Examples V in Arithmetic 2

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Examples V

Note that doing multiplications, we often use a dot · instead of x.
For instance, instead of writing 3 x 5, we just write $3{\cdot}5$. That is, $3 \times 5 = 3{\cdot}5$.
And also, doing multiplications using powers as 2^3, we often omit the dot, too, if it is not ambiguous. For instance, we have $3^2 \times 5^2 = 3^2 \cdot 5^2 = 3^2 5^2$.

Find the GCD (called GCF or GCM, too) and LCM in each of the cases below.

0. 2700 and 2250
1. 25 and 12
2. 7 and 19
3. 8, 9, and 12
4. 24, 45, and 245
5. 60, 75, and 490
6. 4620, 825, and 6930
7. 45045, 225225, and 10395
8. 42042, 315315, 72765, and 231231
9. 58905, 1576575, 20790, 13230, and 114345

Suggestions or Solutions
To the Problems in the Examples V

GCD is the acronym of Greatest Common Divisor, and LCM is the acronym of Least Common Multiple. So the GCD of a set of integers is the largest integer that can divide all the integers in the set, and the LCM is the smallest positive integer that can be divided by all the integers in the set, that is, the smallest multiple common to all the integers.

0. 2700 and 2250

To begin with, factorizing the two integers, we get $2700 = 2^2 3^3 5^2$, and $2250 = 2 \cdot 3^2 5^3$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 2700 and 2250. And all the common factors are 2, 3, and 5.
So the product is $2 \cdot 3 \cdot 5$.

And next, we apply to each common factor the smallest exponent used for the factor.

The smallest exponent applied to the factor 2 is 1, the smallest exponent applied to the factor 3 is 2, and the smallest exponent applied to the factor 5 is 2.

And thus, we get GCD = $2 \cdot 3^2 5^2 = 90$.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, the prime factors of 2700 and the prime factors of 2250.
And all the prime factors are 2, 3, and 5. So the product is $2 \cdot 3 \cdot 5$.

And next, we want to apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 2, the largest exponent applied to the factor 3 is 3, and the largest exponent applied to the factor 5 is 3.
And thus, we get LCM = $2^2 3^3 5^3 = 13500$.

1. 25 and 12

GCD = 1, and LCM = $2^2 3 \cdot 5^2 = 300$.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the two integers, we get $25 = 5^2$, and $12 = 2^2 3$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 25 and 12.

In this case though, no factor is common to both numbers.
That is to say that the only divisor common to both numbers is 1.
So the GCD is 1. And in such a case, the two numbers are said to be prime to each other.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, the prime factors of 25 and the prime factors of 12.
And all the prime factors are 2, 3, and 5. So the product is $2 \cdot 3 \cdot 5$.

And next, we apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 2, the largest exponent applied to the factor 3 is 1, and the largest exponent applied to the factor 5 is 2.

And thus, we get LCM = $2^2 3 \cdot 5^2 = 300$.

2. 7 and 19

GCD = 1, and LCM = $7 \cdot 19 = 133$.

If not sure, follow the steps below.

The two integers are prime integers, and are just called primes, for short.
And no factor is common to primes, which are said to be prime to each other.
That is to say that the only divisor common to both numbers is 1. So the GCD is 1.

And the LCM of primes is just the product of the primes.
So we get LCM = $7 \cdot 19 = 133$.

3. 8, 9, and 12

To begin with, factorizing the 3 integers, we get $8 = 2^3$, $9 = 3^2$, and $12 = 2^2 3$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 8, 9, and 12. In this case though, no factor is common to all the three numbers, which are thus, prime to each other.
That is to say that the only divisor common to all the three numbers is 1.
So the GCD is 1.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 8, 9 and 12.
And all the prime factors are 2 and 3. So the product is $2 \cdot 3$.
And next, we apply to each factor the largest exponent used for the factor.
The largest exponent applied to the factor 2 is 3, and the largest exponent applied to the factor 3 is 2. And thus, we get LCM = $2^3 3^2 = 72$.

4. 24, 45, and 245

To begin with, factorizing the 3 integers, we get $24 = 2^2 3$, $45 = 3^2 5$, and $245 = 5 \cdot 7^2$.
Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 24, 45, and 245. In this case though, no factor is common to all the numbers, which are thus, prime to each other.
That is to say that the only divisor common to all the three numbers is 1.
So the GCD is 1.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 24, 45, and 245.
And all the prime factors are 2, 3, 5, and 7. So the product is $2 \cdot 3 \cdot 5 \cdot 7$.
And next, we apply to each factor the largest exponent used for the factor.
The largest exponent applied to the factor 2 is 2, the largest exponent applied to the factor 3 is 2, the largest exponent applied to 5 is 1, and the largest exponent applied to 7 is 2. And thus, we get LCM = $2^2 3^2 5 \cdot 7^2 = 8820$.

5. 60, 75, and 490

GCD = 5, and LCM = $2^2 3 \cdot 5^2 7^2$ = 14700.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the 3 integers, we get 60 = $2^2 3 \cdot 5$, 75 = $3 \cdot 5^2$, and 490 = $2 \cdot 5 \cdot 7^2$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 60, 75, and 490. And 5 is the only prime factor common.

So the product is 5.

And next, we apply to each common factor the smallest exponent used for the factor. The smallest exponent applied to the factor 5 is 1.

And thus, we get GCD = 5.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 60, 75, and 490. And they are 2, 3, 5, and 7.
So the product is $2 \cdot 3 \cdot 5 \cdot 7$.

And next, we apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 2, the largest exponent applied to the factor 3 is 1, the largest exponent applied to the factor 5 is 2, and the largest exponent applied to the factor 7 is 2.

And thus, we get LCM = $2^2 3 \cdot 5^2 7^2$ = 14700.

6. 4620, 825, and 6930

GCD = $3 \cdot 5 \cdot 11$ = 165, and LCM = $2^2 3^2 5^2 7 \cdot 11$ = 68300.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the 3 integers, we get
4620 = $2^2 3 \cdot 5 \cdot 7 \cdot 11$, 825 = $3 \cdot 5^2 11$, and 6390 = $2 \cdot 3^2 5 \cdot 7 \cdot 11$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 4620, 825, and 6930.

And all the common factors are 3, 5, and 11. So the product is $3 \cdot 5 \cdot 11$.

And next, we apply to each common factor the smallest exponent used for the factor.

The smallest exponent applied to the factor 3 is 1, the smallest exponent applied to the factor 5 is 1, and the smallest exponent applied to the factor 11 is 1.

And thus, we get GCD = $3 \cdot 5 \cdot 11$ = 165.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 4620, 825, and 6930.

And all the prime factors are 2, 3, 5, 7, and 11. So the product is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$.

And next, we want to apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 2, the largest exponent applied to the factor 3 is 2, the largest exponent applied to the factor 5 is 2, the largest exponent applied to the factor 7 is 1, and the largest exponent applied to the factor 11 is 1.

And thus, we get LCM = $2^2 3^2 5^2 7 \cdot 11$ = 68300.

7. 45045, 225225, and 10395

GCD = $3^2 \cdot 5 \cdot 7 \cdot 11$ = 3465, and LCM = $3^3 5^2 7 \cdot 11 \cdot 13$ = 675675.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the 3 integers, we get

44045 = $3^2 5 \cdot 7 \cdot 11 \cdot 13$, 225225 = $3^2 5^2 7 \cdot 11 \cdot 13$, and 10395 = $3^3 5 \cdot 7 \cdot 11$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 45045, 225225, and 10395.

And all the common factors are 3, 5, 7, and 11. So the product is $3 \cdot 5 \cdot 7 \cdot 11$.

And next, we apply to each common factor the smallest exponent used for the factor.

The smallest exponent applied to the factor 3 is 2, the smallest exponent applied to the factor 5 is 1, the smallest exponent applied to the factor 7 is 1, and the smallest exponent applied to the factor 11 is 1.

And thus, we get GCD = $3^2 \cdot 5 \cdot 7 \cdot 11$ = 3465.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 45045, 225225, and 10395.

And all the prime factors are 3, 5, 7, 11, and 13. So the product is $3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$.

And next, we want to apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 3 is 3, the largest exponent applied to the factor 5 is 2, the largest exponent applied to the factor 7 is 1, the largest exponent applied to the factor 11 is 1, and the largest exponent applied to the factor 13 is 1.

And thus, we get LCM = $3^3 5^2 7 \cdot 11 \cdot 13$ = 675675.

8. 42042, 315315, 72765, and 231231

GCD = $3 \cdot 7^2 11$ = 1617, and LCM = $2 \cdot 3^3 5 \cdot 7^2 11^2 13$ = 1891890.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the 3 integers, we get

42042 = $2 \cdot 3 \cdot 7^2 11 \cdot 13$, 315315 = $3^2 5 \cdot 7^2 11 \cdot 13$, 72765 = $3^3 5 \cdot 7^2 11$, and 231231 = $3 \cdot 7^2 11^2 13$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 42042, 315315, 72765, and 231231.

And all the common factors are 3, 7, and 11. So the product is $3 \cdot 7 \cdot 11$.

And next, we apply to each common factor the smallest exponent used for the factor.

The smallest exponent applied to the factor 3 is 1, the smallest exponent applied to the factor 7 is 2, and the smallest exponent applied to the factor 11 is 1.

And thus, we get GCD = $3 \cdot 7^2 11$ = 1617.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 42042, 315315, 72765, and 231231.

And all the prime factors are 2, 3, 5, 7, 11, and 13. So the product is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$.

And next, we want to apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 1, the largest exponent applied to the factor 3 is 3, the largest exponent applied to the factor 5 is 1, the largest exponent applied to the factor 7 is 2, the largest exponent applied to the factor 11 is 2, and the largest exponent applied to the factor 13 is 1.

And thus, we get LCM = $2 \cdot 3^3 5 \cdot 7^2 11^2 13$ = 1891890.

9. 58905, 1576575, 20790, 13230, and 114345

GCD = $3^2 5 \cdot 7$ = 315, and LCM = $2 \cdot 3^3 5^2 7^2 11^2 13 \cdot 17$ = 1768917150.

If not sure of how to get it, follow the steps below.

To begin with, factorizing the 3 integers, we get

58905 = $3^2 5 \cdot 7 \cdot 11 \cdot 17$, 1576575 = $3^2 5^2 7^2 11 \cdot 13$, 20790 = $2 \cdot 3^3 5 \cdot 7 \cdot 11$, 13230 = $2 \cdot 3^3 5 \cdot 7^2$, and 114345 = $3^3 5 \cdot 7 \cdot 11^2$.

Then, finding the GCD, we take first, the product of all the common factors, which are in this case, common to 58905, 1576575, 20790, 13230, and 114345.

And all the common factors are 3, 5, and 7. So the product is $3 \cdot 5 \cdot 7$.

And next, we apply to each common factor the smallest exponent used for the factor.

The smallest exponent applied to the factor 3 is 2, the smallest exponent applied to the factor 5 is 1, and the smallest exponent applied to the factor 7 is 1.

And thus, we get GCD = $3^2 5 \cdot 7$ = 315.

Next, finding the LCM, we take first, the product of all the prime factors, which are in this case, all the prime factors of 58905, 1576575, 20790, 13230, and 114345.

And all the prime factors are 2, 3, 5, 7, 11, 13, and 17.
So the product is $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$.

And next, we want to apply to each factor the largest exponent used for the factor.

The largest exponent applied to the factor 2 is 1, the largest exponent applied to the factor 3 is 3, the largest exponent applied to the factor 5 is 2, the largest exponent applied to the factor 7 is 2, the largest exponent applied to the factor 11 is 2, the largest exponent applied to the factor 13 is 1, and the largest exponent applied to the factor 17 is 1.

And thus, we get LCM = $2 \cdot 3^3 5^2 7^2 11^2 13 \cdot 17 = 1768917150$.