13 \cdot 17 \cdot 23^2 29 \cdot 31$

$= 2^{-1} 13 \cdot 17 \cdot 23^2 29 \cdot 31$.

And thus, $\sqrt{\mathbf{0.525505955}}\ \mathbf{=}\ 0.525505955^{1/2} = (2^{-1} 13 \cdot 17 \cdot 23^2 29 \cdot 31)^{1/2}$

$= 2^{-1/2} 13^{1/2} 17^{1/2} 23^{2/2} 29^{1/2} 31^{1/2} = 2^{-1/2} 13^{1/2} 17^{1/2} 23^1 29^{1/2} 31^{1/2} = 13^{1/2} 17^{1/2} 23 \cdot 29^{1/2} 31^{1/2} / 2^{1/2}$

6. $\sqrt[4]{\mathbf{65.816751}}$

$65816751 = 3 \cdot 7^2 11 \cdot 13 \cdot 31 \cdot 101$.

So $65.816751 = 3 \cdot 7^2 11 \cdot 13 \cdot 31 \cdot 101 \cdot 10^{-2} = 2^{-2} 3 \cdot 5^{-2} 7^2 11 \cdot 13 \cdot 31 \cdot 101$.

And thus, $\sqrt[4]{\mathbf{65.816751}}\ \mathbf{=}\ 65.816751^{1/4} = (2^{-2} 3 \cdot 5^{-2} 7^2 11 \cdot 13 \cdot 31 \cdot 101)^{1/4}$

$= 2^{-2/4} 3^{1/4} 5^{-2/4} 7^{2/4} 11^{1/4} 13^{1/4} 31^{1/4} 101^{1/4} = 2^{-1/2} 3^{1/4} 5^{-1/2} 7^{1/2} 11^{1/4} 13^{1/4} 31^{1/4} 101^{1/4}$

$= 3^{1/4} 7^{1/2} 11^{1/4} 13^{1/4} 31^{1/4} 101^{1/4} / (2^{1/2} 5^{1/2})$

7. $\sqrt[5]{\mathbf{-98013.2580713}}$

$980132580713 = 11\cdot13\cdot17^2 23\cdot29\cdot31^2 37.$

So $98013.2580713 = 11\cdot13\cdot17^2 23\cdot29\cdot31^2 37\cdot10^{-7} = 2^{-7}5^{-7}11\cdot13\cdot17^2 23\cdot29\cdot31^2 37.$

Thus, we get $98013.2580713^{1/5} = (2^{-7}5^{-7}11\cdot13\cdot17^2 23\cdot29\cdot31^2 37)^{1/5}$

$= 2^{-7/5}5^{-7/5}11^{1/5}13^{1/5}17^{2/5}23^{1/5}29^{1/5}31^{2/5}37^{1/5} = 11^{1/5}13^{1/5}17^{2/5}23^{1/5}29^{1/5}31^{2/5}37^{1/5} / (2^{7/5}5^{7/5})$

And thus, $\sqrt[5]{\mathbf{-98013.2580713}}\ \mathbf{=} -98013.2580713^{1/5}$

$= -11^{1/5}13^{1/5}17^{2/5}23^{1/5}29^{1/5}31^{2/5}37^{1/5} / 2^{7/5}5^{7/5}.$

And of course, we can put it the way below, too.

$\sqrt[5]{\mathbf{-98013.2580713}}\ \mathbf{=} -98013.2580713^{1/5} = -\{11\cdot13\cdot17^2 23\cdot29\cdot31^2 37 / (2^7 5^7)\}^{1/5}$ or

$-(11\cdot13\cdot17^2 23\cdot29\cdot31^2 37)^{1/5} / (2^7 5^7)^{1/5}.$

2. Put in ascending order, the numbers as follows. $\mathbf{3^{55}}$, $\mathbf{4^{44}}$, and $\mathbf{5^{33}}$.

We have $55 = 11\cdot5$, $44 = 11\cdot4$, and $33 = 11\cdot3$.

So we can get $3^{55} = (3^5)^{11} = 243^{11}$, $4^{44} = (4^4)^{11} = 256^{11}$, and $5^{33} = (5^3)^{11} = 125^{11}$.

And we know $125 < 243 < 256$.

Thus, we get $5^{33} < 3^{55} < 4^{44}$.