

Sense of Arithmetic 4

Copyright © 2018 by Seong Ryeol Kim. All rights reserved

Click this to start.

Read the calculations below if you want to. And if you read them, just keep reading them at your pace. It can help increase sense of arithmetic, and mental math.

You don't have to read them all at once. Each reading can take 5, 10, or 15 minutes at a time, or as much as you can concentrate. And having read them all, read them again when you feel like it. **Note however**, you don't have to do calculations the way below.

Each calculation just shows that **there are many ways we can calculate.**

$$0 = 1 \times 0 = 0 \times 1 = 0 + 0 = 2 \times 0 = 0 \times 2 = 0 + 0 + 0 = 3 \times 0 = 0 \times 3$$

$$= 987 \times 0 = 0 \times 987 = 0 + 0 + 0 + 0 + \dots = 0$$

$$1 = 1 \times 1, 0.1 = 0.1 \times 1 = 1 \times 0.1, \text{ and } 0.2 = 0.1 + 0.1 = 0.1 \times 2 = 2 \times 0.1$$

$$0.3 = 0.1 + 0.1 + 0.1 = 0.1 \times 3 = 3 \times 0.1$$

$$= 0.1 + 0.2 = 0.1 + (0.1 + 0.1) = 0.1 + 0.1 \times 2 = 0.1 + 2 \times 0.1$$

$$= 0.2 + 0.1 = (0.1 + 0.1) + 0.1 = 0.1 \times 2 + 0.1 = 2 \times 0.1 + 0.1$$

$$= 0 + 0.3$$

$$0.4 = 0.1 + 0.1 + 0.1 + 0.1 = 0.1 \times 4$$

$$= 0.1 + 0.3 = 0.1 + 0.1 \times 3$$

$$= 0.3 + 0.1 = 0.1 \times 3 + 0.1$$

$$= 0.2 + 0.2 = 0.2 \times 2$$

$$0.5 = 0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 0.1 \times 5 = 5 \times 0.1$$

$$= 0.1 + 0.4 = 0.4 + 0.1 = 0.2 + 0.3 = 0.3 + 0.2$$

$$0.6 = 0.1 \times 6 = 6 \times 0.1$$

$$= 0.1 + 0.5 = 0.5 + 0.1$$

$$= 0.2 + 0.4 = 0.2 + 0.2 \times 2$$

$$= 0.4 + 0.2 = 0.2 \times 2 + 0.2$$

$$= (0.2 + 0.2) + 0.2 = 0.2 + 0.2 + 0.2 = 0.2 \times 3 = 3 \times 0.2$$

$$0.7 = 0.1 \times 7 = 7 \times 0.1$$

$$= 0.2 + 0.5 = 0.5 + 0.2$$

$$= 0.1 + 0.6 = 0.1 + 0.2 \times 3 = 0.1 + 3 \times 0.2$$

$$= 0.3 + 0.4 = 0.3 + 0.2 \times 2 = 0.1 \times 3 + 1 \times 0.2 \times 2$$

$$= 1 \times 0.1 \times 3 + 1 \times 0.2 \times 2 \times 1$$

$$0.8 = 8 \times 0.1 = 0.1 + 0.1 + 0.1 + \dots + 0.1 = 0.1 \times 8$$

$$= 0.2 + 0.2 + 0.2 + 0.2 = 0.2 \times 4$$

$$= 0.4 + 0.4 = 0.4 \times 2$$

$$= 0.3 + 0.5 = 0.5 + 0.3$$

$$0.9 = 0.9 \times 1 = 0.1 + 0.1 + 0.1 + \dots + 0.1 = 0.1 \times 9$$

$$= 0.4 + 0.5 = 0.5 + 0.4$$

$$= 0.4 + (0.4 + 0.1) = 0.4 + 0.4 + 0.1 = 0.4 \times 2 + 0.1 = 2 \times 0.4 + 0.1$$

$$= 0.8 + 0.1 = \mathbf{2 \times 0.4 + 0.1 = 4 \times 0.2 + 0.1}$$

$$= 0.3 + 0.3 + 0.3 = 0.3 \times 3 = 0.3 + 0.6$$

$$= 0.3 + (0.3 + 0.3) = 0.3 + 0.3 \times 2 = 0.3 \times 1 + 0.3 \times 2$$

$$= 0.3 \times (1 + 2) = 0.3 \times 3 = 0.6 + 0.3$$

$$= (0.3 + 0.3) + 0.3 = 0.3 \times 2 + 0.3 = 0.3 \times 2 + 0.3 \times 1$$

$$= 0.3 \times (2 + 1) = 0.3 \times 3 = 0.2 + 0.7 = 0.7 + 0.2$$

$$= 0.2 + (0.2 + 0.5) = 0.2 + 0.2 + 0.5$$

$$= 0.2 \times 2 + 0.5 = 0.2 \times 2 + (0.2 + 0.3)$$

$$= 0.2 \times 2 + 0.2 + 0.3 = 0.2 \times 2 + 0.2 \times 1 + 0.3$$

$$= 0.2 \times (2 + 1) + 0.3 = \mathbf{0.2 \times 3 + 0.3 = 0.3 \times 2 + 0.3 \times 1}$$

$$= 0.3 \times (2 + 1) = 0.3 \times 3 = (0.2 + 0.1) \times (2 + 1)$$

$$= (0.2 + 0.1) \times 2 + (0.2 + 0.1) \times 1 = (0.2 \times 2 + 0.1 \times 2) + (0.2 \times 1 + 0.1 \times 1)$$

$$= 0.2 \times 2 + 0.2 + 0.2 + 0.1 = 0.4 + 0.2 + 0.2 + 0.1$$

$$= 0.4 + 0.2 \times 2 + 0.1 = 0.4 + 0.4 + 0.1$$

$$= 0.4 \times 2 + 0.1 = 0.8 + 0.1$$

$$1 = 10 \times 0.1 + 1 \times 0$$

$$= 0.1 + 0.9 = 0.9 + 0.1$$

$$= 0.2 + 0.8 = 0.8 + 0.2 = 0.8 + 0.1 + 0.1 = 0.8 + 0.1 \times 2$$

$$= 0.3 + 0.7 = 0.7 + 0.3 = 0.7 + 0.2 + 0.1$$

$$= 0.4 + 0.6 = 0.6 + 0.4 = 0.6 + 0.2 + 0.2 = 0.6 + 0.2 \times 2$$

$$= 0.5 + 0.5 = \mathbf{0.5 \times 2 = 0.2 \times 5}$$

$$= 0.1 + 0.9 = 0.1 \times 1 + 0.9 \times 1 = 1 \times 0.1 + 0.1 \times 9$$

$$= 0.1 + 0.1 + 0.8 = 0.1 \times 2 + 0.8 \times 1 = 2 \times 0.1 + 0.1 \times 8$$

$$= 0.1 + 0.1 + 0.1 + 0.7 = 0.1 \times 3 + 0.7 \times 1$$

$$= 0.3 \times 1 + 0.1 \times 7 = 0.1 + 0.2 + 0.7$$

$$= 0.1 + 0.1 + 0.1 + 0.1 + 0.6 = 0.2 + 0.2 + 0.6$$

$$= 0.2 \times 2 + 0.3 + 0.3$$

$$= 0.3 \times 2 + 2 \times 0.2 = 0.2 \times 3 + 0.2 \times 2 = 0.2 \times (3 + 2) = 0.2 \times 5$$

$$= 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.5$$

$$= 0.1 \times 5 + 0.5 \times 1 = 5 \times 0.1 + 5 \times 0.1 = 5 \times (0.1 + 0.1)$$

$$= 5 \times 0.2 = 2 \times 0.5$$

$$= (0.1 + 0.1) + (0.1 + 0.1) + (0.1 + 0.1) + (0.1 + 0.1) + (0.1 + 0.1)$$

$$= 0.2 + 0.2 + 0.2 + 0.2 + 0.2$$

$$= 0.2 \times 5 = 0.5 \times 2$$

$$= (0.1 + 0.1 + 0.1 + 0.1 + 0.1) + (0.1 + 0.1 + 0.1 + 0.1 + 0.1)$$

$$= 0.5 + 0.5 = 5 \times 0.1 + 5 \times 0.1 = 2 \times 5 \times 0.1 = 10 \times 0.1$$

$$= 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1$$

$$= 0.1 + (0.1 + 0.1) + (0.1 + 0.1 + 0.1) + (0.1 + 0.1 + 0.1 + 0.1)$$

$$= 0.1 + 0.2 + 0.3 + 0.4 = (0.1 + 0.2) + (0.3 + 0.4) = 0.3 + 0.7$$

$$= 0.1 \times 3 + 0.1 \times 7 = 0.1 \times (3 + 7) = 0.1 \times 10$$

$$= 0.1 + 0.2 + 0.3 + 0.4$$

$$= 0.2 + 0.4 + 0.1 + 0.3$$

$$= 0.2 + 0.1 \times (4 + 1) + 0.3$$

$$= 0.2 + 0.5 + 0.3 = 0.2 + 0.1 \times (5 + 3) = 0.2 + 0.8$$

$$1.1 = 1 + 0.1 = 0.5 \times 2 + 0.1 = 0.5 + 0.5 + 0.1$$

$$= 0.1 \times 5 + 0.1 \times 5 + 0.1 = 0.1 \times (5 + 5 + 1)$$

$$= 0.1 \times 11$$

$$\begin{aligned}
1.2 &= 1 + 0.2 = 0.5 \times 2 + 0.2 = 0.2 \times 5 + 0.2 \times 1 \\
&= 0.2 \times (5 + 1) = 0.2 \times 6 = 0.6 \times 2 = 0.6 + 0.6 \\
&= 0.4 + 0.2 + 0.4 + 0.2 = 0.4 \times 2 + 0.2 + 2 = 0.4 \times 2 + 0.4 = 0.4 \times (2 + 1) \\
&= 0.4 \times 3 = 0.1 \times 4 \times 3 = 0.1 \times 3 \times 4 = 0.3 \times 4
\end{aligned}$$

$$\begin{aligned}
1.3 &= 1 + 0.3 = 0.5 \times 2 + 0.3 = 5 \times 0.2 + 0.2 + 0.1 \\
&= (5 + 1) \times 0.2 + 0.1 = 6 \times 0.2 + 0.1 = 1.2 + 0.1 \\
&= 0.1 \times 12 + 0.1 \times 1 = 0.1 \times (12 + 1) = 0.1 \times 13
\end{aligned}$$

$$\begin{aligned}
1.4 &= 1 + 0.4 = 0.5 \times 2 + 0.4 \\
&= 0.2 \times 5 + 0.2 \times 2 = 0.2 \times (5 + 2) = 0.2 \times 7 = 0.7 \times 2 = 0.1 \times 7 \times 2 = 0.1 \times 14
\end{aligned}$$

$$\begin{aligned}
1.5 &= 1 + 0.5 = 0.5 \times 2 + 0.5 = 0.5 \times (2 + 1) = 0.5 \times 3 \\
&= 0.1 \times 5 \times 3 = 0.1 \times 15 = 15 \times 0.1
\end{aligned}$$

$$\begin{aligned}
1.6 &= 1 + 0.6 = 1 + 0.5 + 0.1 = 0.5 \times 2 + 0.5 + 0.1 = 0.5 + 0.5 + 0.5 + 0.1 \\
&= 0.5 + 0.1 + 0.4 + 0.5 + 0.1 = 0.6 + 0.4 + 0.6 = 0.6 \times 2 + 0.4 \\
&= 0.1 \times 6 \times 2 + 0.4 = 0.1 \times 12 + 0.4 = 1.2 + 0.4 = (1 + 0.2) + 0.4
\end{aligned}$$

$$= 1 + 0.2 + 0.4 = 0.5 + 0.5 + 0.2 + 0.4 = 0.4 + 0.1 + 0.4 + 0.1 + 0.2 + 0.4$$

$$= 0.4 + 0.4 + 0.4 + 0.1 + 0.1 + 0.2 = 0.4 + 0.4 + 0.4 + 0.4 = 0.4 \times 4$$

$$= 0.4 \times 3 + 0.1 + 0.1 + 0.2 = 0.4 \times 3 + 0.4 = 0.4 \times (3 + 1)$$

$$= 0.4 \times (1 + 2 + 1) = 0.4 \times 1 + 0.4 \times 2 + 0.4 \times 1$$

$$= 0.4 + 0.8 + 0.4 = 0.4 \times 2 + 0.8 = 0.8 + 0.8 = 0.8 \times 2$$

$$= 0.1 \times 8 \times 2 = 0.1 \times 2 \times 8 = 0.2 \times 8 = 0.1 \times 2 \times 8 = 0.1 \times 16 = 0.1 \times (4 + 12)$$

$$= 0.4 + 1.2 = 0.4 + 0.6 + 0.6 = 0.2 + 0.2 + 0.6 + 0.6 = 0.2 \times 2 + 0.6 \times 2$$

$$= 2 \times 0.2 + 6 \times 0.2 = 8 \times 0.2 = 0.2 \times 8$$

$$= 0.2 \times (7 + 1) = 0.1 \times 2 \times 7 + 0.1 \times 2 = 0.1 \times (14 + 2)$$

$$1.7 = 1 + 0.7 = 0.5 \times 2 + 0.7$$

$$= 0.5 \times 2 + (0.5 + 0.2) = 0.5 \times 3 + 0.2 = 0.1 \times 5 \times 3 + 0.1 \times 2 = 0.1 \times (15 + 2)$$

$$1.8 = 1 + 0.8 = 0.5 \times 2 + 0.8 = 0.5 \times 2 + 0.4 \times 2 = (0.5 + 0.4) \times 2$$

$$= 0.9 \times 2 = 2 \times 0.9 = 0.1 \times 10 + 0.8 = 0.1 \times (6 + 4) + 0.8 = 0.6 + 0.4 + 0.8$$

$$= 0.6 + 0.4 + 0.6 + 0.2 = 0.6 + 0.6 + 0.4 + 0.2 = 0.6 + 0.6 + 0.6$$

$$= 0.6 \times 3 = 0.3 \times 6 = 0.1 \times 3 \times 6 = 0.1 \times 3 \times 3 \times 2 = 0.1 \times 2 \times 9 = 0.2 \times 9$$

$$1.9 = 1 + 0.9 = 0.5 \times 2 + 0.5 + 0.4$$

$$= 0.5 \times 3 + 0.4 = 5 \times 0.3 + 0.4 = 0.1 \times 5 \times 3 + 0.1 \times 4 = 0.1 \times (15 + 4)$$

$$2 = 1 \times 2 = (0.5 \times 2) \times 2 = 0.5 \times 2 \times 2 = 0.5 \times 4$$

$$= 0.4 \times 5 = 0.1 \times 4 \times 5 = 0.1 \times 20$$

$$2.1 = 1 \times 2 + 0.1$$

$$2.2 = 2 + 0.2 = 1 \times 2 + 0.1 \times 2 = (1 + 0.1) \times 2 = 1.1 \times 2$$

$$= 0.1 \times 11 \times 2 = 0.1 \times 2 \times 11 = 0.2 \times 11$$

$$2.3 = 2 + 0.3 = 1 \times 2 + 0.1 \times 3$$

$$9.8 = 1 \times 9 + 0.8$$

$$9.9 = 1 \times 9 + 0.9$$

$$0.1 \times 10 = 1, \text{ and } 10 \times 0.1 = 1.$$

$$10 \div 10 = 1 \div 1 = 2 \div 2 = 5 \div 5 = 47 \div 47 = 7.2 \div 7.2 = 198 \div 198 = \dots = 1$$

$$0.1 \times 100 = 0.1 \times 10 \times 10 = 1 \times 10 = 10$$

$$100 \times 0.1 = 10 \times 10 \times 0.1 = 10 \times 1 = 10$$

$10 \div 1 = 10$ because 10 has 10 of 1s, that is, $10 \times 1 = 10$.

And 10 has 1 of 10s, so $1 \times 10 = 10$. Thus, $1 \times 10 = 10 \times 1 = 10$.

$100 \div 10 = 10$ because 100 has 10 of 10s, that is, $10 \times 10 = 100$.

$1000 \div 100 = 10$, for 1000 has 10 of 100s, that is, $10 \times 100 = 1000$.

And 1000 has 100 of 10s, so $100 \times 10 = 1000$.

Thus, $100 \times 10 = 10 \times 100 = 1000$.

$10000 \div 1000 = 10$, because 10000 has 10 of 1000s,

that is, $10 \times 1000 = 10000$.

And 10000 has 1000 of 10s, so $1000 \times 10 = 10000$.

So $1000 \times 10 = 10 \times 1000 = 10000$.

$30 \div 3 = 10$, because 30 has 10 of 3s, that is, $10 \times 3 = 30$.

And 30 has 3 of 10s, so $3 \times 10 = 30$. Thus, $3 \times 10 = 10 \times 3 = 30$.

$50 \div 5 = 10$, because 50 has 10 of 5s, that is, $10 \times 5 = 50$.

And 50 has 5 of 10s, so $5 \times 10 = 50$. Thus, $5 \times 10 = 10 \times 5 = 50$.

$20 \div 2 = 10$, because 20 has 10 of 2s, that is, $10 \times 2 = 20$.

And 20 has 2 of 10s, so $2 \times 10 = 20$. Thus, $2 \times 10 = 10 \times 2 = 20$.

$170 \div 17 = 10$ because 170 has 10 of 17s, that is, $10 \times 17 = 170$.

And 170 has 17 of 10s, so $17 \times 10 = 170$. Thus, $17 \times 10 = 10 \times 17 = 170$.

$17 \times 10 = (10 + 7) \times 10 = 10 \times 10 + 7 \times 10 = 100 + 70 = 170$.

$3750 \div 375 = 10$, because 3750 has 10 of 375s, that is, $3750 = 10 \times 375$.

And 3750 has 375 of 10s, so $375 \times 10 = 3750$.

Thus, $375 \times 10 = 10 \times 375 = 3750$.

$375 \times 10 = (300 + 70 + 5) \times 10 = 300 \times 10 + 70 \times 10 + 5 \times 10$

$= 3000 + 700 + 50 = 3750$.

$1 \div 0.1 = 10$, because 1 has 10 of 0.1s, that is, $1 = 10 \times 0.1$.

And 1 is one tenth of 10, that is, 1 is 0.1 of 10, so $0.1 \times 10 = 1$.

Thus, $10 \times 0.1 = 0.1 \times 10 = 1$.

$2 \div 0.2 = 10$, because 2 has 10 of 0.2s, that is, $2 = 10 \times 0.2$.

And 2 is two tenths of 10, that is, 2 is 0.2 of 10, so $0.2 \times 10 = 2$.

Thus, $0.2 \times 10 = 10 \times 0.2 = 2$.

$0.2 \times 10 = (0.1 + 0.1) \times 10 = 0.1 \times 10 + 0.1 \times 10 = 1 + 1 = 2$.

$5 \div 0.5 = 10$, because 5 has 10 of 0.5s, that is, $10 \times 0.5 = 5$.

And 5 is five tenths of 10, that is, 5 is 0.5 of 10, so $0.5 \times 10 = 5$.

Thus, $0.5 \times 10 = 10 \times 0.5 = 5$.

$0.5 \times 10 = 0.1 \times 5 \times 10 = 0.1 \times 10 \times 5 = 1 \times 5 = 5$.

$19 \div 1.9 = 10$, because 19 has 10 of 1.9s, that is, $10 \times 1.9 = 19$.

And 19 is one and nine tenths of 10, in other words, 19 is $(1 + 0.9)$ of 10.

So $(1 + 0.9) \times 10 = 1.9 \times 10 = 19$. Thus, $1.9 \times 10 = 10 \times 1.9 = 19$.

$1.9 \times 10 = 0.1 \times 19 \times 10 = 0.1 \times 10 \times 19 = 1 \times 19 = 19$.

$174 \div 17.4 = 10$, because 174 has 10 of 17.4s, that is, $10 \times 17.4 = 174$.

And 174 is seventeen and four tenths of 10,

in other words, 174 is $(17 + 0.4)$ of 10.

So $(17 + 0.4) \times 10 = 17.4 \times 10 = 174$. Thus, $17.4 \times 10 = 10 \times 17.4 = 174$.

$$17.4 \times 10 = (10 + 7 + 0.4) \times 10 = 10 \times 10 + 7 \times 10 + 0.4 \times 10$$

$$= 100 + 70 + 0.1 \times 4 \times 10 = 100 + 70 + 0.1 \times 10 \times 4$$

$$= 100 + 70 + 1 \times 4 = 174.$$

Now, $0.1 \times 100 = 0.1 \times 10 \times 10 = 1 \times 10 = 10$,

$$100 \times 0.1$$

$$= 10 \times 10 \times 0.1 = 10 \times 1 = 10, \text{ and } 10 \div 1 = 100 \div 10$$

$$= 1000 \div 100 = 10000 \div 1000$$

$$= 30 \div 3 = 50 \div 5 = 20 \div 2 = 170 \div 17 = 3750 \div 375$$

$$= 1 \div 0.1 = 2 \div 0.2 = 5 \div 0.5 = 19 \div 1.9 = 174 \div 17.4 = 10.$$

And we often use $/$ (a slash) as a division sign instead of $\div$.

So we can do this:

$$10 / 1$$

$$= 100 / 10$$

$$= 1000 / 100$$

$$= 10000 / 1000$$

$$= 30 / 3$$

$$= 50 / 5$$

$$= 20 / 2$$

$$= 170 / 17$$

$$= 3750 / 375$$

$$= 1 / 0.1$$

$$= 2 / 0.2$$

$$= 5 / 0.5$$

$$= 19 / 1.9$$

$$= 174 / 17.4$$

$$= 10.$$

Now, we have $1 \times 0.1 = 0.1$, and have $1/10 = 0.1$, since $1/10 = 1 \div 10 = 0.1$.

So we can get these:

$$2/10 = (1 + 1)/10 = 1/10 + 1/10 = 2 \times (1/10) = 2 \times 0.1 = 0.2.$$

$$3/10 = (1 + 2)/10 = 1/10 + 2/10 = 0.1 + 0.2 = 0.3$$

$$4/10 = (1 + 3)/10 = 1/10 + 3/10 = 0.1 + 0.3 = 0.4$$

...

$$9/10 = (1 + 8)/10 = 1/10 + 8/10 = 0.1 + 0.8 = 0.9$$

And next, we have $1/100 = 0.01$. So we can get

$$2/100 = (1 + 1)/100 = 1/100 + 1/100 = 0.01 + 0.01 = 0.02$$

$$3/100 = (1 + 2)/100 = 1/100 + 2/100 = 0.01 + 0.02 = 0.03$$

$$4/100 = (1 + 3)/100 = 1/100 + 3/100 = 0.01 + 0.03 = 0.04$$

...

$$9/100 = (1 + 8)/100 = 1/100 + 8/100 = 0.01 + 0.08 = 0.09$$

And next, we have $0.1/10 = 0.01$. So we can get these:

$$0.2/10 = (0.1 + 0.1)/10 = 0.1/10 + 0.1/10 = 2 \times (0.1/10) = 2 \times 0.01 = 0.02.$$

$$0.3/10 = (0.1 + 0.2)/10 = 0.1/10 + 0.2/10 = 0.01 + 0.02 = 0.03$$

$$0.4/10 = (0.1 + 0.3)/10 = 0.1/10 + 0.3/10 = 0.01 + 0.03 = 0.04$$

...

$$0.9/10 = (0.1 + 0.8)/10 = 0.1/10 + 0.8/10 = 0.01 + 0.08 = 0.09$$

Now, we have $1/10 = 0.1$, $1/100 = 0.01$, and $0.1/10 = 0.01$.

And also, we know $0.1 \times 1 = 1 \times 0.1 = 0.1$, and $0.1 \times 10 = 10 \times 0.1 = 1$.

So we can get $10 \times 0.1 = 10 \times (1/10) = 10/10 = 1$.

Also, $0.1 \times 0.1 = 0.1/10 = 0.01$, since $0.1/10 = 0.1 \div 10$.

Thus, $0.1 \times 0.1 = 0.01$.

So we can get $0.01 = 0.1 \times 0.1 = (1/10) \times (1/10) = (1/10)/10$.

And next, we have $(1/10)/10 = 1/10/10$.

And we have $0.1 = 1/10$, and $0.1/10 = 0.01$.

So we get $1/10/10 = 0.1/10 = 0.01$.

Thus, we get $1/10/10 = 0.01 = 1/100$. So we get $1/10/10 = 1/100$.

What is $1/10$ though?

Dividing 1 into ten (10) equal parts, and taking one of the parts,
we get $1/10 = 0.1$. What then is $(1/10)/10$?

Dividing $1/10$ into ten (10) equal parts, and taking one of the parts,
we get $(1/10)/10$, which is $1/10/10$.

And dividing 0.1 into 10 equal parts, and taking one of the parts,
we get 0.01, which is $1/100$.

We know $1/10 = 0.1$. So we get $1/10/10 = 1/100$.

And also, dividing 1 into a hundred equal parts, and taking one of the parts,
we get $1/100$. Thus, we get $(1/10)/10 = 1/10/10 = 0.1/10 = 1/100 = 0.01$.

And by the same token, we can get

$$1/10/10/10 = 1/100/10 = 1/1000 = 0.001,$$

$$1/10/10/10/10 = 1/10/100/10 = 1/10/1000 = 1/1000/10 = 1/10000 = 0.0001.$$

And again, by the same token, we can get

$$3/10 = 0.3, 3/10/10 = 3/100 = 0.03, 3/10/10/10 = 3/10/100 = 3/1000 = 0.003,$$

...

Also, we can get

$$3/2/7 = 3/14$$

$$3/2/7/5 = 3/14/5 = 3/2/35 = 3/70$$

$$10 = (0.1 + 0.9) \times 10 = (0.9 + 0.1) \times 10 = 9 + 1$$

$$= (0.2 + 0.8) \times 10 = (0.8 + 0.2) \times 10 = 8 + 1 + 1$$

$$= (0.3 + 0.7) \times 10 = (0.7 + 0.3) \times 10 = 7 + 2 + 1$$

$$= (0.4 + _) \times 10 = (0.6 + _) \times 10 = 6 + 2 + 2 = 6 + 2 \times _$$

$$= (0.5 + _) \times 10 = 0.5 \times 2 \times _ = 0.1 \times 2 \times 5 \times _$$

$$= (0.1 + _) \times 10 = 0.1 \times 10 + 0.9 \times _ = 1 + _ \times 9$$

$$= (0.1 + 0.1 + 0.8) \times 10 = (0.1 \times 2 + 0.8 \times 1) \times 10 = (0.2 \times 1 + 0.1 \times 8) \times 10$$

$$= (0.1 + 0.1 + 0.1 + 0.7) \times 10 = (0.1 \times 3 + 0.7 \times 1) \times 10$$

$$= (3 \times 0.1 + 0.1 \times 7) \times 10 = (0.3 \times 1 + 7 \times 0.1) \times 10 = (0.1 + 0.2 + 0.7) \times 10$$

$$= (0.1 + 0.1 + 0.1 + 0.1 + 0.6) \times 10 = (0.2 + 0.2 + 0.6) \times 10$$

$$= (0.2 \times 2 + 0.3 + 0.3) \times 10 = (0.3 \times 2 + 0.2 \times 2) \times 10$$

$$= (2 \times 0.3 + 2 \times 0.2) \times 10 = 2 \times (0.3 + 0.2) \times 10 = 2 \times 0.5 \times 10$$

$$= (0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.5) \times 10$$

$$= (0.1 \times 5 + 0.5 \times 1) \times 10$$

$$= (5 \times 0.1 + 5 \times 0.1) \times 10 = (5 \times 0.1) \times 2 \times 10$$

$$= 0.5 \times (1 + 1) \times 10 = 0.5 \times 2 \times 10 = (0.5 + 0.5) \times 10$$

$$= \{(0.1 + 0.1 + 0.1 + 0.1 + 0.1) + (0.1 + 0.1 + 0.1 + 0.1 + 0.1)\} \times 10$$

$$= \{(0.2 + 0.3) + (0.1 + 0.4)\} \times 10 = (0.2 + 0.3) \times 10 + (0.1 + 0.4) \times 10$$

$$= 2 + 3 + 1 + 4 = 10 = 0.1 \times 10 \times 10$$

$$= (0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1) \times 10$$

$$= \{0.1 + (0.1 + 0.1) + (0.1 + 0.1 + 0.1) + (0.1 + 0.1 + 0.1 + 0.1)\} \times 10$$

$$= (0.1 + 0.2 + 0.3 + 0.4) \times 10$$

$$= \{(0.1 + 0.2) + (0.3 + 0.4)\} \times 10 = (0.3 + 0.7) \times 10$$

$$= 1 \times 10 = (0.1 + 0.2 + 0.3 + 0.4) \times 10 = (0.2 + 0.4 + 0.1 + 0.3) \times 10$$

$$= \{0.2 + (0.4 + 0.1) + 0.3\} \times 10 = (0.2 + 0.5 + 0.3) \times 10$$

$$= \{0.2 + (0.5 + 0.3)\} \times 10 = (0.2 + 0.8) \times 10 = 1 \times 10$$

$$1.01 = 1 + 0.01 = 1 \times 1 + 0.01 \times 1 = 1 \times 1 + 0.1 \times 0 + 0.01 \times 1$$

$$1.02 = 1 + 0.02 = 1 \times 1 + 0.01 \times 2 = 1 \times 1 + 0.1 \times 0 + 0.01 \times 2$$

$$1.03 = 1 + 0.03 = 1 \times 1 + 0.01 \times 3 = 1 \times 1 + 0.1 \times 0 + 0.01 \times 3$$

$$1.08 = 1 + 0.08 = 1 \times 1 + 0.01 \times 8 = 1 \times 1 + 0.1 \times 0 + 0.01 \times 8$$

$$1.09 = 1 + 0.09 = 1 \times 1 + 0.1 \times 0 + 0.01 \times 9$$

$$0.11 = 0.1 + 0.01 = 1 \times 0 + 0.1 \times 1 + 0.01 \times 1$$

$$0.111 = 0.1 + 0.01 + 0.001 = 0.1 \times 1 + 0.01 \times 1 + 0.001 \times 1$$

$$0.112 = 0.1 + 0.01 + 0.002 = 0.1 \times 1 + 0.01 \times 1 + 0.001 \times 2$$

$$0.113 = 0.1 \times 1 + 0.01 \times 1 + 0.001 \times 3 = 1 \times 0.1 + 1 \times 0.01 + 3 \times 0.001$$

$$0.114 = 0.1 + 0.01 + 0.004 = 1 \times 0.1 + 1 \times 0.01 + 4 \times 0.001$$

$$0.117 = 0.1 + 0.01 + 0.007 = 1 \times 0.1 + 1 \times 0.01 + 7 \times 0.001$$

$$\begin{aligned} 0.127 &= 0.1 + 0.02 + 0.007 = 0.1 \times 1 + 0.01 \times 2 + 0.001 \times 7 \\ &= 1 \times 0.1 + 2 \times 0.01 + 7 \times 0.001 \end{aligned}$$

$$\begin{aligned} 0.128 &= 0.1 + 0.02 + 0.008 = 0.1 \times 1 + 0.01 \times 2 + 0.001 \times 8 \\ &= 1 \times 0.1 + 2 \times 0.01 + 8 \times 0.001 \end{aligned}$$

$$\begin{aligned} 0.129 &= 0.1 + 0.02 + 0.009 = 0.1 \times 1 + 0.01 \times 2 + 0.001 \times 9 \\ &= 1 \times 0.1 + 2 \times 0.01 + 9 \times 0.001 \end{aligned}$$

$$0.13 = 0.1 + 0.03 = 0.1 \times 1 + 0.01 \times 3 + 0.001 \times 0$$

$$= 1 \times 0.1 + 3 \times 0.01 + 0 \times 0.001$$

$$0.131 = 0.1 + 0.03 + 0.001 = 0.1 \times 1 + 0.01 \times 3 + 0.001 \times 1$$

$$= 1 \times 0.1 + 3 \times 0.01 + 1 \times 0.001$$

$$0.132 = 0.1 + 0.03 + 0.002 = 0.1 \times 1 + 0.01 \times 3 + 0.001 \times 2$$

$$= 1 \times 0.1 + 3 \times 0.01 + 2 \times 0.001 = 1 \times 0.1 + 3 \times 0.1 \times 0.1 + 2 \times 0.1 \times 0.01$$

$$= 1 \times 0.1 + 3 \times 0.1 \times 0.1 + 2 \times 0.1 \times 0.1 \times 0.1$$

$$0.199 = 0.1 + 0.09 + 0.009 = 0.1 \times 1 + 0.01 \times 9 + 0.001 \times 9$$

$$= 1 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 = 1 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.01$$

$$= 1 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1$$

$$0.2 = \dots + 100 \times 0 + 10 \times 0 + 1 \times 0 + 0.1 \times 2 + 0.01 \times 0 + 0.001 \times 0 + \dots$$

$$= \dots + 0 \times 100 + 0 \times 10 + 0 \times 1 + 2 \times 0.1 + 0 \times 0.01 + 0 \times 0.001 + \dots$$

$$= \dots + 0 \times 10 \times 10 + 0 \times 10 + 0 \times 1 + 1 \times 0.1 + 10 \times 0.1 \times 0.1$$

$$+ 0 \times 0.1 \times 0.1 \times 0.1 + \dots$$

0.201

$$= 0.2 + 0.001$$

$$= 0.1 \times 2 + 0.01 \times 0 + 0.001 \times 1$$

$$= 2 \times 0.1 + 0 \times 0.01 + 1 \times 0.001$$

$$= 2 \times 0.1 + 0 \times 0.1 \times 0.1 + 1 \times 0.1 \times 0.1 \times 0.1$$

0.248

$$= 0.2 + 0.04 + 0.008$$

$$= 0.1 \times 2 + 0.01 \times 4 + 0.001 \times 8$$

$$= 2 \times 0.1 + 4 \times 0.01 + 8 \times 0.001$$

$$= 2 \times 0.1 + 4 \times 0.1 \times 0.1 + 8 \times 0.1 \times 0.1 \times 0.1$$

0.249

$$= 0.2 + 0.04 + 0.009$$

$$= 0.1 \times 2 + 0.01 \times 4 + 0.001 \times 9$$

$$= 2 \times 0.1 + 4 \times 0.01 + 9 \times 0.001$$

$$= 2 \times 0.1 + 4 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1$$

$$0.794$$

$$= 0.7 + 0.09 + 0.004$$

$$= 0.1 \times 7 + 0.01 \times 9 + 0.001 \times 4$$

$$= 7 \times 0.1 + 9 \times 0.01 + 4 \times 0.001$$

$$= 7 \times 0.1 + 9 \times 0.1 \times 0.1 + 4 \times 0.1 \times 0.1 \times 0.1$$

$$0.795$$

$$= 0.7 + 0.09 + 0.005$$

$$= 0.1 \times 7 + 0.01 \times 9 + 0.001 \times 5$$

$$= 7 \times 0.1 + 9 \times 0.01 + 5 \times 0.001$$

$$= 7 \times 0.1 + 9 \times 0.1 \times 0.1 + 5 \times 0.1 \times 0.1 \times 0.1$$

$$0.796$$

$$= 0.7 + 0.09 + 0.006$$

$$= 0.1 \times 7 + 0.01 \times 9 + 0.001 \times 6$$

$$= 7 \times 0.1 + 9 \times 0.01 + 6 \times 0.001$$

$$= 7 \times 0.1 + 9 \times 0.1 \times 0.1 + 6 \times 0.1 \times 0.1 \times 0.1$$

$$0.997$$

$$= 0.9 + 0.09 + 0.007$$

$$= 0.1 \times 9 + 0.01 \times 9 + 0.001 \times 7$$

$$= 9 \times 0.1 + 9 \times 0.01 + 7 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 7 \times 0.1 \times 0.1 \times 0.1$$

Note that $10 = 10^1$, $0.1 = 0.1^1$, and $1/10 = 1/10^1 = (1/10)^1$.

$$\{10, 100, 1000 \dots\} = \{10, 10 \times 10, 10 \times 10 \times 10 \dots\}$$

$$= \{10^1, 10^2, 10^3 \dots\}$$

$$\{0.1, 0.01, 0.001 \dots\} = \{0.1, 0.1 \times 0.1, 0.1 \times 0.1 \times 0.1 \dots\}$$

$$= \{0.1^1, 0.1^2, 0.1^3 \dots\} = \{1/10, 1/100, 1/1000 \dots\}$$

$$= \{1/10^1, 1/10^2, 1/10^3 \dots\} = \{1/10, 1/10/10, 1/10/10/10 \dots\}$$

$$= \{1/10, (1/10) \times (1/10), (1/10) \times (1/10) \times (1/10) \dots\}$$

$$= \{(1/10)^1, (1/10)^2, (1/10)^3 \dots\}$$

$$0.998$$

$$= 0.9 + 0.09 + 0.009$$

$$= 0.1 \times 9 + 0.01 \times 9 + 0.001 \times 8$$

$$= 9 \times 0.1 + 9 \times 0.01 + 8 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 8 \times 0.1 \times 0.1 \times 0.1$$

$$0.999$$

$$= 0.9 + 0.09 + 0.009$$

$$= 0.1 \times 9 + 0.01 \times 9 + 0.001 \times 9$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1$$

$$1$$

$$= 0.999 + 0.001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1 + 1 \times 0.1 \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + (9 + 1) \times 0.1 \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 10 \times 0.1 \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 1 \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + (9 + 1) \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + 10 \times 0.1 \times 0.1$$

$$= 9 \times 0.1 + 1 \times 0.1 = (9 + 1) \times 0.1 = 10 \times 0.1 = 0.1 \times 10$$

$$1 = 0.01 \times 100$$

$$= (0.001 + 0.009) \times 100$$

$$= 0.001 \times (1 + 9) \times 100 = 0.001 \times 10 \times 100 = 0.001 \times 1000$$

$$= (0.002 + 0.008) \times 100 = (0.003 + 0.007) \times 100$$

$$= (0.004 + 0.006) \times 100 = (0.005 + 0.005) \times 100$$

$$= 0.1 \times 10$$

$$= (0.01 + 0.09) \times 10 = 0.01 \times (1 + 9) \times 10 = 0.01 \times 10 \times 10 = 0.01 \times 100$$

$$= (0.02 + 0.08) \times 10 = (0.03 + 0.07) \times 10$$

$$= (0.04 + 0.07) \times 10 = (0.05 + 0.05) \times 10$$

$$= 0.1 + 0.9 = 0.1 \times (1 + 9) = 0.1 \times 10 = 0.2 + 0.8 = 0.3 + 0.7$$

$$= 0.4 + 0.6 = 0.5 + 0.5$$

0.1985

$$= 0.1 + 0.09 + 0.008 + 0.0005$$

$$= 0.1 \times 1 + 0.01 \times 9 + 0.001 \times 8 + 0.0001 \times 5$$

$$= 1 \times 0.1 + 9 \times 0.01 + 8 \times 0.001 + 5 \times 0.0001$$

0.3092

$$= 0.3 + 0.009 + 0.0002$$

$$= 0.1 \times 3 + 0.01 \times 0 + 0.001 \times 9 + 0.0001 \times 2$$

$$= 3 \times 0.1 + 9 \times 0.001 + 2 \times 0.0001$$

0.8964

$$= 0.8 + 0.09 + 0.006 + 0.0004$$

$$= 8 \times 0.1 + 9 \times 0.01 + 6 \times 0.001 + 4 \times 0.0001$$

0.8974

$$= 0.8 + 0.0974 = 0.8 + 0.09 + 0.0074 = 0.8 + 0.09 + 0.007 + 0.0004$$

$$= 0.89 + 0.007 + 0.0004 = 0.897 + 0.0004 = 0.807 + 0.0904$$

$$= 0.8004 + 0.097 = 0.0004 + 0.897$$

$$0.9999$$

$$= 0.9 + 0.09 + 0.009 + 0.0009$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + 9 \times 0.0001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.01 + 9 \times 0.1 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.01$$

$$= 9 \times 0.1 + 9 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1 + 9 \times 0.1 \times 0.1 \times 0.1 \times 0.1$$

$$1 = 0.9999 + 0.0001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + 9 \times 0.0001 + 0.0001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + (9 + 1) \times 0.0001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + 10 \times 0.0001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + 10 \times 0.1 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 9 \times 0.001 + 1 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.01 + (9 + 1) \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 10 \times 0.001$$

$$= 9 \times 0.1 + 9 \times 0.01 + 1 \times 0.01$$

$$= 9 \times 0.1 + (9 + 1) \times 0.01 = 9 \times 0.1 + 10 \times 0.01$$

$$= 9 \times 0.1 + 1 \times 0.1 = (9 + 1) \times 0.1 = 10 \times 0.1 = 1$$

$$6974.835$$

$$= 6000 + 900 + 70 + 4 + 0.8 + 0.03 + 0.005$$

$$= 6 \times 1000 + 9 \times 100 + 7 \times 10 + 4 \times 1 + 8 \times 0.1 + 3 \times 0.01 + 5 \times 0.001$$

$$= 6 \times 10 \times 100 + 9 \times 10 \times 10 + 7 \times 10 + 4 \times (10/10)$$

$$+ 8 \times 0.1 + 3 \times 0.1 \times 0.1 + 5 \times 0.1 \times 0.01$$

$$= 6 \times 10 \times 10 \times 10 + 9 \times 10 \times 10 + 7 \times 10$$

$$+ 4 \times (10/10) + 8 \times 0.1 + 3 \times 0.1 \times 0.1 + 5 \times 0.1 \times 0.1 \times 0.1$$

$$= 6 \times 10 \times 10 \times 10 + 9 \times 10 \times 10 + 7 \times 10 + 4 \times (10/10) + 8 \times (1/10)$$

$$+ 3 \times (1/10) \times (1/10) + 5 \times (1/10) \times (1/10) \times (1/10)$$

$$= 6 \times 1000 + 9 \times 100 + 7 \times 10 + 4 \times 1 + 8 \times (1/10)$$

$$+ 3 \times (1/100) + 5 \times (1/1000)$$

$$= 6 \times 10^3 + 9 \times 10^2 + 7 \times 10^1 + 4 \times 10^0 + 8 \times 10^{-1} + 3 \times 10^{-2} + 5 \times 10^{-3}$$