

# Examples 1 in Rationals in Arithmetic 3

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**Examples 1 in Rationals**

Find the larger of the two in each of the pairs as follows.

Note: show your work.

0.  $\frac{1}{2}$  and  $\frac{3}{4}$

1.  $\frac{1}{2}$  and  $\frac{1}{3}$

2.  $\frac{3}{5}$  and  $\frac{3}{7}$

3.  $\frac{5}{4}$  and  $\frac{7}{4}$

4.  $\frac{3}{4}$  and  $\frac{7}{9}$

5.  $\frac{4}{5}$  and  $\frac{5}{6}$

6.  $\frac{3}{5}$  and  $\frac{5}{7}$

7.  $\frac{9}{10}$  and  $\frac{11}{12}$

8.  $\frac{7}{5}$  and  $\frac{5}{3}$

9.  $\frac{15}{11}$  and  $\frac{11}{7}$

A.  $-\frac{3}{5}$  and  $-\frac{2}{5}$

B.  $-\frac{7}{9}$  and  $\frac{1}{100}$

C.  $-\frac{11}{12}$  and  $-\frac{15}{16}$

D.  $\frac{78943828}{78943880}$  and  $\frac{987656789}{987656792}$

E.  $\frac{78943828}{987656789}$  and  $\frac{78943830}{987656792}$

## Suggestions or Solutions To the Examples 1 in Rationals

How can we see if one of two numbers is greater than the other?

Basically, we have two ways. One is taking the difference between the two, and the other is taking the ratio between the two.

First, if we subtract one from the other and the difference is positive, the one is greater.

So for instance, if we get  $A - B > 0$ , we get  $A > B$ . If  $A - B = 0$ , we get  $A = B$ .  
And of course, if  $A - B < 0$ , we get  $A < B$ .

Next, if the ratio of one to the other is greater than 1, the one is greater.

So for instance, if we get  $\frac{A}{B} > 1$ , we get  $A > B$ . If  $\frac{A}{B} = 1$ , we get  $A = B$ .  
And of course, if  $\frac{A}{B} < 1$ , we get  $A < B$ .

What then about fractional numbers as  $1/2$  and  $2/3$ ?

If the denominators are the same, we can see which is bigger taking the difference or the ratio between the numerators. What if however, the denominators are different?

We can make the denominators the same. That is, taking the common denominator, and then, comparing the numerators using the ways above, we can see which is the bigger.

### 0. $1/2$ and $3/4$

$1/2 = 2/4$ , so  $3/4$  is larger.

**1.  $\frac{1}{2}$  and  $\frac{1}{3}$** 

We can see that the numerators are the same. The larger the denominator, the smaller the fraction if the fraction is positive. So  $\frac{1}{2}$  is the larger.

**2.  $\frac{3}{5}$  and  $\frac{3}{7}$** 

We can see that the numerators are the same. The larger the denominator, the smaller the fraction if the fraction is positive. So  $\frac{3}{5}$  is the larger.

**3.  $\frac{5}{4}$  and  $\frac{7}{4}$** 

The denominators are the same. The larger the numerator, the larger the fraction if the fraction is positive. So  $\frac{7}{4}$  is the larger.

**4.  $\frac{3}{4}$  and  $\frac{7}{9}$** 

Taking the common denominator, we get  $\frac{3}{4} = \frac{27}{36}$ , and  $\frac{7}{9} = \frac{28}{36}$ .  
So  $\frac{7}{9}$  is the larger.

**5.  $\frac{4}{5}$  and  $\frac{5}{6}$** 

$\frac{4}{5} = \frac{24}{30}$ , and  $\frac{5}{6} = \frac{25}{30}$ . So  $\frac{5}{6}$  is the larger.

**6.  $\frac{3}{5}$  and  $\frac{5}{7}$** 

$\frac{3}{5} = \frac{21}{35}$ , and  $\frac{5}{7} = \frac{25}{35}$ . So  $\frac{5}{7}$  is the larger.

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### **7. 9/10 and 11/12**

$9/10 = 108/120$ , and  $11/12 = 110/120$ . So  $11/12$  is the larger.

And we can put it the way below, too.

$1 - 9/10 = 1/10$ ,  $1 - 11/12 = 1/12$ , and  $1/12$  is smaller than  $1/10$ . So  $11/12$  is the larger.

### **8. 7/5 and 5/3**

$7/5 = 21/15$ , and  $5/3 = 25/15$ . So  $5/3$  is larger.

### **9. 15/11 and 11/7**

$15/11 = 105/77$ , and  $11/7 = 121/77$ . So  $11/7$  is larger.

### **A. -3/5 and -2/5**

The larger the magnitude, the smaller the negative number. So  $-2/5$  is larger.

### **B. -7/9 and 1/100**

A positive number is larger than negative number. So  $1/100$  is larger.

### **C. -11/12 and -15/16**

The difference between the numerator and the denominator is 1.

However, the numbers are negative. So  $-11/12$  is the larger.

D.  $\frac{78943828}{78943880}$  and  $\frac{987656789}{987656792}$

For  $\frac{78943828}{78943880}$ , the difference between the denominator and the numerator is 2.

And for  $\frac{987656789}{987656792}$ , the difference is 3.

And we can see that both fractions are less than 1.

Next, the denominator in the second fraction is much bigger than the one in the first.

So we can say that the second fraction is much closer to 1 than the first.

Thus,  $\frac{987656789}{987656792}$  is the larger. In fact,

$$\frac{78943828}{78943880} = 0.99999997446655311757739648557715$$

$$\frac{987656789}{987656792} = 0.99999999696250759950223680535374$$

And we can get the same the way below, too.

First, we have  $78943828 = 78943830 - 2$ .

$$\text{So we get } \frac{78943828}{78943880} = \frac{78943830}{78943880} - \frac{2}{78943880} = 1 - \frac{2}{78943880}.$$

Next, we have  $987656789 = 987656792 - 3$ .

$$\text{So we get } \frac{987656789}{987656792} = 1 - \frac{3}{987656792}.$$

And we can easily see that  $\frac{2}{78943880} > \frac{3}{987656792}$ , since the denominator in  $\frac{2}{78943880}$  is much smaller,

and both numerators are about the same. So we get

$$1 - \frac{2}{78943880} < 1 - \frac{3}{987656792} \Rightarrow \frac{78943828}{78943880} < \frac{987656789}{987656792}.$$

E.  $\frac{78943828}{987656789}$  and  $\frac{78943830}{987656792}$

Looking at the denominators, we can see both are about the same.

And looking at the numerators, we can see both are about the same, too.

And taking the difference between the two, we get this:

$$\begin{aligned} \frac{78943828}{987656789} - \frac{78943830}{987656792} &= \frac{78943828 \times 987656792}{987656789 \times 987656792} - \frac{78943830 \times 987656789}{987656789 \times 987656792} \\ &= \frac{78943828 \times 987656792 - 78943830 \times 987656789}{987656789 \times 987656792} \end{aligned}$$

So without using a calculator, we cannot see quickly enough if is positive, 0, or negative.

What then can we do?

To begin with, we can have  $78943828 = 78943830 - 2$

So we get this:

$$\begin{aligned} &78943828 \times 987656792 - 78943830 \times 987656789 \\ &= (78943830 - 2)987656792 - 78943830 \times 987656789 \\ &= 78943830 \times 987656792 - 2 \times 987656792 - 78943830 \times 987656789 \\ &= 78943830(987656792 - 987656789) - 2 \times 987656792 \\ &= 78943830 \times 3 - 2 \times 987656792 \end{aligned}$$

And next, we can see this:

$78943830 \times 3 < 80000000 \times 3$ , which has 9 digits, and  $2 \times 987656792$  has 10 digits.

So we can see  $78943830 \times 3 - 2 \times 987656792 < 0$ .

That is, the difference is negative. Thus,  $\frac{78943830}{987656792}$  is the larger.