Examples 7 in Rationals in Arithmetic 3

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Examples 7 in Rationals

0. Convert the following decimals into the fractional numbers.

0.0. 0.375375375…
0.1. 0.476547654765…
0.2. 2.0496496496…
0.3. 7.2060706070607…

1. Some fractional numbers can be put in decimals. For instance,

1/7 = 0.142857142857142857…
2/7 = 0.285714285714285714…
3/7 = 0.428571428571428571…

1.0. Find the 17th digit below the point in the decimal equivalent of 8/7.
1.1. Find the 24th digit below the point in the decimal equivalent of 30/7.

2. Convert the following fractional numbers into their decimal equivalents, and find the thousandth digit below the point in each of the decimals.

2.0. 5/33
2.1. 29/222

3. Find the thousandth digit below the point in the product of the two decimals in each case below. Use a calculator.

3.0. 0.4122412241224… and 0.9443594435944359…
3.1. 1.124545454… and 4.2879269269269…

Suggestions or Solutions
To the Examples 7 in Rationals

0. Convert each decimal into its fractional equivalent.

0.0. 0.375375375…

We can see some digits repeating, and they are 375.
So assuming $\boldsymbol{A = 0.375375375\ldots}$, we can set $\mathbf{1000}\boldsymbol{A} \mathbf{= 375.375375375\ldots}$.
That is, we get $\mathbf{1000}\boldsymbol{A} \mathbf{= 375 + 0.375375375\ldots}$.

And we know $\boldsymbol{A = 0.375375375\ldots}$.
So we get $\mathbf{1000}\boldsymbol{A} \mathbf{= 375 + 0.375375375\ldots = 375 +} \boldsymbol{A} \Rightarrow \mathbf{1000}\boldsymbol{A} \mathbf{= 375 +} \boldsymbol{A}$.

Thus, we get $\mathbf{1000}\boldsymbol{A} - \boldsymbol{A} \mathbf{= 375} \Rightarrow \mathbf{999}\boldsymbol{A} \mathbf{= 375} \Rightarrow \boldsymbol{A} = \dfrac{\mathbf{375}}{\mathbf{999}}$.

So we get $\mathbf{0.375375375\ldots} = \dfrac{\mathbf{375}}{\mathbf{999}}$.

0.1. 0.476547654765…

We can see that the four digits 4765 are repeating.
So assuming $\boldsymbol{A = 0.476547654765\ldots}$, we can set $\mathbf{10000}\boldsymbol{A} \mathbf{= 4765.476547654765\ldots}$.
That is, we get $\mathbf{10000}\boldsymbol{A} \mathbf{= 4765 + 0.476547654765\ldots}$.

So we get $\mathbf{10000}\boldsymbol{A} \mathbf{= 4765 +} \boldsymbol{A} \Rightarrow \mathbf{10000}\boldsymbol{A} - \boldsymbol{A} \mathbf{= 9999}\boldsymbol{A} \mathbf{= 4765} \Rightarrow \boldsymbol{A} = \dfrac{\mathbf{4765}}{\mathbf{9999}}$.

Thus, we get $\mathbf{0.476547654765\ldots} = \dfrac{\mathbf{4765}}{\mathbf{9999}}$.

0.2. 2.0496496496…

Assuming first, $\boldsymbol{A = 2.0496496496\ldots}$, we can put it this way: $\boldsymbol{A} \mathbf{= 2 + 0.0496496496\ldots}$.

And assuming next, $\boldsymbol{B} = \mathbf{0.496496496\ldots}$, we can put $\boldsymbol{A}$ this way, too: $\boldsymbol{A} = \mathbf{2} + \dfrac{\boldsymbol{B}}{\mathbf{10}}$.

And in $\boldsymbol{B}$, we can see the three digits 496 repeating.

So to begin with, we can get $\mathbf{1000}\boldsymbol{B} = \mathbf{496.496496496\ldots} = \mathbf{496} + \boldsymbol{B}$.

Thus, we get $\mathbf{999}\boldsymbol{B} = \mathbf{496} \Rightarrow \boldsymbol{B} = \dfrac{\mathbf{496}}{\mathbf{999}}$.

And we have $\boldsymbol{A} = \mathbf{2} + \dfrac{\boldsymbol{B}}{\mathbf{10}}$. So we get $\boldsymbol{A} = \mathbf{2} + \dfrac{\mathbf{496}}{\mathbf{9990}}$.

Meanwhile,

$$\mathbf{2} + \frac{\mathbf{496}}{\mathbf{9990}} = \frac{\mathbf{2(10000-10)}}{\mathbf{9990}} + \frac{\mathbf{496}}{\mathbf{9990}} = \frac{\mathbf{20000-20+496}}{\mathbf{9990}}$$

$$\frac{\mathbf{19980+496}}{\mathbf{9990}} = \frac{\mathbf{20000+476}}{\mathbf{9990}} = \frac{\mathbf{20476}}{\mathbf{9990}}$$

Thus, we get $\boldsymbol{A} = \dfrac{\mathbf{20476}}{\mathbf{9990}}$.

And of course, we can put it this way, too: $\boldsymbol{A} = \dfrac{\mathbf{10238}}{\mathbf{4995}}$.

0.3. 7.2060706070607…

Assuming first, $\boldsymbol{A} = \mathbf{7.2060706070607\ldots}$, we can put it this way:

$\boldsymbol{A} = \mathbf{7.2} + \mathbf{0.0060706070607\ldots}$.

And assuming next, $\boldsymbol{B} = \mathbf{0.0060706070607\ldots}$, we can set $\boldsymbol{A} = \mathbf{7.2} + \boldsymbol{B}$, and we can see four digits repeating in $\boldsymbol{B}$, and they are 0607.

So assuming now, $\boldsymbol{C} = \mathbf{0.060706070607\ldots}$, we can set $\boldsymbol{B} = \dfrac{\boldsymbol{C}}{\mathbf{10}}$.

Then, we can get $\boldsymbol{A} = \mathbf{7.2} + \mathbf{0.0060706070607\ldots} = \mathbf{7.2} + \dfrac{\boldsymbol{B}}{\mathbf{10}} \Rightarrow \boldsymbol{A} = \mathbf{7.2} + \dfrac{\boldsymbol{B}}{\mathbf{10}}$.

So beginning with C, we get $\mathbf{10000C = 607.060706070607\ldots = 607 + C}$.

Thus, we get $\mathbf{10000C - C = 9999C = 607 \Rightarrow C = \frac{607}{9999}}$.

Next, we know $\mathbf{B = \frac{C}{10}}$. So we get $\mathbf{B = \frac{607}{99990}}$.

Next, we have $\mathbf{A = 7.2 + \frac{B}{10}}$. Thus, we get $\mathbf{A = 7.2 + \frac{607}{999900}}$.

Meanwhile,

$7.2 + \frac{607}{999900} = 7 + 0.2 + \frac{607}{999900}$.

$7 \cdot 999900 = 7(1000000 - 100) = 7 \cdot 10^6 - 700$.

$0.2 \cdot 999900 = 2 \cdot 99990 = 2(100000 - 10) = 2 \cdot 10^5 - 20$.

$-700 - 20 + 607 = -100 - 20 + 7 = -113$, and $200 - 113 = 87$.

So we get $7 \cdot 10^6 - 700 + 2 \cdot 10^5 - 20 + 607 = 7 \cdot 10^6 + 2 \cdot 10^5 - 113$

$= 7 \cdot 10^6 + 1 \cdot 10^5 + 99800 + 200 - 113 = 7199887$.

So we get $\mathbf{A = \frac{7199887}{999900}}$.

1. Some fractional numbers can be put in decimals. For instance,

1/7 = 0.142857142857142857…
2/7 = 0.285714285714285714…
3/7 = 0.428571428571428571…

1.0. Find the 17th digit below the point in the decimal equivalent of 8/7.

We know $\frac{8}{7} = 1 + \frac{1}{7}$, and we have $\frac{1}{7} = 0.142857142857142857\ldots$

So we get $\frac{8}{7} = 1.142857142857142857\ldots$

And the seventeenth digit below the dot is 5.

1.1. Find the 24th digit below the point in the decimal equivalent of 30/7.

We know $\frac{30}{7} = 4 + \frac{2}{7}$, and we have $\frac{2}{7}$ = 0.285714285714285714…

So we get $\frac{30}{7}$ = 4.$\underline{285714}$285714285714…

And we can see six digits repeating, which are 285714.

And 4 divides 24, and the last digit in the six digit is 4.

So the twenty fourth digit below the dot is 4.

2.0 Convert 5/33 into the decimal equivalent, and find the thousandth digit below the point in the decimal.

Doing the direct division, we get $\frac{5}{33}$ = 0.15151515…

And we can see two digits repeating, which are 15.

We know 2 divides 1000, and the last of the two digits repeating is 5.
So the thousandth digit below the dot is 5.

2.1 Convert 29/222 into the decimal equivalent, and find the thousandth digit below the point in the decimal.

Doing the direct division, we get $\frac{29}{222}$ = 0.1306306306…

And we can see three digits repeating, which are 306.

The repeating begins though, right after the first digit below the dot.
So we are looking at the 999^{th} digit after the first digit.

And we know 3 divides 999, and the last digit in the three digits repeating is 6.
So the thousandth digit below the dot is 6.

3.0. Find the thousandth digit below the point in the product of 0.4122412241224… and 0.9443594435944359… Use a calculator.

We can get the digit the way below.

Convert the two decimals into their fractional equivalents.
Take the product of the two fractional numbers, and then, put the product into its decimal equivalent.
And then, find the digit.
So assuming first, $\boldsymbol{A} = \mathbf{0.4122412241224\ldots}$, we get

$$\mathbf{10000}\boldsymbol{A} = \mathbf{4122.412241224122\ldots = 4122} + \boldsymbol{A} \Rightarrow \mathbf{9999}\boldsymbol{A} = \mathbf{4122} \Rightarrow \boldsymbol{A} = \frac{\mathbf{4122}}{\mathbf{9999}}.$$

And assuming next, $\boldsymbol{B} = \mathbf{0.9443594435944359\ldots}$, we get

$$\mathbf{100000}\boldsymbol{B} = \mathbf{94435.944359443594435\ldots = 94435} + \boldsymbol{B} \Rightarrow \mathbf{99999}\boldsymbol{B} = \mathbf{94435}.$$

Thus next, taking the product of $\boldsymbol{A}$ and $\boldsymbol{B}$, and putting the product into the decimal using a calculator, we get

$$\boldsymbol{AB} = \frac{\mathbf{4122}}{\mathbf{9999}} \cdot \frac{\mathbf{94435}}{\mathbf{99999}} = \frac{\mathbf{4122} \cdot \mathbf{94435}}{\mathbf{9999} \cdot \mathbf{99999}} = \frac{\mathbf{389261070}}{\mathbf{999890001}} = \mathbf{0.\underline{38930}3893038930\ldots}$$

We can see five digits repeating, which are 38930.
And we know 5 divides 1000, and the last of the digits repeating is 0.

So the thousandth digit below the dot is 0.

3.1. Find the thousandth digit below the point in the product of 1.124545454… and 4.2879269269269… Use a calculator.

Assuming first, $\boldsymbol{A} = \mathbf{1.124545454\ldots}$, and $\boldsymbol{B} = \mathbf{0.004545454\ldots}$, we get $\boldsymbol{A} = \mathbf{1.12} + \boldsymbol{B}$.

And assuming next, $\boldsymbol{C} = \mathbf{0.4545454\ldots}$, we can set $\boldsymbol{B} = \dfrac{\boldsymbol{C}}{\mathbf{1000}}$, and we get

$\mathbf{100}\boldsymbol{C} = \mathbf{45.454545\ldots} = \mathbf{45} + \boldsymbol{C} \Rightarrow \mathbf{99}\boldsymbol{C} = \mathbf{45} \Rightarrow \boldsymbol{C} = \frac{\mathbf{45}}{\mathbf{99}}$.

So we get $\boldsymbol{B} = \frac{\boldsymbol{C}}{\mathbf{1000}} = \frac{\mathbf{45}}{\mathbf{99000}}$. And we have $\boldsymbol{A} = \mathbf{1.12} + \boldsymbol{B}$.

So we get $\boldsymbol{A} = \mathbf{1.12} + \frac{\mathbf{45}}{\mathbf{99000}} = \frac{\mathbf{112}}{\mathbf{100}} + \frac{\mathbf{45}}{\mathbf{99000}} = \frac{\mathbf{112 \cdot 900 + 45}}{\mathbf{99000}} = \frac{\mathbf{110880 + 45}}{\mathbf{99000}} = \frac{\mathbf{110925}}{\mathbf{99000}}$.

Next, assuming $\boldsymbol{D} = \mathbf{4.2879269269269\ldots}$, and $\boldsymbol{E} = \mathbf{0.000926926\ldots}$,we get

$\boldsymbol{D} = \mathbf{4.287} + \boldsymbol{E}$.

And assuming next, $\boldsymbol{F} = \mathbf{0.926926\ldots}$, we can set $\boldsymbol{E} = \frac{\boldsymbol{F}}{\mathbf{1000}}$, and we get

$\mathbf{1000}\boldsymbol{F} = \mathbf{926.926926\ldots} = \mathbf{926} + \boldsymbol{F} \Rightarrow \mathbf{999}\boldsymbol{F} = \mathbf{926} \Rightarrow \boldsymbol{F} = \frac{\mathbf{926}}{\mathbf{999}}$.

So we get $\frac{\boldsymbol{F}}{\mathbf{1000}} = \frac{\mathbf{926}}{\mathbf{999000}}$. And we have $\boldsymbol{D} = \mathbf{4.287} + \boldsymbol{E}$.

So we get $\boldsymbol{D} = \mathbf{4.287} + \frac{\mathbf{926}}{\mathbf{999000}} = \frac{\mathbf{4287}}{\mathbf{1000}} + \frac{\mathbf{926}}{\mathbf{999000}} = \frac{\mathbf{4287 \cdot 999 + 926}}{\mathbf{999000}} = \frac{\mathbf{4283639}}{\mathbf{999000}}$.

Thus next, taking the product of ***A*** and ***D***, and putting the product into the decimal using a calculator, we get

$\boldsymbol{AD} = \mathbf{(110925/99000)(4283639/999000) = 475162656075/98901000000}$
$\mathbf{= 4.804427\underline{215852}2158522158522\ldots}$

We can see six digits repeating, which are 215852, which begin repeating however, right after the sixth digit below the dot.
So we are looking at the 994^{th} digit after the sixth digit.

And we have $994 = 165 \cdot 6 + 4$.
So the digit we are after is the fourth digit in 215852.

And thus, the thousandth digit below the dot is 8.

Some rational numbers look like irrational numbers when they get converted to decimal equivalents. That's because the calculator has limits in the number of the digits that can be displayed.

For instance, $\mathbf{\frac{49356012946}{3332666700000}}$ is a rational number, and has to have repeating digits, because 3 divides the denominator, but cannot divide the numerator.

Using a calculator, we can get

$\mathbf{\frac{49356012946}{3332666700000} = 0.0148097656888401111338256537925 02}$, which seems to have no repeating digits, and thus, looks like an irrational number.

It is rational, of course. That's simply because a rational number is a ratio between two integers, and 3332666700000 and 49356012946 are integers.