

Examples 2 in Rationals in Arithmetic 3

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Examples 2 in Rationals

0. $\frac{3}{7} + \frac{4}{9}$

2. $\frac{12}{15} + \frac{19}{23}$

4. $\frac{8}{9} + \frac{11}{12}$

6. $\frac{12}{35} + \frac{4}{15}$

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C. $\frac{24}{88} + \frac{32}{36}$

E. $\frac{60}{105} + \frac{252}{168}$

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K. $\frac{51}{61} + \frac{61}{51} + 2$

M. $\frac{11}{111} + \frac{111}{11} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{22}{32}$

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N. $\frac{22}{111} - \frac{111}{22}$

P. $\frac{2525}{101} - \frac{3}{25}$

R. $\frac{12321}{111} - \frac{1210}{11}$

**Suggestions or Solutions
To the Examples 2 in Rationals**

$$0. \quad \frac{3}{7} + \frac{4}{9} = \frac{3 \cdot 9}{7 \cdot 9} + \frac{4 \cdot 7}{9 \cdot 7} = \frac{27}{63} + \frac{28}{63} = \frac{27+28}{63} = \frac{55}{63}$$

$$1. \quad \frac{3}{7} - \frac{4}{9} = \frac{3 \cdot 9}{7 \cdot 9} - \frac{4 \cdot 7}{9 \cdot 7} = \frac{27}{63} - \frac{28}{63} = \frac{27-28}{63} = \frac{-1}{63} = -\frac{1}{63}$$

$$2. \quad \frac{12}{15} + \frac{19}{23} = \frac{4 \cdot 3}{5 \cdot 3} + \frac{19}{23} = \frac{4}{5} + \frac{19}{23} = \frac{4 \cdot 23}{5 \cdot 23} + \frac{19 \cdot 5}{23 \cdot 5} = \frac{92}{115} + \frac{95}{115} = \frac{92+95}{115} = \frac{187}{115}$$

$$3. \quad \frac{12}{15} - \frac{19}{23} = \frac{4 \cdot 3}{5 \cdot 3} - \frac{19}{23} = \frac{4}{5} - \frac{19}{23} = \frac{4 \cdot 23}{5 \cdot 23} - \frac{19 \cdot 5}{23 \cdot 5} = \frac{92}{115} - \frac{95}{115} = \frac{92-95}{115} = \frac{-3}{115} = -\frac{3}{115}$$

$$4. \quad \frac{8}{9} + \frac{11}{12} = \frac{8 \cdot 12}{9 \cdot 12} + \frac{11 \cdot 9}{12 \cdot 9} = \frac{96}{108} + \frac{99}{108} = \frac{96+99}{108} = \frac{195}{108} = \frac{65 \cdot 3}{36 \cdot 3} = \frac{65}{36}$$

And we can put it the way below, too.

$$\text{First, we can get } \frac{8}{9} + \frac{1}{9} = 1 \Rightarrow \frac{8}{9} = 1 - \frac{1}{9}, \text{ and } \frac{11}{12} + \frac{1}{12} = 1 \Rightarrow \frac{11}{12} = 1 - \frac{1}{12}.$$

You could do it by heart.

$$\text{Then, we can get } \frac{8}{9} + \frac{11}{12} = 1 - \frac{1}{9} + 1 - \frac{1}{12} = 2 - \left(\frac{1}{9} + \frac{1}{12}\right).$$

$$\text{Meanwhile, } \frac{1}{9} + \frac{1}{12} = \frac{4}{9 \cdot 4} + \frac{3}{12 \cdot 3} = \frac{4}{36} + \frac{3}{36} = \frac{7}{36}.$$

$$\text{So we get } \frac{8}{9} + \frac{11}{12} = 2 - \left(\frac{1}{9} + \frac{1}{12}\right) = 2 - \frac{7}{36} = \frac{72-7}{36} = \frac{65}{36}$$

$$5. \quad \frac{11}{12} - \frac{8}{9} = \frac{11 \cdot 3}{12 \cdot 3} - \frac{8 \cdot 4}{9 \cdot 4} = \frac{33}{36} - \frac{32}{36} = \frac{1}{36}.$$

And we can do it the way below, too, of course.

$$\frac{11}{12} - \frac{8}{9} = \frac{11 \cdot 9}{12 \cdot 9} - \frac{8 \cdot 12}{9 \cdot 12} = \frac{99}{108} - \frac{96}{108} = \frac{3}{108} = \frac{3}{36 \cdot 3} = \frac{1}{36}.$$

6. $\frac{12}{35} + \frac{4}{15}$

First, we can get $\frac{12}{35} + \frac{4}{15} = \frac{4 \cdot 3}{35} + \frac{4 \cdot 1}{15} = 4\left(\frac{3}{35} + \frac{1}{15}\right)$.

And next, we get $\frac{3}{35} + \frac{1}{15} = \frac{3 \cdot 3}{35 \cdot 3} + \frac{1 \cdot 7}{15 \cdot 7} = \frac{9}{105} + \frac{7}{105} = \frac{16}{105}$.

So we get $\frac{12}{35} + \frac{4}{15} = 4\left(\frac{3}{35} + \frac{1}{15}\right) = 4 \cdot \frac{16}{105} = \frac{64}{105}$.

Of course, we can get the same result the way below, too.

$$\frac{12}{35} + \frac{4}{15} = \frac{12 \cdot 15}{35 \cdot 15} + \frac{4 \cdot 35}{15 \cdot 35} = \dots$$

7. $\frac{12}{27} - \frac{132}{15}$

We have $\frac{12}{27} = \frac{3 \cdot 4}{3 \cdot 9} = \frac{4}{9}$, and $\frac{132}{15} = \frac{3 \cdot 44}{3 \cdot 5} = \frac{44}{5}$.

So we get $\frac{12}{27} - \frac{132}{15} = \frac{4}{9} - \frac{44}{5} = 4\left(\frac{1}{9} - \frac{11}{5}\right)$, and $\frac{1}{9} - \frac{11}{5} = \frac{5 - 99}{45} = \frac{-94}{45}$.

Thus, we get $\frac{12}{27} - \frac{132}{15} = 4\left(\frac{1}{9} - \frac{11}{5}\right) = 4 \cdot \left(-\frac{94}{45}\right) = -\frac{376}{45}$

And of course, we can do it the way below, too.

$$\frac{12}{27} - \frac{132}{15} = \frac{12 \cdot 15}{27 \cdot 15} - \frac{132 \cdot 27}{15 \cdot 27} = \dots$$

8. $\frac{35}{225} + \frac{105}{625}$

To begin with, we get $\frac{35}{225} + \frac{105}{625} = \frac{7 \cdot 5}{45 \cdot 5} + \frac{21 \cdot 5}{125 \cdot 5} = \frac{7}{45} + \frac{21}{125} = 7\left(\frac{1}{45} + \frac{3}{125}\right)$.

And we can get $\frac{1}{45} + \frac{3}{125} = \frac{1}{9 \cdot 5} + \frac{3}{25 \cdot 5} = \frac{1 \cdot 25}{9 \cdot 5 \cdot 25} + \frac{3 \cdot 9}{25 \cdot 5 \cdot 9} = \frac{25 + 27}{1125} = \frac{52}{1125}$.

So we get $\frac{35}{225} + \frac{105}{625} = 7\left(\frac{1}{45} + \frac{3}{125}\right) = 7 \cdot \frac{52}{1125} = \frac{364}{1125}$.

9. $\frac{45}{1225} - \frac{70}{75}$

First, we have $45 = 5 \times 9 = 5 \times 3^2$, and $1225 = 5 \times 245 = 5 \times 5 \times 49 = 5^2 \times 7^2$.

So we get $\frac{45}{1225} = \frac{5 \cdot 3^2}{5^2 7^2} = \frac{3^2}{5 \cdot 7^2}$.

Next, we have $70 = 7 \times 2 \times 5$, and $75 = 5 \times 15 = 5 \times 5 \times 3 = 5^2 \times 3$.

So we get $\frac{70}{75} = \frac{2 \cdot 7}{3 \cdot 5}$.

Thus, we get $\frac{45}{1225} - \frac{70}{75} = \frac{3^2}{5 \cdot 7^2} - \frac{2 \cdot 7}{3 \cdot 5} = \frac{3^2 \cdot 3}{5 \cdot 7^2 \cdot 3} - \frac{2 \cdot 7 \cdot 7^2}{3 \cdot 5 \cdot 7^2} = \frac{27 - 686}{3 \cdot 5 \cdot 7^2} = \frac{-659}{735}$.

A. $\frac{13}{19} + \frac{23}{247} = \frac{13}{19} + \frac{23}{13 \cdot 19} = \frac{13 \cdot 13}{19 \cdot 13} + \frac{23}{13 \cdot 19} = \frac{169 + 23}{19 \cdot 13} = \frac{192}{247}$.

B. $\frac{23}{47} - \frac{47}{23} = \frac{23^2 - 47^2}{47 \cdot 23} = \frac{529 - 2209}{1081} = -\frac{1680}{1081}$.

C. $\frac{24}{88} + \frac{32}{36} = \frac{3 \cdot 8}{11 \cdot 8} + \frac{4 \cdot 8}{4 \cdot 9} = \frac{3}{11} + \frac{8}{9} = \frac{27 + 88}{99} = \frac{115}{99}$.

D. $\frac{72}{216} - \frac{42}{252}$

First, we have $72 = 8 \times 9 = 2^3 \times 3^2$, and $216 = 2 \times 108 = 2^2 \times 54 = 2^2 \times 6 \times 9 = 2^3 \times 3^3$.

So we get $\frac{72}{216} = \frac{1}{3}$.

Next, we have $42 = 2 \times 3 \times 7$, and $252 = 2 \times 126 = 2 \times 2 \times 63 = 2^2 \times 3^2 \times 7$.

So we get $\frac{42}{252} = \frac{1}{6}$.

Thus, we get $\frac{72}{216} - \frac{42}{252} = \frac{1}{3} - \frac{1}{6} = \frac{2-1}{6} = \frac{1}{6}$.

E. $\frac{60}{105} + \frac{252}{168}$

We have $60 = 5 \times 12 = 2^2 \times 3 \times 5$, and $105 = 5 \times 21 = 3 \times 5 \times 7$

So we get $\frac{60}{105} = \frac{4}{7}$.

Next, we have $252 = 2^2 \times 3^2 \times 7$, and $168 = 2 \times 84 = 2^2 \times 42 = 2^3 \times 21 = 2^3 \times 3 \times 7$

So we get $\frac{252}{168} = \frac{3}{2}$.

Thus, we get $\frac{60}{105} + \frac{252}{168} = \frac{4}{7} + \frac{3}{2} = \frac{4 \cdot 2 + 3 \cdot 7}{14} = \frac{29}{14}$.

F. $\frac{144}{468} - \frac{54}{244}$

We have $144 = 2 \cdot 72 = 2^2 36 = 2^4 3^2 = 12^2 = (4 \cdot 3)^2 = (2^2 3)^2 = 2^4 3^2$, and
 $468 = 2 \cdot 234 = 2^2 117 = 2^2 3 \cdot 39 = 2^2 3^2 13$.

So we get: $\frac{144}{468} = \frac{2^2}{13}$.

Next, we have: $54 = 2 \cdot 27 = 2 \cdot 3^3$, and $244 = 2 \cdot 122 = 2^2 61$.

So we get $\frac{54}{244} = \frac{3^3}{2 \cdot 61}$.

Thus, we get $\frac{144}{468} - \frac{54}{244} = \frac{2^2}{13} - \frac{3^3}{2 \cdot 61} = \frac{2^2 2 \cdot 61}{2 \cdot 13 \cdot 61} - \frac{3^3 13}{2 \cdot 3 \cdot 61} = \frac{2^3 \cdot 61 - 3^3 13}{2 \cdot 3 \cdot 61} = \frac{137}{1586}$.

G. $\frac{3}{4} + \frac{4}{3} + 2$

First, we get $\frac{3}{4} + \frac{4}{3} = \frac{3^2 + 4^2}{12}$, and $2 = \frac{2 \cdot 3 \cdot 4}{12}$.

So we get $\frac{3}{4} + \frac{4}{3} + 2 = \frac{3^2 + 4^2}{12} + \frac{2 \cdot 3 \cdot 4}{12} = \frac{25 + 24}{12} = \frac{49}{12}$.

Note that we can have $3^2 + 2 \cdot 3 \cdot 4 + 4^2 = (3 + 4)^2$.

H. $\frac{4}{5} + \frac{5}{4} - 2 = \frac{4^2 + 5^2}{20} - \frac{2 \cdot 4 \cdot 5}{20} = \frac{16 + 25 - 40}{20} = \frac{1}{20}$

Note that we can have $4^2 - 2 \cdot 4 \cdot 5 + 5^2 = (4 - 5)^2 = (5 - 4)^2$.

$$\text{I. } \frac{13}{29} + \frac{29}{13} - 2 = \frac{13^2 + 29^2}{29 \cdot 13} - \frac{2 \cdot 13 \cdot 29}{29 \cdot 13} = \frac{16^2}{377} = \frac{256}{377}.$$

Note that we can have: $13^2 + 29^2 - 2 \times 13 \times 29 = (29 - 13)^2 = (13 - 29)^2 = 16^2$.

$$\text{J. } \frac{161}{101} - \frac{101}{161} = \frac{161^2 - 101^2}{101 \cdot 161} = \frac{25921 - 10201}{16261} = \frac{15720}{16261}.$$

$$\text{K. } \frac{51}{61} + \frac{61}{51} + 2 = \frac{51^2 + 61^2 + 2 \cdot 51 \cdot 61}{61 \cdot 51} = \frac{(51 + 61)^2}{61 \cdot 51} = \frac{112^2}{3111} = \frac{12544}{3111}.$$

$$\text{L. } \frac{151}{161} - \frac{1510}{1610}$$

First, we have $1510 = 151 \times 10$ and $1610 = 161 \times 10$. So we get $\frac{1510}{1610} = \frac{151}{161}$.

Thus, we get $\frac{151}{161} - \frac{1510}{1610} = 0$.

$$\text{M. } \frac{11}{111} + \frac{111}{11} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{22}{32}$$

First, we have $\frac{3}{4} = \frac{24}{32}$, $\frac{3}{8} = \frac{12}{32}$, and $\frac{3}{16} = \frac{6}{32}$.

So we get $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{22}{32} = \frac{24 + 12 + 6 + 22}{32} = \frac{64}{32} = 2$.

Thus, we get $\frac{11}{111} + \frac{111}{11} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{22}{32} = \frac{11}{111} + \frac{111}{11} + 2 = \frac{(11 + 111)^2}{111 \cdot 11} = \frac{122^2}{1221} = \frac{14884}{1221}$.

Note that we can have $11^2 + 111^2 + 2 \times 11 \times 111 = (11 + 111)^2 = 122^2$.

$$\text{N. } \frac{22}{111} - \frac{111}{22} = \frac{22^2 - 111^2}{111 \cdot 22} = \frac{484 - 12321}{2442} = -\frac{11837}{2442}.$$

$$\text{O. } \frac{1010}{101} + \frac{1}{10} = \frac{101 \cdot 10}{101} + \frac{1}{10} = 10 + \frac{1}{10} = \frac{100 + 1}{10} = \frac{101}{10}.$$

$$\text{P. } \frac{2525}{101} - \frac{3}{25} = \frac{25 \cdot 101}{101} - \frac{3}{25} = 25 - \frac{3}{25} = \frac{25^2 - 3}{25} = \frac{625 - 3}{25} = \frac{622}{25}.$$

Q. $\frac{675675}{1001} + \frac{4040}{101}$

We have $675675 = 675 \times 1001$, and $4040 = 4 \times 1010 = 40 \times 101$

So we get $\frac{675675}{1001} + \frac{4040}{101} = 675 + 40 = 715$.

R. $\frac{12321}{111} - \frac{1210}{11}$

We have $12321 = 111^2$, and $121 = 11^2$

So we get $\frac{12321}{111} - \frac{1210}{11} = \frac{111^2}{111} - \frac{11^2 \cdot 10}{11} = 111 - 11 \cdot 10 = 111 - 110 = 1$.