Examples 6 in Irrationals in Arithmetic 3

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Examples 6 in Irrationals

Simplify each radical below or put each in a mixed radical if it can be done so.
A mixed radical is a radical multiplied by a rational such as $\mathbf{2\sqrt{3}}$ and $\mathbf{4\sqrt[3]{2}}$.

0. $\sqrt{x^2}$ 1. $\sqrt{x^3}$ 2. $\sqrt{x^4}$ 3. $\sqrt{x^5}$ 4. $\sqrt{xy^2}$

5. $\sqrt{x^2y^2}$ 6. $\sqrt{x^5y^2}$ 7. $\sqrt{y^6}$ 8. $\sqrt{x^5y^7}$ 9. $\sqrt{|x|^2}$

A. $\sqrt{|x|^2\ y^2}$ B. $\sqrt[3]{x^3}$ C. $\sqrt[3]{x^4}$ D. $\sqrt[3]{x^5}$ E. $\sqrt[3]{x^7}$

F. $\sqrt[3]{-x^{23}}$ G. $\sqrt[3]{-x^3y^3}$ H. $\sqrt[3]{x^4y^4}$ I. $\sqrt[3]{-x^4y^4}$ J. $\sqrt{\frac{1}{x^2}}$

K. $\sqrt{\frac{1}{x^3}}$ L. $\sqrt{\frac{-2}{x^3}}$ M. $\sqrt[3]{\frac{8}{-x^3}}$ N. $\sqrt[3]{-\frac{4y^3}{x^6}}$ O. $\sqrt{\frac{7y^3}{8x^2}}$

P. $\sqrt{\frac{3x^2}{-4y}}$ Q. $\sqrt[3]{\frac{64y^2}{9x^3}}$ R. $\sqrt[3]{-\frac{16y^3z^4}{81x^4}}$ S. $\sqrt{\frac{4y^3z^3}{27x^3}}$ T. $\sqrt[3]{\frac{27z^5}{4x^4y^5}}$

Suggestions or Solutions
To the Examples 6 in Irrationals

0. $\sqrt{x^2} = x$ if $x \geq 0$. If however, $x < 0$, we get $\sqrt{x^2} = -x$, because $\sqrt{x^2} > 0$.

So in sum, we get $\sqrt{x^2} = |x|$. What do we mean by $|x|$ though?

$|x|$ is the absolute value of, that is, the magnitude of x.
So for instance, we have |-2| = 2, |0| = 0, and |3| = 3.

And we want to keep in mind that not only what's inside a square root sign but the square root, too, is ≥ 0.

So if for instance, $r = \sqrt{t}$, we get $r \geq 0$, that is, $\sqrt{t} \geq 0$ as well as $t \geq 0$.

1. $\sqrt{x^3} = \sqrt{x^2 x} = x\sqrt{x}$ for $x \geq 0$.   Why not $\sqrt{x^2 x} = |x|\sqrt{x}$, though?

It is OK, too, to set $\sqrt{x^2 x} = |x|\sqrt{x}$ for $x \geq 0$, because if $x \geq 0$, $|x|\sqrt{x} = x\sqrt{x}$.

And in $\sqrt{x^3}$, x cannot be negative, because what's inside a square root sign cannot be negative, that is, it can only be positive or 0.

So it is necessary to specify $x \geq 0$ in this case.

Just setting $\sqrt{x^3} = \sqrt{x^2 x} = x\sqrt{x}$, we mean x can be any of all real numbers including negative numbers, that is, it can be negative as well as positive or 0.

2. $\sqrt{x^4} = \sqrt{(x^2)^2} = x^2$. Why not $|x^2|$ though?

We have $|x^2| = x^2$, since $|x^2| \geq 0$, and so is x^2. So the absolute value sign is unnecessary.

3. $\sqrt{x^5} = \sqrt{x^4 x} = x^2\sqrt{x}$ for $x \geq \mathbf{0}$.

4. $\sqrt{xy^2} = \sqrt{y^2}\sqrt{x} = |y|\sqrt{x}$ for $x \geq \mathbf{0}$.

5. $\sqrt{x^2 y^2} = \sqrt{x^2}\sqrt{y^2} = |x|\cdot|y| = |xy|$.

Note however, it is ***not always*** the case where $|x - y| = |x| - |y|$. So for instance, we have

3 – 2| = |1| = 1, and |3| – |2| = 3 – 2 = 1.

2 – 3| = |-1| = 1, but |2| – |3| = 2 – 3 = -1.

So we have $|x - y| \neq |x| - |y|$.

6. $\sqrt{x^5 y^2} = \sqrt{y^2}\sqrt{x^5} = |y|\sqrt{x^5} = |y|x^2\sqrt{x}$ for $x \geq \mathbf{0}$.

7. $\sqrt{y^6} = \sqrt{(y^3)^2} = |y^3|$. That's because

if $y \geq \mathbf{0}$, we get $y^3 \geq \mathbf{0}$. So it is the case where $\sqrt{y^6} = y^3$, because $\sqrt{y^6} \geq \mathbf{0}$.

If however, $y < \mathbf{0}$, we get $y^3 < \mathbf{0}$, so we cannot just set $\sqrt{y^6} = y^3$, because $\sqrt{y^6} > \mathbf{0}$.

And thus, we get $\sqrt{y^6} = |\, y^3 \,|$.

• So we want to be careful with the sign of the number or the variable when we take it out of a square root sign.

8. $\sqrt{x^5 y^7} = \sqrt{x^5}\sqrt{y^7} = \sqrt{x^4 x}\sqrt{y^6 y} = \sqrt{x^4}\sqrt{x}\sqrt{y^6}\sqrt{y} = x^2\sqrt{x}\cdot y^3\sqrt{y}$

$= x^2 y^3\sqrt{x}\sqrt{y} = x^2 y^3\sqrt{xy}$ for x and y both $\geq \mathbf{0}$.

Why not $\sqrt{y^6}\sqrt{y} = |\, y^3 \,|\sqrt{y}$, though?

Just setting $\sqrt{y^6}\sqrt{y} = \ |y^3|\sqrt{y}$, we mean y can be any of all real numbers including negative numbers, that is, it can be negative as well as positive or 0.

And we can get the same result the way below, too.

$\sqrt{x^5y^7} = \sqrt{x^4y^6xy} = \sqrt{(x^2y^3)^2xy} = \ x^2y^3\sqrt{xy}$ for x and y both $\geq \mathbf{0}$.

9. $\sqrt{|x|^2}$ is nothing but $\sqrt{x^2}$, so we get $\sqrt{|x|^2} = |x|$.

A. $\sqrt{|x|^2\ y^2}$ is nothing but $\sqrt{x^2y^2}$, so we get $\sqrt{|x|^2\ y^2} = \sqrt{x^2y^2} = |xy|$.

B. $\sqrt[3]{x^3} = x$

C. $\sqrt[3]{x^4} = \sqrt[3]{x^3x} = \sqrt[3]{x^3}\cdot\sqrt[3]{x} = x\sqrt[3]{x}$

D. $\sqrt[3]{x^5} = \sqrt[3]{x^3}\cdot\sqrt[3]{x^2} = x\sqrt[3]{x^2}$

E. $\sqrt[3]{x^7} = \sqrt[3]{x^6x} = \sqrt[3]{x^6}\cdot\sqrt[3]{x} = \sqrt[3]{(x^2)^3}\cdot\sqrt[3]{x} = x^2\sqrt[3]{x}$

F. $\sqrt[3]{-x^{23}} = -\sqrt[3]{x^{21}x^2} = -\sqrt[3]{x^{21}}\cdot\sqrt[3]{x^2} = -\sqrt[3]{(x^7)^3}\cdot\sqrt[3]{x^2} = -x^7\sqrt[3]{x^2}$

G. $\sqrt[3]{-x^3y^3} = -\sqrt[3]{x^3y^3} = -\sqrt[3]{(xy)^3} = -xy$

H. $\sqrt[3]{x^4y^4} = \sqrt[3]{x^3y^3xy} = \sqrt[3]{x^3y^3}\cdot\sqrt[3]{xy} = \sqrt[3]{(xy)^3}\cdot\sqrt[3]{xy} = xy\sqrt[3]{xy}$

I. $\sqrt[3]{-x^4y^4} = -\sqrt[3]{x^4y^4} = -xy\sqrt[3]{xy}$

J. $\sqrt{\frac{1}{x^2}} = \frac{1}{\sqrt{x^2}} = \frac{1}{|x|} = |\frac{1}{x}|$ for $x \neq 0$.

K. $\sqrt{\frac{1}{x^3}} = \frac{1}{\sqrt{x^3}} = \frac{1}{\sqrt{x^2x}} = \frac{1}{\sqrt{x^2}\sqrt{x}} = \frac{1}{x\sqrt{x}} = \frac{\sqrt{x}}{x\sqrt{x}\sqrt{x}} = \frac{\sqrt{x}}{x^2}$ for $x > 0$.

And we can put it this way, too: $\sqrt{\frac{1}{x^3}} = \sqrt{\frac{x}{x^3x}} = \frac{\sqrt{x}}{\sqrt{x^4}} = \frac{\sqrt{x}}{x^2}$ for $x > 0$.

L. $\sqrt{\frac{-2}{x^3}} = \sqrt{\frac{-2}{x}\cdot\frac{1}{x^2}} = \sqrt{\frac{-2}{x}}\sqrt{\frac{1}{x^2}} = \sqrt{\frac{-2}{x}}\sqrt{(\frac{1}{x})^2} = -\frac{1}{x}\sqrt{\frac{-2}{x}}$ for $x < 0$. How com?

What's inside a square root sign is ≥ 0, which means in this case, x has to be < **0**, and it cannot be 0, because no denominator can be 0. So x can only be negative in this case.

M. $\sqrt[3]{\frac{8}{-x^3}} = -\sqrt[3]{(\frac{2}{x})^3} = -\frac{2}{x}$ for $x \neq 0$.

N. $\sqrt[3]{-\frac{4y^3}{x^6}} = -\sqrt[3]{4\cdot\frac{y^3}{x^6}} = -\sqrt[3]{4}\cdot\sqrt[3]{\frac{y^3}{x^6}} = -\sqrt[3]{4}\cdot\sqrt[3]{(\frac{y}{x^2})^3} = -\sqrt[3]{4}\cdot\frac{y}{x^2} = -\frac{y\sqrt[3]{4}}{x^2}$ for $x \neq 0$.

O. $\sqrt{\frac{7y^3}{8x^2}} = \frac{\sqrt{7y^3}}{\sqrt{8x^2}} = \frac{\sqrt{7y^2y}}{\sqrt{4\cdot 2x^2}} = \frac{y\sqrt{7y}}{2|x|\sqrt{2}} = \frac{y\sqrt{7y}\sqrt{2}}{2|x|\sqrt{2}\sqrt{2}} = \frac{y\sqrt{14y}}{4|x|}$ for $x \neq 0$, and $y \geq 0$.

Note that $4|x| = |4x|$.

P. $\sqrt{\frac{3x^2}{-4y}} = \sqrt{\frac{x^2}{4}}\sqrt{\frac{3}{-y}} = \frac{|x|}{2}\sqrt{\frac{3}{-y}} = \frac{|x|}{2}\sqrt{\frac{-3y}{yy}} = \frac{|x|}{2}\sqrt{\frac{1}{y^2}}\sqrt{-3y} = \frac{|x|}{-2y}\sqrt{-3y}$ for $y < 0$.

Note that in the case above, y can only be negative, because no denominator can be 0, and what's inside a square root sign is ≥ **0**.

Q. $\sqrt[3]{\frac{64y^2}{9x^3}} = \sqrt[3]{\frac{4^3}{x^3}} \cdot \sqrt[3]{\frac{y^2}{3^2}} = \frac{4}{x}\sqrt[3]{\frac{3y^2}{3^2 3}} = \frac{4}{3x}\sqrt[3]{3y^2} = \frac{4\sqrt[3]{3y^2}}{3x}$ for $x \neq 0$.

R. $\sqrt[3]{-\frac{16y^3z^4}{81x^4}} = -\sqrt[3]{\frac{8\cdot 2y^3z^3z}{27\cdot 3x^3x}} = -\sqrt[3]{\frac{(2yz)^3 2z}{(3x)^3 3x}} = -\sqrt[3]{\frac{(2yz)^3}{(3x)^3}} \cdot \sqrt[3]{\frac{2z}{3x}} = -\frac{2yz}{3x}\sqrt[3]{\frac{2z}{3x}}$ for $x \neq 0$.

S. $\sqrt{\frac{4y^3z^3}{27x^3}} = \sqrt{\frac{2^2y^3z^3}{3^3x^3}} = \sqrt{\frac{(2yz)^2yz}{(3x)^2 3x}} = \frac{2yz}{3x}\sqrt{\frac{yz}{3x}} = \frac{2yz}{3x}\sqrt{\frac{yz3x}{3^2x^2}} = \frac{2yz}{3x}\frac{\sqrt{3xyz}}{3x} = \frac{2yz\sqrt{3xyz}}{9x^2}$ for $x > 0$, and y and z both ≥ 0.

T. $\sqrt[3]{\frac{27z^5}{4x^4y^5}} = \sqrt[3]{\frac{2\cdot 3^3z^3z^2}{8x^3y^3xy^2}} = \sqrt[3]{\frac{3^3z^3}{2^3x^3y^3}} \cdot \sqrt[3]{\frac{2z^2}{xy^2}} = \frac{3z}{2xy}\sqrt[3]{\frac{2z^2}{xy^2}} = \frac{3z}{2xy}\sqrt[3]{\frac{2yz^2}{xy^3}} = \frac{3z}{2xy^2}\sqrt[3]{\frac{2yz^2}{x}}$

$= \frac{3z}{2xy^2}\sqrt[3]{\frac{2x^2yz^2}{x^3}} = \frac{3z}{2x^2y^2}\sqrt[3]{2x^2yz^2} = \frac{3z\sqrt[3]{2x^2yz^2}}{2x^2y^2}$ for x and y both $\neq 0$.