Examples X in Arithmetic 2

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Examples X

0. Assuming that $\boldsymbol{a}$ is a positive integer, and that dividing 40 by $\boldsymbol{a}$, we get 8 as a remainder, and dividing 50 by $\boldsymbol{a}$, we get 2 as a remainder, find $\boldsymbol{a}$.

1. Assuming $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers positive, $\boldsymbol{\frac{a}{b} = \frac{12}{5}}$, and the sum of their GCD (GCF) and LCM is 305, find $\boldsymbol{a}$ and $\boldsymbol{b}$.

2. Assuming $\boldsymbol{a}$ is a positive integer, GCD of $\boldsymbol{a}$ and 32 is 8, and LCM is 160, find $\boldsymbol{a}$.

3. Assuming we have 60 apples and 90 bananas, find the maximum number of people to whom we can give the same number of apples and the same number of bananas.

4. Assuming we have 60 apples, 90 bananas, 120 pears, and 150 oranges, find the maximum number of people to whom we can give the same number of each fruit evenly. Assuming for instance, we have 3 apples and 6 bananas, we can give 1 apple and 2 bananas to 3 people evenly, and thus, the maximum number of people is 3.

5. Suppose we have 27,225 apples, 370,260 pears, and 1,262,250 bananas. Suppose also, we want to deliver the fruits to a number of stores, but each store needs to get as many of each fruit as possible. And each of the stores needs to get the same amount for each fruit. For instance, if one store gets 5 apples, each of all the others has to get 5 apples each, too.

How many of each fruit then, do we need to deliver to each store maximizing the number of stores?

And how many of each fruit, do we need to deliver to each store minimizing the number of stores? The minimum is greater than 1, of course.

Suggestions or Solutions
To the Examples X

0. Assuming that *a* is a positive integer, and that dividing 40 by *a*, we get 8 as a remainder, and dividing 50 by *a*, we get 2 as a remainder, find *a*.

$\boldsymbol{a = 16}$.	*If not sure of how to get it, follow the steps below.*

To begin with, dividing 40 by $\boldsymbol{a}$, we get 8 as a remainder.
So we get $\mathbf{40 = \boldsymbol{ka} + 8}$ where $\boldsymbol{k}$ is an integer, and $\boldsymbol{a} > \mathbf{8}$ since the remainder is less than $\boldsymbol{a}$.
Thus, we get $\boldsymbol{ka} = \mathbf{40 - 8 = 32} \Rightarrow \boldsymbol{ka} = \mathbf{32}$.

So we can say that $\boldsymbol{a}$ divides 32. That is to say that $\boldsymbol{a}$ is a divisor of 32.

Next, dividing 50 by $\boldsymbol{a}$, we get 2 as a remainder.
So we get $\mathbf{50 = \boldsymbol{qa} + 2}$ where $\boldsymbol{q}$ is an integer, and $\boldsymbol{a} > \mathbf{2}$ since the remainder is less than $\boldsymbol{a}$.
Thus, we get $\boldsymbol{qa} = \mathbf{50 - 2 = 48} \Rightarrow \boldsymbol{qa} = \mathbf{48}$.

So we can say that $\boldsymbol{a}$ divides 48, too. That is, $\boldsymbol{a}$ is a divisor of 48.
What then, can we say about $\boldsymbol{a}$?

We know $\boldsymbol{a}$ divides 32 as well as 48.
So we can say that $\boldsymbol{a}$ is a common divisor of 32 and 48.
And the GCD of 32 and 48 is the greatest common divisor of 32 and 48.
That is to say that of all divisors common to 32 and 48, the GCD is the largest.
What then, $\boldsymbol{a}$ can be?

We can say that $\boldsymbol{a}$ is one of divisors of the GCD.

That's because a divisor of the GCD is a common divisor of 32 and 48, too.
In other words, a factor of the GCD is a factor common to 32 and 48, too.

So let's now find the GCD.

$32 = 2^5$, and $48 = 2 \cdot 24 = 6 \cdot 8 = 3 \cdot 16 = 2^4 3$. So the GCD is 2^4, which is 16.

And thus, $\boldsymbol{a}$ can be one of 1, 2, 4, 8, and 16, which are divisors of 16.
And we know $\boldsymbol{a > 8}$. So we get $\boldsymbol{a = 16}$.

9. Assuming *a* and *b* are integers positive, $\boldsymbol{\frac{a}{b} = \frac{12}{5}}$, and the sum of their GCD (GCF) and LCM is 305, find *a* and *b*.

$\boldsymbol{a = 60}$, and $\boldsymbol{b = 25}$.

If not sure of how to get it, follow the steps below.

Assuming first, $\boldsymbol{G}$ is the GCD of $\boldsymbol{a}$ and $\boldsymbol{b}$, we can set $\boldsymbol{a = 12G}$, and $\boldsymbol{b = 5G}$.

It's because 12 and 5 are prime to each other, and if they are not, $\boldsymbol{G}$ is not the GCD of $\boldsymbol{a}$ and $\boldsymbol{b}$, either.

And since $\boldsymbol{G}$ is the largest divisor common to $\boldsymbol{a}$ and $\boldsymbol{b}$, we get $\boldsymbol{\frac{a}{b} = \frac{12G}{5G} = \frac{12}{5}}$.

And assuming next, $\boldsymbol{L}$ is the LCM of $\boldsymbol{a}$ and $\boldsymbol{b}$, we can set $\boldsymbol{L = 12 \cdot 5G = 60G}$.

That's because 12 and 5 are prime to each other, $\boldsymbol{a = 12G}$, $\boldsymbol{b = 5G}$, and $\boldsymbol{L}$ is the smallest multiple common to $\boldsymbol{a}$ and $\boldsymbol{b}$, and thus, $\boldsymbol{L}$ is the smallest that $\boldsymbol{a}$ and $\boldsymbol{b}$ both can divide.

And next, we have $\boldsymbol{G + L = 305}$. What then, do we get?

We have $\boldsymbol{L = 60G}$, too.

So we get $\boldsymbol{G + L = G + 60G = 61G = 305 \Rightarrow G = 5}$.

Thus, we get $\boldsymbol{a = 12G = 60}$, and $\boldsymbol{b = 5G = 25}$.

A. **Assuming a is a positive integer, GCD of a and 32 is 8, and LCM is 160, find a.**

$a = 40$.

If not sure of how to get it, follow the steps below.

Assuming first, G is the GCD of a and 32, we can set $a = kG$, where k is an integer.

Next, we have $32 = 4 \times 8 = 4G$, since $G = 8$.

How then, can we put the LCM of a and 32 in terms of G?

We have $a = kG$, and $32 = 4G$, where G is the GCD of a and 32.

So we can say that k and 4 are prime to each other.
And thus, assuming L is the LCM of a and 32, we can set $L = 4kG$.

And we know $L = 160$, and $G = 8$.

So we get $L = 4kG \Rightarrow 160 = 4 \cdot k \cdot 8 = 32k \Rightarrow k = 160/32 = 5$.

And we know $a = kG$, where $k = 5$, and $G = 8$. So we get $a = 40$.

B. **Assuming we have 60 apples and 90 bananas, find the maximum number of people to whom we can give the same number of apples and the same number of bananas.**

30.

If not sure of how to get it, follow the steps below.

Suppose in a division, the divisor is a positive integer.
Then, the larger the divisor, the larger the number of equal pieces.
Dividing for instance, 1 by 3, we divide 1 into three equal pieces. And each piece is $\frac{1}{3}$.

And dividing 10 by 5, we divide 10 into 5 equal parts. And each part is 2.

So for instance, giving apples to people, and dividing the number of apples by the number of people, we get the number of apples each person gets. And the larger the number of people, the larger the divisor, because the divisor is the number of people.

And the same is true for bananas, too.

So the maximum number of people in this problem is the greatest common divisor of the number of apples and the number of bananas.
In short, the maximum number is the GCD of 60 and 90.

So first, factorizing 60 and 90, we get
$60 = 2 \cdot 30 = 2 \cdot 2 \cdot 15 = 2^2 3 \cdot 5$, and $90 = 2 \cdot 45 = 2 \cdot 5 \cdot 9 = 2 \cdot 3^2 5$.

Thus next, the GCD $= 2 \cdot 3 \cdot 5 = 30$, which is the maximum number of people.

And we have $60 = 2 \cdot 30$, and $90 = 3 \cdot 30$.
So 30 people can get 2 apples each and 3 bananas each.

This is one of the uses of GCD (the greatest common divisor).

The GCD of a set of amounts can indicate the largest number of equal parts each of all the amounts can be divided into.

So in this case, we can divide 60 apples into up to 30 equal parts, each part has 2 apples, and we can divide 90 bananas into 30 equal parts, and each part has 3 bananas.

C. Assuming we have 60 apples, 90 bananas, 120 pears, and 150 oranges, find the maximum number of people to whom we can give the same number of each fruit evenly. Assuming for instance, we have 3 apples and 6 bananas, we can give 1 apple and 2 bananas to 3 people evenly, and thus, the maximum number of people is 3.

30. *If not sure of how to get it, follow the steps below.*

This is no other than the problem **B** above.
So the maximum number of people is the greatest common divisor of the four numbers of the fruits.

In short, the maximum number is the GCD of 60, 90, 120, and 150.

So first, factorizing 60, 90, 120, and 150, we get
$60 = 2^2 3 \cdot 5$, $90 = 2 \cdot 3^2 5$, $120 = 2^3 3 \cdot 5$, and $150 = 2 \cdot 3 \cdot 5^2$.

Thus next, the GCD $= 2 \cdot 3 \cdot 5 = 30$, which is the maximum number of people.
And we have $60 = 2 \cdot 30$, and $90 = 3 \cdot 30$.

So 30 people can get 2 apples each and 3 bananas each.
The maximum number of people is the greatest common divisor of the number of apples and the number of bananas.

In short, the maximum number is the GCD of 60 and 90.

So first, factorizing 60 and 90, we get

$60 = 2 \cdot 30 = 2 \cdot 2 \cdot 15 = 2^2 3 \cdot 5$, and $90 = 2 \cdot 45 = 2 \cdot 5 \cdot 9 = 2 \cdot 3^2 5$.

Thus next, the GCD $= 2 \cdot 3 \cdot 5 = 30$, which is the maximum number of people.

And we have $60 = 2 \cdot 30$, $90 = 3 \cdot 30$, $120 = 4 \cdot 30$, and $150 = 5 \cdot 30$.

So 30 people can get 2 apples each, 3 bananas each, 4 pears each, and 5 oranges each.

And notice that 2, 3, 4, and 5 are prime to each other.
That is, other than 1, the four integers have no common divisor.
And saying no common divisor, we mean the GCD is 1.

D. Suppose we have 27,225 apples, 370,260 pears, and 1,262,250 bananas. Suppose also, we want to deliver the fruits to a number of stores, but each store needs to get as many of each fruit as possible. And each of the stores needs to get the same amount for each fruit. For instance, if one store gets 5 apples, each of all the others has to get 5 apples each, too.

How many of each fruit then, do we need to deliver to each store maximizing the number of stores?

And how many of each fruit, do we need to deliver to each store minimizing the number of stores? The minimum is greater than 1, of course.

The first question is no other than the problem **C** above.
So the maximum number of stores is the greatest common divisor of the 3 numbers of the fruits.
In short, the maximum number is the GCD of 27225, 370260, and 1262250.

So first, factorizing the 3 numbers, we get

$27225 = 3^2 5^2 11^2$, $370260 = 2^2 3^2 5 \cdot 11^2 17$, and $1262250 = 2 \cdot 3^3 \cdot 5^3 11 \cdot 17$.

Thus next, the GCD $= 3^2 \cdot 5 \cdot 11 = 495$, which is the maximum number of stores.
And assuming ***G* = GCD**, we have

$\mathbf{27225 = 11}\boldsymbol{G}$, $\mathbf{370260 = 2^2 \cdot 11 \cdot 17}\boldsymbol{G}$, and $\mathbf{1262250 = 2 \cdot 3 \cdot 5^2 \cdot 17}\boldsymbol{G}$.

So 495 stores can get 11 apples each, $2^2 \cdot 11 \cdot 17$ pears each, and $2 \cdot 3 \cdot 5^2 \cdot 17$ bananas each.

And notice that 11, $2^2 \cdot 11 \cdot 17$, and $2 \cdot 3 \cdot 5^2 \cdot 17$ are prime to each other.
That is, no factor is common to all the three numbers.

And next, we want to find the number of each fruit we need to deliver to each store minimizing the number of stores.

What then, do we need to find for the three numbers 27225, 370260, and 1262250?

We want to find not the greatest but the smallest common divisor.
What then, is the smallest common divisor?

We have $27225 = 3^2 5^2 11^2$, $370260 = 2^2 3^2 5 \cdot 11^2 17$, and $1262250 = 2 \cdot 3^3 \cdot 5^3 11 \cdot 17$.

So we can see that 3 is the divisor the smallest common to all the three numbers.
And we can put the three numbers the way below.

$27225 = 3 \cdot 3 \cdot 5^2 11^2$, $370260 = 3 \cdot 2^2 3 \cdot 5 \cdot 11^2 17$, and $1262250 = 3 \cdot 2 \cdot 3^2 \cdot 5^3 11 \cdot 17$.

So 3 stores can get $3 \cdot 5^2 11^2$ apples each, $2^2 3 \cdot 5 \cdot 11^2 17$ pears each, and $2 \cdot 3^2 \cdot 5^3 11 \cdot 17$ bananas each.