Examples 1 in Irrationals in Arithmetic 3

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Examples 1 in Irrationals

Simplify each radical below or put each in a mixed radical if it can be done so.
A mixed radical is a radical multiplied by a rational such as $\mathbf{2\sqrt{3}}$ and $\mathbf{4\sqrt[3]{2}}$.

0. $\sqrt{\mathbf{4}}$ 1. $\sqrt{\mathbf{8}}$ 2. $\sqrt{\mathbf{-2}}$ 3. $\sqrt{\mathbf{12}}$ 4. $\sqrt[3]{\mathbf{8}}$

5. $\sqrt[3]{\mathbf{32}}$ 6. $\sqrt[3]{\mathbf{-8}}$ 7. $\sqrt{\mathbf{1.69}}$ 8. $\sqrt[3]{\mathbf{5.6}}$ 9. $\sqrt[3]{\mathbf{0.016}}$

A. $\sqrt[3]{\mathbf{-3.2}}$ B. $\sqrt[3]{\mathbf{270}}$ C. $\sqrt[3]{\mathbf{0.081}}$ D. $\sqrt[3]{\mathbf{-16}}$

Suggestions or Solutions
To the Examples 1 in Irrationals

0. $\sqrt{\mathbf{4}} = \sqrt{\mathbf{2^2}} = \mathbf{2}$, because 1 x $\sqrt{\mathbf{4}}$ x $\sqrt{\mathbf{4}} = \mathbf{4}$, which is what's inside the square root sign, and we have $2^2 = 4$, too, which means we have **1·2·2 = 4**. So we get $\sqrt{\mathbf{4}}$ **= 2**.
What then, about **1·(-2)·(-2) = 4**?

We know $\sqrt{\mathbf{4}}$ is positive. So we don't get this $\sqrt{\mathbf{4}}$ **= -2**.

And by the same token, we get $\sqrt{\mathbf{9}} = \sqrt{\mathbf{3^2}} = \mathbf{3}$, $\sqrt{\mathbf{16}} = \sqrt{\mathbf{4^2}} = \mathbf{4}$, $\sqrt{\mathbf{25}} = \sqrt{\mathbf{5^2}} = \mathbf{5}$, etc.

1. $\sqrt{\mathbf{8}} = \sqrt{\mathbf{4 \cdot 2}} = \sqrt{\mathbf{4}}\sqrt{\mathbf{2}} = \mathbf{2}\sqrt{\mathbf{2}}$. How do we though, get this: $\sqrt{\mathbf{8}} = \sqrt{\mathbf{4 \cdot 2}} = \sqrt{\mathbf{4}}\sqrt{\mathbf{2}}$?

By definition, we get **1** x $\sqrt{\mathbf{8}}$ x $\sqrt{\mathbf{8}} = \mathbf{8}$.
And by definition again, we get **1** x $\sqrt{\mathbf{4 \cdot 2}}$ x $\sqrt{\mathbf{4 \cdot 2}} = \mathbf{4 \cdot 2 = 8}$.
Also, we get **1** x $\sqrt{\mathbf{4}}\sqrt{\mathbf{2}}$ x $\sqrt{\mathbf{4}}\sqrt{\mathbf{2}} = \sqrt{\mathbf{4}}$ x $\sqrt{\mathbf{4}}$ x $\sqrt{\mathbf{2}}$ x $\sqrt{\mathbf{2}} = \mathbf{4}$ x **2** = **8**.
So we get $\sqrt{\mathbf{8}} = \sqrt{\mathbf{4 \cdot 2}} = \sqrt{\mathbf{4}}\sqrt{\mathbf{2}}$.

And by the same token, we get

$\sqrt{\mathbf{21}} = \sqrt{\mathbf{3 \cdot 7}} = \sqrt{\mathbf{3}}\sqrt{\mathbf{7}}$, $\sqrt{\mathbf{15}} = \sqrt{\mathbf{3 \cdot 5}} = \sqrt{\mathbf{5}}\sqrt{\mathbf{3}}$, $\sqrt{\mathbf{54}} = \sqrt{\mathbf{6 \cdot 9}} = \sqrt{\mathbf{9}}\sqrt{\mathbf{6}} = \mathbf{3}\sqrt{\mathbf{6}}$

$\sqrt{\mathbf{21}} = \sqrt{\mathbf{3 \cdot 7}} = \sqrt{\mathbf{3}}\sqrt{\mathbf{7}}$, $\sqrt{\mathbf{24}} = \sqrt{\mathbf{6 \cdot 4}} = \sqrt{\mathbf{6}}\sqrt{\mathbf{4}} = \mathbf{2}\sqrt{\mathbf{6}}$, $\sqrt{\mathbf{72}} = \sqrt{\mathbf{36 \cdot 2}} = \sqrt{\mathbf{36}}\sqrt{\mathbf{2}} = \mathbf{6}\sqrt{\mathbf{2}}$

2. $\sqrt{\mathbf{-2}}$ cannot be made within the set of all real numbers, and thus, is not allowed, because no real number can be squared to be a negative number.

3. $\sqrt{\mathbf{12}} = \sqrt{\mathbf{4 \cdot 3}} = \sqrt{\mathbf{4}}\sqrt{\mathbf{3}} = \mathbf{2}\sqrt{\mathbf{3}}$ And by the same token, we get

$\sqrt{\mathbf{18}} = \sqrt{\mathbf{9 \cdot 2}} = \sqrt{\mathbf{9}}\sqrt{\mathbf{2}} = \mathbf{3}\sqrt{\mathbf{2}}$, $\sqrt{\mathbf{50}} = \sqrt{\mathbf{25 \cdot 2}} = \mathbf{5}\sqrt{\mathbf{2}}$, $\sqrt{\mathbf{48}} = \sqrt{\mathbf{16 \cdot 3}} = \mathbf{4}\sqrt{\mathbf{3}}$

$\sqrt{\mathbf{242}} = \sqrt{\mathbf{121 \cdot 2}} = \sqrt{\mathbf{11^2}}\sqrt{\mathbf{2}} = \mathbf{11}\sqrt{\mathbf{2}}$, $\sqrt{\mathbf{432}} = \sqrt{\mathbf{144 \cdot 3}} = \sqrt{\mathbf{12^2}}\sqrt{\mathbf{3}} = \mathbf{12}\sqrt{\mathbf{3}}$, etc.

4. $\sqrt[3]{\mathbf{8}} = \sqrt[3]{\mathbf{2^3}} = \mathbf{2}$, because 1 x $\sqrt[3]{\mathbf{8}}$ x $\sqrt[3]{\mathbf{8}}$ x $\sqrt[3]{\mathbf{8}} = \mathbf{8}$, which is what's inside the cube root sign, and we have $2^3 = 8$, too, which means we have **1·2·2·2 = 8**. So we get $\sqrt[3]{\mathbf{8}}$ **= 2**.

And by the same token, we get
$\sqrt[3]{\mathbf{27}} = \sqrt[3]{\mathbf{3^3}} = \mathbf{3}$, $\sqrt[3]{\mathbf{64}} = \sqrt[3]{\mathbf{4^3}} = \mathbf{4}$, $\sqrt[3]{\mathbf{125}} = \sqrt[3]{\mathbf{5^3}} = \mathbf{5}$, $\sqrt[3]{\mathbf{216}} = \sqrt[3]{\mathbf{6^3}} = \mathbf{6}$, etc.

5. $\sqrt[3]{\mathbf{32}} = \sqrt[3]{\mathbf{8 \cdot 4}} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{\mathbf{4}} = \mathbf{2}\sqrt[3]{\mathbf{4}}$ And by the same token, we get

$\sqrt[3]{\mathbf{40}} = \sqrt[3]{\mathbf{5 \cdot 8}} = \sqrt[3]{\mathbf{5}} \cdot \sqrt[3]{\mathbf{8}} = \mathbf{2}\sqrt[3]{\mathbf{5}}$, $\sqrt[3]{\mathbf{54}} = \sqrt[3]{\mathbf{27 \cdot 2}} = \sqrt[3]{\mathbf{27}} \cdot \sqrt[3]{\mathbf{2}} = \mathbf{3}\sqrt[3]{\mathbf{2}}$

$\sqrt[3]{\mathbf{72}} = \sqrt[3]{\mathbf{9 \cdot 8}} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{\mathbf{9}} = \mathbf{2}\sqrt[3]{\mathbf{9}}$, $\sqrt[3]{\mathbf{81}} = \sqrt[3]{\mathbf{27 \cdot 3}} = \sqrt[3]{\mathbf{27}} \cdot \sqrt[3]{\mathbf{3}} = \mathbf{3}\sqrt[3]{\mathbf{3}}$, $\sqrt[3]{\mathbf{162}} = \sqrt[3]{\mathbf{27 \cdot 6}} = \mathbf{3}\sqrt[3]{\mathbf{6}}$

6. $\sqrt[3]{\mathbf{-8}} = \sqrt[3]{\mathbf{(-2)^3}} = \mathbf{-2}$, because 1 x $\sqrt[3]{\mathbf{-8}}$ x $\sqrt[3]{\mathbf{-8}}$ x $\sqrt[3]{\mathbf{-8}} = \mathbf{-8}$, which is what's inside the cube root sign, and we have $(-2)^3 = -8$, too, which means we have **1·(-2)·(-2)·(-2) = -8**.

So we get $\sqrt[3]{\mathbf{-8}}$ **= -2**. And by the same token, we get

$\sqrt[3]{\mathbf{-27}} = \sqrt[3]{\mathbf{(-3)^3}} = \mathbf{-3}$, $\sqrt[3]{\mathbf{-64}} = \sqrt[3]{\mathbf{(-4)^3}} = \mathbf{-4}$, $\sqrt[3]{\mathbf{-0.008}} = \sqrt[3]{\mathbf{(-0.2)^3}} = \mathbf{-0.2}$, etc.

7. $\sqrt{\mathbf{1.69}} = \sqrt{\mathbf{(1.3)^2}} = \mathbf{1.3}$

8. $\sqrt[3]{\mathbf{5.6}} = \sqrt[3]{\mathbf{8 \cdot 0.7}} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{\mathbf{0.7}} = \mathbf{2}\sqrt[3]{\mathbf{0.7}}$

9. $\sqrt[3]{\mathbf{0.016}} = \sqrt[3]{\mathbf{8 \cdot 0.002}} = \sqrt[3]{\mathbf{8}} \cdot \sqrt[3]{\mathbf{0.001}} \cdot \sqrt[3]{\mathbf{2}} = \mathbf{2 \cdot 0.1}\sqrt[3]{\mathbf{2}} = \mathbf{0.2}\sqrt[3]{\mathbf{2}}$

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A. $\sqrt[3]{-3.2} = \sqrt[3]{-8 \cdot 0.4} = \sqrt[3]{-8} \cdot \sqrt[3]{0.4} = -2\sqrt[3]{0.4}$

B. $\sqrt[3]{270} = \sqrt[3]{27 \cdot 10} = \sqrt[3]{27}\sqrt[3]{10} = 3\sqrt[3]{10}$

C. $\sqrt[3]{0.081} = \sqrt[3]{0.001 \cdot 81} = \sqrt[3]{0.001}\sqrt[3]{81} = 0.1\sqrt[3]{27 \cdot 3} = 0.1\sqrt[3]{3^3 3} = 0.3\sqrt[3]{3}$

D. $\sqrt[3]{-160} = \sqrt[3]{-8 \cdot 20} = \sqrt[3]{-2^3 20} = -2\sqrt[3]{20}$

Note that $-2^3 = (-2)^3 = -8$, but $-2^2 \neq (-2)^2$, because $-2^2 = -4$, and $(-2)^2 = 4$.