Examples 7 in Irrationals in Arithmetic 3

Click this to start.

Examples 7 in Irrationals

Simplify each expression below.

0. $\sqrt{(2-\sqrt{5})^2}$
1. $\sqrt{(\sqrt{3}-1)^2}$
2. $\sqrt{(x^2-2x+4)^2}$
3. $\sqrt{(x^2-3x+2)^2}$
4. $\sqrt{(3x-2-x^2)^2}$
5. $\sqrt{(2x-4-x^2)^2}$
6. $\sqrt{x^2-2xy+y^2}$
7. $\sqrt{(x-2)^2}+|2-x|$

Suggestions or Solutions
To the Examples 7 in Irrationals

0. Simplify $\sqrt{(2-\sqrt{5})^2}$

We know $\sqrt{2^2} = 2$, so assumig $N = \sqrt{(2-\sqrt{5})^2}$, do we just get $N = 2 - \sqrt{5}$?

First, we have $5 > 4 \Rightarrow \sqrt{5} > \sqrt{4} = 2 \Rightarrow \sqrt{5} > 2 \Rightarrow 2 - \sqrt{5} < 0$.

Next, taking a square root of a number, we can only get a number positive or 0. And the number is the square root. That is to say that the square root is positive or 0.

So we cannot have $N < 0$. That is, $N \neq 2 - \sqrt{5}$. What then, is N?

We have $N = \sqrt{(2-\sqrt{5})^2} = \sqrt{(\sqrt{5}-2)^2}$.

And we know $\sqrt{5} - 2 > 0$. So we get $N = \sqrt{5} - 2$.

Note that we have in fact, a fact that $\sqrt[n]{k} \geq 0$ for any $k \geq 0$ if n is even.

If however, n is odd, we get $\sqrt[n]{k} < 0$ if $k < 0$.

For instance, we have $\sqrt[3]{-3} < 0$, and $\sqrt[8]{2} > 0$.

1. Simplify $\sqrt{(\sqrt{3}-1)^2}$

We have $\mathbf{3 > 1 \Rightarrow \sqrt{3} > \sqrt{1} = 1 \Rightarrow \sqrt{3} > 1 \Rightarrow \sqrt{3} - 1 > 0}$. So we get $\sqrt{(\sqrt{3}-1)^2} = \sqrt{3}-1$.

0.2. Simplify $\sqrt{(x^2-2x+4)^2}$

Assuming first, $N = \sqrt{(x^2-2x+4)^2}$, we need to have this first: $N \geq 0$.

That is, it is not always the case where we just get $N = x^2 - 2x + 4$.

Thus next, we want check to see if $(x^2 - 2x + 4)$ can be negative.

So dong the check, we get $x^2 - 2x + 4 = x^2 - 2x + 1 + 3 = (x - 1)^2 + 3 > 0$ for all x.

And thus, we can now say that $N = x^2 - 2x + 4$.

0.3. $\sqrt{(x^2-3x+2)^2}$

Assuming first, $N = \sqrt{(x^2-3x+2)^2}$, we can say for now, that $N \geq 0$.

Thus next, we want to check to see if $(x^2 - 3x + 2)$ can be negative.

We can get $x^2 - 3x + 2 = (x - 1)(x - 2) \geq 0 \Rightarrow x \leq 1$ or $x \geq 2$.
That is to say that we get $x^2 - 3x + 2 < 0$ if $1 < x < 2$.

So we get $N = x^2 - 3x + 2$ for $x \leq 1$ or $x \geq 2$, or $N = -(x^2 - 3x + 2)$ for $1 < x < 2$.

And we can put it this way, too: $N = |x^2 - 3x + 2|$.

That is to say that we can set $\sqrt{(x^2-3x+2)^2} = |x^2-3x+2|$.

0.4. $\sqrt{(3x-2-x^2)^2}$

Assuming first, $N = \sqrt{(3x-2-x^2)^2}$, we can say for now, that $N \geq 0$.

Thus next, we want to check to see if $(3x - 2 - x^2)$ can be negative.

We can get $\mathbf{3x - 2 - x^2 = (1 - x)(x - 2) \geq 0 \Rightarrow (x - 1)(x - 2) \leq 0 \Rightarrow 1 \leq x \leq 2}$.
That is, we get $\mathbf{3x - 2 - x^2 < 0}$ if $\mathbf{x < 1}$ or $\mathbf{x > 2}$.

So we get $\mathbf{N = 3x - 2 - x^2}$ for $\mathbf{1 \leq x \leq 2}$, or $\mathbf{N = x^2 - 3x + 2}$ for $\mathbf{x < 1}$ or $\mathbf{x > 2}$.

And we can put it this way, too: $\mathbf{N = |3x - 2 - x^2|}$.

That is, we can set $\sqrt{(3x-2-x^2)^2} = |3x - 2 - x^2| = |x^2 - 3x + 2|$.

0.5. $\sqrt{(2x-4-x^2)^2}$

Assuming first, $N = \sqrt{(2x-4-x^2)^2}$, we can say for now, that $N \geq 0$.

So next, we want to check to see if $\mathbf{(2x - 4 - x^2)}$ can be negative.

We can get $\mathbf{2x - 4 - x^2 = -x^2 + 2x - 4 = -(x^2 - 2x + 4) = -(x^2 - 2x + 1 + 3)}$
$\mathbf{= -\{(x - 1)^2 + 3\} = -(x - 1)^2 - 3 \leq -3}$, which is negative for all $\mathbf{x}$.

So we get $\mathbf{N = -(2x - 4 - x^2) = x^2 - 2x + 4}$.

And we can put it this way, too: $\mathbf{N = |2x - 4 - x^2|}$.

So we can see that $\sqrt{(2x-4-x^2)^2} = |2x - 4 - x^2| = x^2 + 2x - 4$.

0.6. $\sqrt{x^2-2xy+y^2}$

Assuming first, $N = \sqrt{x^2-2xy+y^2}$, we can say for now, that $N \geq 0$.

So next, we want to check to see if what's inside the square root can be negative.

We can get $\mathbf{x^2 - 2xy + y^2 = (x - y)^2}$. So we get

$\mathbf{N = (x^2 - 2xy + y^2)^{1/2} = \{(x - y)^2\}^{1/2} = x - y}$ for $\mathbf{x \geq y}$, or $\mathbf{N = -(x - y) = y - x}$ for $\mathbf{x < y}$, And we can put it this way, too: $N = |x - y| = |y - x|$.

That is, we can set $\sqrt{x^2 - 2xy + y^2} = |x - y| = |y - x|$.

0.7. $\sqrt{(x-2)^2} + |2 - x|$

Assuming first, $N = \sqrt{(x-2)^2} + |2 - x|$, we can say for now, that $N \geq 0$.

And next, we get $x - 2 \geq 0 \Rightarrow x \geq 2$. And we get $x - 2 < 0 \Rightarrow x < 2$.

So assuming $M = \sqrt{(x-2)^2}$, we get $M = x - 2$ for $x \geq 2$, or $M = 2 - x$ for $x < 2$.

What then, about $|2 - x|$?

The two vertical bars are called absolute value sign.
And we get $|y| \geq 0$ for all y. So for instance, $|-3| = 3$.

Thus first, we get $2 - x \geq 0 \Rightarrow x \leq 2$. So we get $|2 - x| = 2 - x$ if $x \leq 2$.
So next, if $x > 2$, we get $|2 - x| = -(2 - x) = x - 2$.

Assuming thus, $K = |2 - x|$, we get $K = x - 2$ for $x > 2$, or $K = 2 - x$ for $x \leq 2$.

And thus, we get

$N = M + K = x - 2 - (2 - x) = 2x - 4$ if $x \geq 2$.

$N = M + K = 2 - x + (2 - x) = 4 - 2x$ if $x < 2$.