Examples W in Arithmetic 2

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Examples W

0. Assuming that $\boldsymbol{a}$ is an integer positive, and that if we divide it by 7, the quotient is 5, and the remainder is 6, find $\boldsymbol{a}$.

1. Assuming $\boldsymbol{a}$ is a positive integer, and if we divide it by 7, the remainder is 6, find $\boldsymbol{a}$.

2. How many two-digit positive integers can we divide by 3 and 5?

3. Find the smallest integer $\boldsymbol{a}$ satisfying $\boldsymbol{b} = \frac{4}{5}\boldsymbol{a}$ where $\boldsymbol{b}$ is a positive integer.

4. Find the largest integer $\boldsymbol{a}$ satisfying $\boldsymbol{b} = \frac{2}{3}\boldsymbol{a}$ where $\boldsymbol{b}$ is a negative integer.

5. Assuming the GCD (GCF) of five integers is 24, find all the common divisors of the integers.

6. Assuming $\boldsymbol{m}$ is between 0 and 1000, and is a common multiple of 4, 12, and 15, find the number of all the numbers that can be $\boldsymbol{m}$.

7. Assuming $\boldsymbol{a} = \frac{\mathbf{8}}{\mathbf{5}} \cdot \frac{\boldsymbol{x}}{\boldsymbol{y}}$, and $\boldsymbol{b} = \frac{\mathbf{70}}{\mathbf{3}} \cdot \frac{\boldsymbol{x}}{\boldsymbol{y}}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are positive integers, find $\boldsymbol{x}$ and $\boldsymbol{y}$ that are positive integers and prime to each other.

Suggestions or Solutions
To the Examples W

0. Assuming that *a* is an integer positive, and that if we divide it by 7, the quotient is 5, and the remainder is 6, find *a*.

$\boldsymbol{a} = \mathbf{41}$. *If not sure of how to get it, follow the steps below.*

First, dividing $\boldsymbol{a}$ by 7, we get $\dfrac{\boldsymbol{a}}{\mathbf{7}}$.

Next, if the quotient is 5 and the remainder is 6, which of the two below do we get?

$\dfrac{\boldsymbol{a}}{\mathbf{7}} = \mathbf{5} + \mathbf{6}$ and $\dfrac{\boldsymbol{a}}{\mathbf{7}} = \mathbf{5} + \dfrac{\mathbf{6}}{\mathbf{7}}$

We get $\dfrac{\boldsymbol{a}}{\mathbf{7}} = \mathbf{5} + \dfrac{\mathbf{6}}{\mathbf{7}}$. So we get $\boldsymbol{a} = \mathbf{5 \cdot 7 + 6 = 41}$.

1. Assuming *a* is a positive integer, and if we divide it by 55, the remainder is 7, find *a*.

$\boldsymbol{a} = \mathbf{55}\boldsymbol{q} + \mathbf{7}$, where $\boldsymbol{q}$ is a nonnegative integer.

If not sure of how to get it, follow the steps below.

First, dividing $\boldsymbol{a}$ by 55, we get $\dfrac{\boldsymbol{a}}{\mathbf{55}}$.

Next, if the quotient is $\boldsymbol{q}$ and the remainder is 7, which of the two below do we get?

$\dfrac{\boldsymbol{a}}{\mathbf{55}} = \boldsymbol{q} + \mathbf{7}$ and $\dfrac{\boldsymbol{a}}{\mathbf{55}} = \boldsymbol{q} + \dfrac{\mathbf{7}}{\mathbf{55}}$.

We get $\boldsymbol{\frac{a}{55} = q + \frac{7}{55}}$.   What then, is ***q***? Which one of the three below?

A. ***q*** is an integer.   B. ***q*** is a positive integer.   C. ***q*** is a nonnegative integer.

q is one of 0, 1, 2, 3, …, and thus, is one of nonnegative integers.
So we get ***a* = 55*q* + 7**, where ***q*** is a nonnegative integer.
That is, ***a*** can be any of 55·0 + 7 = 7, 55 + 7 = 62, 55·2 + 7 = 117, and so forth.

2. How many 3-digit positive integers can we divide by 4 and 6?

75.   *If not sure of how to get it, follow the steps below.*

First, what integers can 4 and 6 divide?

They are common multiples of 4 and 6. So 4 and 6 both can divide a common multiple of 4 and 6. How large then, can it be?

It can be infinitely large. How small then, can it be?

It can be as small as not 24 but 12, which is therefore, the smallest common multiple. And we often call it the least common multiple, and usually call it the LCM, which is the acronym of the **l**east **c**ommon **m**ultiple.

What then, are numbers that 4 and 6 can divide?

They are multiples of the LCM of the two integers 4 and 6. And the LCM is 12.
So they are multiples of 12. And assuming $\boldsymbol{m}$ is such a multiple, we can put it this way:
$\boldsymbol{m = 12n}$ where $\boldsymbol{n}$ is an integer.

Now, we know that the numbers we want are 3-digit integers positive.
And of all the 3-digit positive integers, the smallest is 100, and the largest is 999.
How then, can we get the number of 3-digit positive integers that 12 can divide?

We can get it finding $\boldsymbol{n}$ that satisfies the relation as follows. $\boldsymbol{100 \le 12n \le 999}$.
And it is the value of $\boldsymbol{n}$, of course. So finding $\boldsymbol{n}$, we get

$$\boldsymbol{100 \le 12n \le 999 \Rightarrow \tfrac{100}{12} \le n \le \tfrac{999}{12} \Rightarrow 8 + \tfrac{4}{12} \le n \le \tfrac{333}{4}}$$
$$\boldsymbol{\Rightarrow 8 + \tfrac{1}{3} \le n \le 83 + \tfrac{1}{4} \Rightarrow 9 \le n \le 83}.$$

That is to say that $\boldsymbol{n}$ can be any integer between 8 and 84.
In other words, $\boldsymbol{n}$ can be any of all the integers from 9 to 83.   How com though?

We know $\boldsymbol{12n}$ has to be a 3-digit positive integer. And we have $12 \cdot 8 = 96$, which is just a 2-digit integer, $12 \cdot 9 = 108$, which is a 3-digit integer, $12 \cdot 83 = 996$, which is a 3-digit integer, and $12 \cdot 84 = 1008$, which is a 4-digit integer.

And thus, the number of all 3-digit positive integers 3 and 6 divide is $83 - 9 + 1 = 75$.

3. Find the smallest integer $\boldsymbol{a}$ satisfying $\boldsymbol{b = \tfrac{4}{5}a}$ where $\boldsymbol{b}$ is a positive integer.

We have $\boldsymbol{b = \tfrac{4}{5}a}$. And we can put it this way, too: $\boldsymbol{\dfrac{b}{a} = \dfrac{4}{5}}$.
So the ratio of $\boldsymbol{b}$ to $\boldsymbol{a}$ is $\boldsymbol{\tfrac{4}{5}}$. In other words, $\boldsymbol{b : a = 4 : 5}$.
So the smallest integer $\boldsymbol{a}$ is 5, since $\boldsymbol{b}$ is a positive integer.

4. Find the largest integer $\boldsymbol{a}$ satisfying $\boldsymbol{b} = \frac{2}{3}\boldsymbol{a}$ where $\boldsymbol{b}$ is a negative integer.

We have $\boldsymbol{b} = \frac{2}{3}\boldsymbol{a}$. And we can put it this way, too: $\dfrac{\boldsymbol{b}}{\boldsymbol{a}} = \dfrac{\mathbf{2}}{\mathbf{3}}$.

So the ratio of $\boldsymbol{b}$ to $\boldsymbol{a}$ is $\frac{2}{3}$. In other words, $\boldsymbol{b} : \boldsymbol{a} = \mathbf{2 : 3}$. So the largest integer $\boldsymbol{a}$ is –3, since $\boldsymbol{b}$ is a negative integer. And in this case, $\boldsymbol{b} = \mathbf{-2}$, of course.

5. Assuming the GCD (GCF) of five integers is 24, find all the common divisors of the integers.

1, 2, 8, 3, 6, 12, and 24.

If not sure of how to get it, follow the steps below.

We know that the GCD of a set of integers is the greatest common divisor of all the integers in the set. And we know that a factor is a divisor. So GCD is called GCF, too, which is the acronym of Greatest Common Factor.

And thus, the GCD can divide all the integers in the set.
So if an integer can divide the GCD, the integer can divide all the integers in the set.
That is to say that a divisor of the GCD can divide all the integers in the set.
In other words, a divisor of the GCD is a common divisor of all the integers in the set.
And thus, all the divisors of the GCD can divide all the integers in the set.
That is to say that all the divisors of the GCD are all the common divisors of all the integers in the set.

Now, the GCD of the five integers is the greatest common divisor of all the five integers.
And thus, the GCD can divide all the integers in the set. We know that the GCD is 24.
So if an integer can divide the GCD 24, the integer can divide all the five integers.
That is to say that a divisor of 24 can divide all the five integers.
And thus, all the divisors of 24 can divide all the five integers.

So first, factorizing the GCD, we get $24 = 8{\cdot}3 = 2^3{\cdot}3$.
Thus next, finding all the divisors of 24, we get
1, 2, $2^2 = 4$, $2^3 = 8$, 3, $2{\cdot}3 = 6$, $2^2{\cdot}3 = 12$, and $2^3{\cdot}3 = 24$.

6. Assuming *m* is between 0 and 1000, and is a common multiple of 4, 12, and 15, find the number of all the numbers that can be *m*.

16. *If not sure of how to get it, follow the steps below.*

We know that the LCM of a set of integers is the least common multiple of all the integers in the set.

So all the multiples of the LCM are common multiples of all the integers in the set.

Now, in this case, all the integers in the set are 4, 12, and 15.

And finding the LCM, we factorize the three integers first.

So factorizing them, we get $4 = 2^2$, $12 = 4 \cdot 3 = 2^2 3$, and $15 = 3 \cdot 5$.

And next, we put all the factors in a product form.
So getting the product, we get $2 \cdot 3 \cdot 5$.

And next, finding the LCM, we apply to each factor the largest exponent used for the factor. Thus, we get LCM $= 2^2 3 \cdot 5 = 60$.

Now, we know ***m*** is a multiple of the LCM.

So we can set $\boldsymbol{m = 60k}$ where ***k*** is an integer.

And we want ***m*** to be between 0 and 1000.

So we get $\boldsymbol{0 < 60k < 1000 \Rightarrow 0 < k < \frac{1000}{60} = \frac{100}{6} = \frac{50}{3} = 16 + \frac{2}{3} \Rightarrow 1 \leq k \leq 16}$.

And thus, the number of all the numbers that can be m is $16 - 1 + 1 = 16$.

7. Assuming $a = \frac{8}{5} \cdot \frac{x}{y}$, and $b = \frac{70}{3} \cdot \frac{x}{y}$ where a and b are positive integers, find x and y that are positive integers and prime to each other.

(x, y) = (15*k*, 1) where ***k*** is a positive integer.
(x, y) = (30*m* + 15, 2) where ***m*** is a nonnegative integer, that is, ***m*** = 0, 1, 2, etc.

If not sure of how to get it, follow the steps below.

First, we know ***a*** is a positive integer.
So we can say that ***x*** is a multiple of 5, and that ***y*** is a divisor of 8.

Next, ***b*** is a positive integer, too. So we can say that ***x*** is a multiple of 3, too, and that ***y*** is a divisor of 70, also. What then, can we say about ***x*** and ***y***?

We can say ***x*** is a common multiple of 5 and 3, and ***y*** is a common divisor of 8 and 70. What then, can we say about ***x*** and ***y***?

We know that the smallest common multiple is the LCM of 5 and 3, and that greatest divisor is the GCD of 8 and 70.

So we can say that ***x*** is a multiple of the LCM of 5 and 3, and that ***y*** is a divisor of the GCD of 8 and 70. And thus, we want to get the LCM and the GCD.

So first, the LCM of 5 and 3 is 15.

And next, finding the GCD, we factorize 8 and 70 first.
So factorizing them, we get $8 = 2^3$, and $70 = 2 \cdot 5 \cdot 7$.

Next, we put all common factors in a product form.
In this case, 2 is the only common factor.

Next, we apply to each factor the smallest exponent used for the factor.
Thus, we get the GCD = 2.

Now, we know ***x*** is a multiple of the LCM, which is 15.

So we can set ***x* = 15*k*** where ***k*** is a positive integer.

Next, we know ***y*** is a divisor of the GCD, which is 2. So we get ***y*** = 2 or 1.

That's not it though. What then, is the next?

We know ***x*** and ***y*** are prime to each other. So we can put ***x*** and ***y*** in the two cases below.

• **(*x*, *y*) = (15*k*, 1)** where ***k*** is a positive integer.
So for instance, ***x* = 15** and ***y* = 1**, ***x* = 30** and ***y* = 1**, ***x* = 45** and ***y* = 1**, and so forth.

• **(*x*, *y*) = (15*n*, 2)** where ***n*** is an odd positive integer.
So for instance, ***x* = 15** and ***y* = 2**, ***x* = 45** and ***y* = 2**, ***x* = 75** and ***y* = 2**, and so forth.

And since ***n*** is an odd positive integer, we can put **15*n*** the way below, too.

15*n* = 15(2*m* + 1) = 30*m* + 15 where ***m*** is a nonnegative integer

So we can put the two above the way below, too.

• **(*x*, *y*) = (15*k*, 1)** where ***k*** is a positive integer.
• **(*x*, *y*) = (30*m* + 15, 2)** where ***m*** is a nonnegative integer, that is, ***m*** = 0, 1, 2, etc.