

Examples 3 in Rationals in Arithmetic 3

Copyright © 2018 by Seong Ryeol Kim. All rights reserved

[Click this to start.](#)

Examples 3 in Rationals

$$0. \quad \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$$

$$1. \quad \frac{1}{2} - \frac{3}{4} + \frac{5}{6} - \frac{7}{8}$$

$$2. \quad \frac{8}{9} - \frac{7}{8} + \frac{17}{18} - \frac{15}{16}$$

$$3. \quad \frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81}$$

$$4. \quad \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$$

$$5. \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$6. \quad \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000}$$

$$7. \quad \frac{1}{10} - \frac{1}{100} - \frac{1}{1000} - \frac{1}{10000}$$

$$8. \quad \frac{12}{20} + \frac{20}{30} + \frac{30}{42} + \frac{42}{56}$$

$$9. \quad \frac{121}{11} + \frac{144}{12} + \frac{169}{13} + \frac{196}{14}$$

$$A. \quad \frac{1}{1} - \frac{1}{11} - \frac{11}{121} - \frac{1331}{121} - \frac{14641}{1331}$$

$$B. \quad \frac{55}{111} + \frac{333}{12321} + \frac{444}{24642}$$

$$C. \quad \frac{2}{3} + \frac{3}{2} + \frac{3}{4} + \frac{4}{3} + \frac{4}{5} + \frac{5}{4}$$

$$D. \quad \frac{15}{12} + \frac{20}{15} + \frac{35}{28} + \frac{36}{27}$$

Suggestions or Solutions To the Examples 3 in Rationals

0. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$

We can get first: $\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$.

And next, we can get: $\frac{2}{3} + \frac{4}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{3 \cdot 4}{3 \cdot 5} = \frac{10}{15} + \frac{12}{15} = \frac{10+12}{15} = \frac{22}{15}$.

So we get $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{5}{4} + \frac{22}{15} = \frac{5 \cdot 15}{60} + \frac{22 \cdot 4}{60} = \frac{75+88}{60} = \frac{163}{60}$

1. $\frac{1}{2} - \frac{3}{4} + \frac{5}{6} - \frac{7}{8}$

To begin with, we can get first, $\frac{1}{2} - \frac{3}{4} - \frac{7}{8} = \frac{4}{8} - \frac{6}{8} - \frac{7}{8} = \frac{4-6-7}{8} = -\frac{9}{8}$.

So we get

$$\frac{1}{2} - \frac{3}{4} + \frac{5}{6} - \frac{7}{8} = \frac{5}{6} - \frac{9}{8}$$

$$= \frac{5}{6} - 1 - \frac{1}{8} \quad \left\{ \frac{9}{8} = \frac{8}{8} + \frac{1}{8} = 1 + \frac{1}{8} \right\}$$

$$= \frac{5}{6} - \frac{5}{6} - \frac{1}{6} - \frac{1}{8} \quad \left\{ -1 = -\frac{5}{6} - \frac{1}{6} \right\}$$

$$= -\frac{1}{6} - \frac{1}{8} = -\frac{4}{24} - \frac{3}{24} = -\frac{7}{24}$$

2. $\frac{8}{9} - \frac{7}{8} + \frac{17}{18} - \frac{15}{16}$

We can get first, $\frac{8}{9} + \frac{17}{18} = \frac{16}{18} + \frac{17}{18} = \frac{33}{18}$, and $-\frac{7}{8} - \frac{15}{16} = -\frac{14}{16} - \frac{15}{16} = -\frac{29}{16}$.

So we get

$$\frac{8}{9} - \frac{7}{8} + \frac{17}{18} - \frac{15}{16} = \frac{33}{18} - \frac{29}{16} = \frac{33 \cdot 8}{18 \cdot 8} - \frac{29 \cdot 9}{16 \cdot 9} = \frac{264}{144} - \frac{180 + 81}{144} = \frac{264 - 261}{144} = \frac{3}{144}$$

$$3. \quad \frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81}$$

To begin with, $\frac{2}{3} = \frac{2 \cdot 27}{3 \cdot 27} = \frac{54}{81}$, $\frac{8}{9} = \frac{8 \cdot 9}{9 \cdot 9} = \frac{72}{81}$, and $\frac{26}{27} = \frac{26 \cdot 3}{27 \cdot 3} = \frac{78}{81}$.

So we get

$$\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} = \frac{54}{81} + \frac{72}{81} + \frac{78}{81} + \frac{80}{81} = \frac{126 + 158}{81} = \frac{284}{81}$$

$$4. \quad \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$$

First, we can get $\frac{1}{4} = \frac{64}{4 \cdot 64} = \frac{64}{256}$, $\frac{1}{16} = \frac{16}{16 \cdot 16} = \frac{16}{256}$, and $\frac{1}{64} = \frac{4}{256}$.

So we get $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} = \frac{64 + 16 + 4 + 1}{256} = \frac{85}{256}$

$$5. \quad \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

We can get first: $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$, and $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$.

So we get $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{3}{4} + \frac{8}{15} = \frac{45 + 32}{60} = \frac{77}{60}$

4

$$6. \quad \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000}$$

$$1/10 = 1000/10000$$

$$1/100 = 100/10000$$

$$1/1000 = 10/10000$$

$$\text{So we get } \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} = \frac{1000+100+10+1}{10000} = \frac{1111}{10000}$$

$$7. \quad \frac{1}{10} - \frac{1}{100} - \frac{1}{1000} - \frac{1}{10000} = \frac{1000-100-10-1}{10000} = \frac{1000-111}{10000} = \frac{889}{10000}$$

$$8. \quad \frac{12}{20} + \frac{20}{30} + \frac{30}{42} + \frac{42}{56}$$

To begin with, we get $12/20 = 3/5$, $20/30 = 2/3$, $30/42 = 5/7$, and $42/56 = 6/8 = 3/4$.

Next, we can get $3/5 = 1 - 2/5$, $2/3 = 1 - 1/3$, $5/7 = 1 - 2/7$, and $3/4 = 1 - 1/4$

So we get $3/5 + 2/3 + 5/7 + 3/4 = 4 - (2/5 + 1/3 + 2/7 + 3/4)$

Next, we can get $2/5 + 1/3 = (6 + 5)/15 = 11/15$, and $2/7 + 1/4 = (8 + 7)/28 = 15/28$.

So next, we get

$$\frac{11}{15} + \frac{15}{28} = \frac{11 \cdot 28}{15 \cdot 28} + \frac{15 \cdot 15}{15 \cdot 28} = \frac{280+225}{300+80+40} = \frac{308+225}{420} = \frac{533}{420}$$

And thus, we get

$$\frac{12}{20} + \frac{20}{30} + \frac{30}{42} + \frac{42}{56} = 4 - \frac{533}{420} = 4 - \frac{420}{420} - \frac{113}{420} = 3 - \frac{113}{420} = 2 + \frac{420-113}{420}$$

$$= 2 + \frac{307}{420} = \frac{840+307}{420} = \frac{1147}{420}$$

9. $\frac{121}{11} + \frac{144}{12} + \frac{169}{13} + \frac{196}{14}$

To begin with, we have $121 = 11^2$, $144 = 12^2$, $169 = 13^2$, and $196 = 14^2$.

So we get $\frac{121}{11} + \frac{144}{12} + \frac{169}{13} + \frac{196}{14} = 11 + 12 + 13 + 14 = 50$.

A. $\frac{1}{1} - \frac{1}{11} - \frac{11}{121} - \frac{1331}{121} - \frac{14641}{1331}$

First, we have $121 = 11^2$, $1331 = 11 \times 121$, and $14641 = 1331 \times 11$.

So next, we can get $11/121 = 1/11$, $1331/121 = 1/11$, and $14641/1331 = 1/11$

So we get

$$\frac{1}{1} - \frac{1}{11} - \frac{11}{121} - \frac{1331}{121} - \frac{14641}{1331} = 1 - \frac{1}{11} - \frac{1}{11} - 11 - 11 = -\frac{2}{11} - 21 = -\frac{2+21 \cdot 11}{11} = -\frac{233}{11}$$

B. $\frac{55}{111} + \frac{333}{12321} + \frac{444}{24642}$

To begin with, we have $12321 = 111^2$, and $24642 = 2 \cdot 12321 = 2 \cdot 111^2$.

So we get

$$\frac{55}{111} + \frac{333}{12321} + \frac{444}{24642} = \frac{55}{111} + \frac{3 \cdot 111}{111^2} + \frac{4 \cdot 111}{2 \cdot 111^2} = \frac{55}{111} + \frac{3}{111} + \frac{2}{111} = \frac{60}{111} = \frac{20}{37}$$

C. $\frac{2}{3} + \frac{3}{2} + \frac{3}{4} + \frac{4}{3} + \frac{4}{5} + \frac{5}{4}$

First, we can get $2/3 + 4/3 = 6/3 = 2$, and $3/2 + 3/4 + 5/4 = (6 + 3 + 5)/4 = 14/4 = 7/2$.

So we get $\frac{2}{3} + \frac{3}{2} + \frac{3}{4} + \frac{4}{3} + \frac{4}{5} + \frac{5}{4} = 2 + \frac{7}{2} + \frac{4}{5} = 2 + 3 + \frac{1}{2} + \frac{4}{5} = 5 + \frac{5+8}{10} = 5 + \frac{13}{10} = \frac{63}{10}$

D. $\frac{15}{12} + \frac{20}{15} + \frac{35}{28} + \frac{36}{27}$

To begin with, we can get $15/12 = 5/4$, $20/15 = 4/3$, $35/28 = 5/4$, and $36/27 = 4/3$

So we get

$$\frac{15}{12} + \frac{20}{15} + \frac{35}{28} + \frac{36}{27} = 2 \cdot \frac{5}{4} + 2 \cdot \frac{4}{3} = \frac{5}{2} + \frac{8}{3} = \frac{15+16}{6} = \frac{31}{6}.$$