Examples 4 in Irrationals in Arithmetic 3

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Examples 4 in Irrationals

Simplify each radical below or put each in a mixed radical if it can be done so.
A mixed radical is a radical multiplied by a rational such as $\mathbf{2\sqrt{3}}$ and $\mathbf{4\sqrt[3]{2}}$.

0. $\sqrt{\frac{1}{4}}$ 1. $\sqrt{\frac{1}{8}}$ 2. $\sqrt{\frac{-3}{8}}$ 3. $\sqrt[3]{\frac{8}{-27}}$

4. $\sqrt[3]{-\frac{8}{49}}$ 5. $\sqrt{\frac{7}{8}}$ 6. $\sqrt{\frac{3}{-4}}$ 7. $\sqrt[3]{\frac{2}{27}}$

8. $\sqrt[3]{-\frac{16}{27}}$ 9. $\sqrt{\frac{4}{27}}$ A. $\sqrt[3]{\frac{27}{4}}$ B. $\sqrt[3]{\frac{108}{27}}$

C. $\sqrt{\frac{45}{98}}$ D. $\sqrt[3]{\frac{1}{4}}$ E. $\sqrt[3]{\frac{1}{24}}$ F. $\sqrt[3]{\frac{3}{8}}$

G. $\sqrt[3]{\frac{4}{9}}$ H. $\sqrt{\frac{4}{9}}$ I. $\sqrt{\frac{2}{9}}$ J. $\sqrt{-\frac{4}{49}}$

K. $\sqrt[3]{\frac{15}{18}}$ L. $\sqrt[3]{\frac{-1}{24}}$ M. $\sqrt{\frac{8}{3}}$ N. $\sqrt[3]{\frac{8}{81}}$

O. $\sqrt[3]{\frac{16}{9}}$ P. $\sqrt[3]{\frac{135}{98}}$

Suggestions or Solutions
To the Examples 4 in Irrationals

0. $\sqrt{\frac{1}{4}} = \sqrt{(\frac{1}{2})^2} = \frac{1}{2}$, and we can put it this way, too, of course: $\sqrt{\frac{1}{4}} = \sqrt{\frac{1}{2^2}} = \frac{\sqrt{1}}{\sqrt{2^2}} = \frac{1}{2}$.

1. $\sqrt{\frac{1}{8}} = \sqrt{\frac{1}{4} \cdot \frac{1}{2}} = \sqrt{(\frac{1}{2})^2 \frac{1}{2}} = \frac{1}{2}\sqrt{\frac{1}{2}}$, and this way, too: $\sqrt{\frac{1}{8}} = \sqrt{(\frac{1}{2})^3} = \sqrt{(\frac{1}{2})^2 \frac{1}{2}} = \frac{1}{2}\sqrt{\frac{1}{2}}$.

2. $\sqrt{\frac{-3}{8}}$ cannot be made within the set of all real numbers, and thus, is not allowed, because no real number can be squared to be a negative number.

3. $\sqrt[3]{\frac{8}{-27}} = \sqrt[3]{-\frac{8}{27}} = -\sqrt[3]{\frac{2^3}{3^3}} = -\sqrt[3]{(\frac{2}{3})^3} = -\frac{2}{3}$

4. $\sqrt[3]{-\frac{8}{49}} = -\sqrt[3]{\frac{8}{49}} = -\sqrt[3]{\frac{2^3}{49}} = -\frac{\sqrt[3]{2^3}}{\sqrt[3]{49}} = -\frac{2}{\sqrt[3]{49}}$, which can be put the way below, too:

$-\frac{2}{\sqrt[3]{49}} = -\frac{2\sqrt[3]{7}}{\sqrt[3]{49}\sqrt[3]{7}} = -\frac{2\sqrt[3]{7}}{\sqrt[3]{49 \cdot 7}} = -\frac{2\sqrt[3]{7}}{\sqrt[3]{7^3}} = -\frac{2\sqrt[3]{7}}{7}$

5. $\sqrt{\frac{7}{8}} = \sqrt{\frac{7}{4 \cdot 2}} = \sqrt{\frac{1}{4} \cdot \frac{7}{2}} = \sqrt{(\frac{1}{2})^2 \frac{7}{2}} = \frac{1}{2}\sqrt{\frac{7}{2}}$

6. $\sqrt{\frac{3}{-4}}$ cannot be made within the set of all real numbers, and thus, is not allowed, because no real number can be squared to be a negative number.

7. $\sqrt[3]{\frac{2}{27}} = \sqrt[3]{\frac{2}{3^3}} = \sqrt[3]{\frac{1}{3^3} \cdot 2} = \sqrt[3]{(\frac{1}{3})^3 2} = \sqrt[3]{(\frac{1}{3})^3} \cdot \sqrt[3]{2} = \frac{1}{3}\sqrt[3]{2}$, which equals $\frac{\sqrt[3]{2}}{3}$.

8. $\sqrt[3]{-\frac{16}{27}} = -\sqrt[3]{\frac{8\cdot 2}{3^3}} = -\frac{1}{3}\sqrt[3]{8\cdot 2} = -\frac{1}{3}\sqrt[3]{2^3\cdot 2} = -\frac{1}{3}\sqrt[3]{2^3}\cdot\sqrt[3]{2} = -\frac{2}{3}\sqrt[3]{2}$, which equals $-\frac{2\sqrt[3]{2}}{3}$.

9. $\sqrt{\frac{4}{27}} = \sqrt{\frac{2^2}{3^2\cdot 3}} = \sqrt{\frac{2^2}{3^2}\cdot\frac{1}{3}} = \sqrt{(\frac{2}{3})^2\,\frac{1}{3}} = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3}\cdot\frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3\cdot 3} = \frac{2\sqrt{3}}{9}$

A. $\sqrt[3]{\frac{27}{4}} = \sqrt[3]{\frac{3^3}{4}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{4}} = \frac{3}{\sqrt[3]{4}} = \frac{3\sqrt[3]{2}}{\sqrt[3]{4}\cdot\sqrt[3]{2}} = \frac{3\sqrt[3]{2}}{\sqrt[3]{4\cdot 2}} = \frac{3\sqrt[3]{2}}{\sqrt[3]{2^3}} = \frac{3\sqrt[3]{2}}{2} = \frac{3}{2}\sqrt[3]{2}$

B. $\sqrt[3]{\frac{108}{27}} = \sqrt[3]{\frac{27\cdot 4}{3^3}} = \sqrt[3]{\frac{3^3\cdot 4}{3^3}} = \sqrt[3]{4}$

C. $\sqrt{\frac{45}{98}} = \sqrt{\frac{9\cdot 5}{49\cdot 2}} = \sqrt{\frac{3^2\cdot 5}{7^2\cdot 2}} = \sqrt{\frac{3^2}{7^2}}\sqrt{\frac{5}{2}} = \frac{3}{7}\sqrt{\frac{5}{2}} = \frac{3}{7}\frac{\sqrt{5}}{\sqrt{2}} = \frac{3}{7}\frac{\sqrt{5}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3}{7}\frac{\sqrt{5\cdot 2}}{2} = \frac{3\sqrt{10}}{14}$

D. $\sqrt[3]{\frac{1}{4}}$ does not really have to be simplified, but can be changed to $\sqrt[3]{\frac{2}{8}} = \frac{\sqrt[3]{2}}{\sqrt[3]{8}} = \frac{\sqrt[3]{2}}{2}$.

E. $\sqrt[3]{\frac{1}{24}} = \sqrt[3]{\frac{1}{8\cdot 3}} = \sqrt[3]{\frac{1}{8}}\cdot\sqrt[3]{\frac{1}{3}} = \frac{1}{2}\sqrt[3]{\frac{1}{3}} = \frac{1}{2}\frac{1}{\sqrt[3]{3}} = \frac{1}{2\sqrt[3]{3}} = \frac{\sqrt[3]{3^2}}{2\sqrt[3]{3}\cdot\sqrt[3]{3^2}} = \frac{\sqrt[3]{3^2}}{2\cdot 3} = \frac{\sqrt[3]{9}}{6}$

F. $\sqrt[3]{\frac{3}{8}} = \sqrt[3]{\frac{3}{2^3}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{3}}{2}$

G. $\sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{12}{27}} = \frac{\sqrt[3]{12}}{\sqrt[3]{27}} = \frac{\sqrt[3]{12}}{3}$

H. $\sqrt{\frac{4}{9}} = \sqrt{(\frac{2}{3})^2} = \frac{2}{3}$

I. $\sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{\sqrt{9}} = \frac{\sqrt{2}}{3}$

J. $\sqrt{-\frac{4}{49}}$ cannot be made within the set of all real numbers, and thus, is not allowed, because no real number can be squared to be a negative number.

K. $\sqrt[3]{\frac{15}{18}} = \sqrt[3]{\frac{5}{6}}$, which does not really have to be simplified, but can be changed to $\sqrt[3]{\frac{5 \cdot 36}{6 \cdot 36}} = \frac{\sqrt[3]{180}}{6}$.

L. $\sqrt[3]{\frac{-1}{24}} = -\sqrt[3]{\frac{1}{24}} = -\sqrt[3]{\frac{1}{8 \cdot 3}} = -\sqrt[3]{\frac{1}{8}} \cdot \sqrt[3]{\frac{1}{3}} = -\frac{1}{2}\sqrt[3]{\frac{9}{27}} = -\frac{1}{2}\frac{\sqrt[3]{9}}{\sqrt[3]{27}} = -\frac{1}{2}\frac{\sqrt[3]{9}}{3} = -\frac{\sqrt[3]{9}}{6}$

M. $\sqrt{\frac{8}{3}} = \sqrt{\frac{4 \cdot 2}{3}} = 2\sqrt{\frac{2}{3}} = 2\frac{\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{6}}{3}$

N. $\sqrt[3]{\frac{8}{81}} = \frac{\sqrt[3]{8}}{\sqrt[3]{81}} = \frac{2\sqrt[3]{9}}{\sqrt[3]{81 \cdot 9}} = \frac{2\sqrt[3]{9}}{9}$

O. $\sqrt[3]{\frac{16}{9}} = \sqrt[3]{\frac{16 \cdot 3}{27}} = \frac{\sqrt[3]{48}}{\sqrt[3]{27}} = \frac{\sqrt[3]{48}}{3}$

P. $\sqrt[3]{\frac{135}{98}} = \sqrt[3]{\frac{27 \cdot 5}{49 \cdot 2}} = \sqrt[3]{\frac{3^3 \cdot 5 \cdot 7}{7^3 \cdot 2}} = \frac{3}{7}\sqrt[3]{\frac{5 \cdot 7}{2}} = \frac{3}{7}\sqrt[3]{\frac{5 \cdot 7 \cdot 4}{2^3}} = \frac{3}{14}\sqrt[3]{140} = \frac{3\sqrt[3]{140}}{14}$