

Examples 4 in Rationals in Arithmetic 3

Copyright © 2018 by Seong Ryeol Kim. All rights reserved

[Click this to start.](#)

Examples 4 in Rationals

$$0. \quad \frac{0.4}{0.5} - \frac{0.5}{0.6}$$

$$1. \quad \frac{0.3}{0.4} - \frac{0.3}{0.17}$$

$$2. \quad \frac{5}{6} + \frac{5}{12} + \frac{5}{20} + \frac{5}{30} + \frac{5}{42}$$

$$3. \quad \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143}$$

$$4. \quad \frac{7}{10} + \frac{7}{40} + \frac{7}{88} + \frac{7}{154} + \frac{7}{238}$$

Suggestions or Solutions To the Examples 4 in Rationals

$$0. \quad \frac{0.4}{0.5} + \frac{0.5}{0.6} = \frac{4}{5} + \frac{5}{6} = \frac{24+25}{30} = \frac{49}{30}$$

$$1. \quad \frac{0.3}{0.4} - \frac{0.3}{0.17} = \frac{3}{4} - \frac{30}{17} = \frac{51-120}{68} = \frac{-69}{68}$$

We have $\frac{a}{AB} = \frac{a}{B-A} \left(\frac{1}{A} - \frac{1}{B} \right)$, where $A \neq 0$, $B \neq 0$, and $A \neq B$.

That's simply because $\frac{a}{B-A} \left(\frac{1}{A} - \frac{1}{B} \right) = \frac{a}{B-A} \cdot \frac{B-A}{AB} = \frac{a}{AB}$.

For instance, $\frac{2}{15} = \frac{2}{3 \times 5} = \frac{2}{5-3} \left(\frac{1}{3} - \frac{1}{5} \right)$. So using the fact above, we can get:

$$2. \quad \frac{5}{6} + \frac{5}{12} + \frac{5}{20} + \frac{5}{30} + \frac{5}{42} = \frac{5}{2 \times 3} + \frac{5}{3 \times 4} + \frac{5}{4 \times 5} + \frac{5}{5 \times 6} + \frac{5}{6 \times 7}$$

$$= \frac{5}{3-2} \left(\frac{1}{2} - \frac{1}{3} \right) + \frac{5}{4-3} \left(\frac{1}{3} - \frac{1}{4} \right) + \frac{5}{5-4} \left(\frac{1}{4} - \frac{1}{5} \right) + \frac{5}{6-5} \left(\frac{1}{5} - \frac{1}{6} \right) + \frac{5}{7-6} \left(\frac{1}{6} - \frac{1}{7} \right)$$

$$= 5 \cdot \left(\frac{1}{2} - \frac{1}{3} \right) + 5 \cdot \left(\frac{1}{3} - \frac{1}{4} \right) + 5 \cdot \left(\frac{1}{4} - \frac{1}{5} \right) + 5 \cdot \left(\frac{1}{5} - \frac{1}{6} \right) + 5 \cdot \left(\frac{1}{6} - \frac{1}{7} \right)$$

$$= \frac{5}{2} - \frac{5}{3} + \frac{5}{3} - \frac{5}{4} + \frac{5}{4} - \frac{5}{5} + \frac{5}{5} - \frac{5}{6} + \frac{5}{6} - \frac{5}{7} = \frac{5}{2} - \frac{5}{7} = \frac{35-10}{14} = \frac{25}{14}$$

$$\begin{aligned}
3. \quad & \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} = \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \frac{1}{9 \times 11} + \frac{1}{11 \times 13} \\
&= \frac{1}{5-3} \left(\frac{1}{3} - \frac{1}{5} \right) + \frac{1}{7-5} \left(\frac{1}{5} - \frac{1}{7} \right) + \frac{1}{9-7} \left(\frac{1}{7} - \frac{1}{9} \right) + \frac{1}{11-9} \left(\frac{1}{9} - \frac{1}{11} \right) + \frac{1}{13-11} \left(\frac{1}{11} - \frac{1}{13} \right) \\
&= \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} \right) + \frac{1}{2} \left(\frac{1}{5} - \frac{1}{7} \right) + \frac{1}{2} \left(\frac{1}{7} - \frac{1}{9} \right) + \frac{1}{2} \left(\frac{1}{9} - \frac{1}{11} \right) + \frac{1}{2} \left(\frac{1}{11} - \frac{1}{13} \right) \\
&= \frac{1}{2} \left\{ \left(\frac{1}{3} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{13} \right) \right\} \\
&= \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \frac{1}{9} - \frac{1}{11} + \frac{1}{11} - \frac{1}{13} \right) = \frac{1}{2} \left(\frac{1}{3} - \frac{1}{13} \right) = \frac{1}{2} \cdot \frac{13-3}{39} = \frac{1}{2} \cdot \frac{10}{39} = \frac{5}{39}
\end{aligned}$$

$$\begin{aligned}
4. \quad & \frac{7}{10} + \frac{7}{40} + \frac{7}{88} + \frac{7}{154} + \frac{7}{238} = 7 \left(\frac{1}{2 \times 5} + \frac{1}{5 \times 8} + \frac{1}{8 \times 11} + \frac{1}{11 \times 14} + \frac{1}{14 \times 17} \right) \\
&= \frac{7}{5-2} \left(\frac{1}{2} - \frac{1}{5} \right) + \frac{7}{8-5} \left(\frac{1}{5} - \frac{1}{8} \right) + \frac{7}{11-8} \left(\frac{1}{8} - \frac{1}{11} \right) + \frac{7}{14-11} \left(\frac{1}{11} - \frac{1}{14} \right) + \frac{7}{17-14} \left(\frac{1}{14} - \frac{1}{17} \right) \\
&= \frac{7}{3} \left(\frac{1}{2} - \frac{1}{5} \right) + \frac{7}{3} \left(\frac{1}{5} - \frac{1}{8} \right) + \frac{7}{3} \left(\frac{1}{8} - \frac{1}{11} \right) + \frac{7}{3} \left(\frac{1}{11} - \frac{1}{14} \right) + \frac{7}{3} \left(\frac{1}{14} - \frac{1}{17} \right) \\
&= \frac{7}{3} \left\{ \left(\frac{1}{2} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{11} \right) + \left(\frac{1}{11} - \frac{1}{14} \right) + \left(\frac{1}{14} - \frac{1}{17} \right) \right\} \\
&= \frac{7}{3} \left(\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \frac{1}{11} - \frac{1}{14} + \frac{1}{14} - \frac{1}{17} \right) \\
&= \frac{7}{3} \left(\frac{1}{2} - \frac{1}{17} \right) = \frac{7}{3} \cdot \frac{17-2}{34} = \frac{7}{3} \cdot \frac{15}{34} = \frac{7 \cdot 5}{34} = \frac{35}{34}
\end{aligned}$$