Examples A in Irrationals in Arithmetic 3

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Examples A in Irrationals

0. Assuming $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{x}$, and $\boldsymbol{y}$ are rationals, and $\boldsymbol{C}$ is an irrational, show that $\boldsymbol{a} + \boldsymbol{bC} = \boldsymbol{x} + \boldsymbol{yC} \Rightarrow \boldsymbol{a} = \boldsymbol{x}$, and $\boldsymbol{b} = \boldsymbol{y}$.

1. Assuming $\boldsymbol{a}$ and $\boldsymbol{b}$ are rationals, and $\boldsymbol{C}$ is an irrational, show that $\boldsymbol{a} + \boldsymbol{bC} = \mathbf{0} \Rightarrow \boldsymbol{a} = \boldsymbol{b} = \mathbf{0}$.

2. Assuming $\boldsymbol{a}$ and $\boldsymbol{c}$ are rationals, but $\boldsymbol{B}$ and $\boldsymbol{D}$ are irrationals, show that $\boldsymbol{a} + \boldsymbol{B} = \boldsymbol{c} + \boldsymbol{D} \Rightarrow \boldsymbol{a} = \boldsymbol{c}$, and $\boldsymbol{B} = \boldsymbol{D}$.

3. Assuming $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are rationals, $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are irrationals, and $\boldsymbol{A} \neq \boldsymbol{B} \neq \boldsymbol{C}$, show that $\boldsymbol{aA} + \boldsymbol{bB} + \boldsymbol{cC} = \mathbf{0} \Rightarrow \boldsymbol{a} = \boldsymbol{b} = \boldsymbol{c} = \mathbf{0}$.

Suggestions or Solutions
To the Examples A in Irrationals

0. Assuming a, b, x, and y are rationals, and C is an irrational, show that
$a + bC = x + yC \Rightarrow a = x$, and $b = y$.

To begin with, we can put it this way: $a + bC = x + yC \;\Rightarrow\; a - x + (b - y)C = 0$.

Next, assuming $b - y \neq 0$, we can get $C = \frac{a-x}{-(b-y)} = \frac{a-x}{y-b}$. What then, do we get?

We get a contradiction. Why?

We know a, b, x, and y are rationals. So $a - x$ and $y - b$ are rationals, too.

So $\frac{a-x}{y-b}$ is a rational, too, because dividing a rational by another, we get a rational, too.

However, C is an irrational, which contradicts the fact that C is an irrational.

So it is not the case where $b - y \neq 0$, that is, we get $b - y = 0 \Rightarrow b = y$.
What then, do we get?

We can get $a - x + (b - y)C = 0 \Rightarrow a - x + 0 = 0 \Rightarrow a - x = 0 \Rightarrow a = x$.

1. Assuming a and b are rationals, and C is an irrational, show that
$a + bC = 0 \Rightarrow a = b = 0$.

Assuming first, $\boldsymbol{b \neq 0}$, we can get $\boldsymbol{C = -\frac{b}{a}}$, which is a rational, which however, contradicts the fact that $\boldsymbol{C}$ is an irrational. So we get $\boldsymbol{b = 0}$.

So next, we get $\boldsymbol{a + bC = 0 \Rightarrow a + 0 = 0 \Rightarrow a = 0}$.

2. Assuming *a* and *c* are rationals, but *B* and *D* are irrationals, show that
$\boldsymbol{a + B = c + D \Rightarrow a = c}$, and $\boldsymbol{B = D}$.

To begin with, $\boldsymbol{a - c}$ is a rational.
Next, we can get $\boldsymbol{a + B = c + D \Rightarrow a - c = D - B}$, which is an irrational unless $\boldsymbol{D - B = 0}$, since 0 is a rational. So what?

We know that $\boldsymbol{a - c}$ has to be a rational, and cannot be irrational.
However, $\boldsymbol{D - B}$ can be a rational if $\boldsymbol{D - B = 0}$.

So if it is the case where $\boldsymbol{a - c = D - B}$, we have to get $\boldsymbol{D - B = 0}$.
Thus, we get $\boldsymbol{D - B = 0 \Rightarrow D = B}$.

Then next, we get $\boldsymbol{a - c = D - B = 0 \Rightarrow a - c = 0 \Rightarrow a = c}$.

3. Assuming *a*, *b*, and *c* are rationals, *A*, *B*, and *C* are irrationals, and $\boldsymbol{A \neq B \neq C}$, show that $\boldsymbol{aA + bB + cC = 0 \Rightarrow a = b = c = 0}$.

First, we can get $\boldsymbol{aA + bB + cC = 0 \Rightarrow aA + bB = -cC \Rightarrow (aA + bB)^2 = (-cC)^2}$
$\boldsymbol{\Rightarrow a^2A^2 + 2abAB + b^2B^2 = c^2C^2 \Rightarrow a^2A^2 + 2abAB + b^2B^2 - c^2C^2 = 0}$.

Then, next, we can say that $\boldsymbol{a^2A^2 + b^2B^2 - c^2C^2}$ can be a rational, but $\boldsymbol{2abAB}$ cannot be a rational, and has to be an irrational, because $\boldsymbol{A \neq B}$.

So from the example 1 above, we have to get $\boldsymbol{a^2A^2 + b^2B^2 - c^2C^2 = 0}$, and $\boldsymbol{ab = 0}$.

And we get $\boldsymbol{ab = 0 \Rightarrow a = 0}$ or $\boldsymbol{b = 0}$.

Then, first, we get

$\boldsymbol{a = 0 \Rightarrow a^2A^2 + b^2B^2 - c^2C^2 = 0 \Rightarrow b^2B^2 - c^2C^2 = 0 \Rightarrow (bB - cC)(bB + cC)}$

$\boldsymbol{\Rightarrow bB = cC}$ or $\boldsymbol{bB = -cC}$. So we get

$\boldsymbol{bB = cC \Rightarrow b = c\frac{C}{B} \Rightarrow b - c\frac{C}{B} = 0 \Rightarrow b = c = 0}$ from the example 1, because $\boldsymbol{\frac{C}{B}}$ is an irrational and $\boldsymbol{b}$ and $\boldsymbol{c}$ are rationals.

$\boldsymbol{bB = -cC \Rightarrow b = -c\frac{C}{B} \Rightarrow b + c\frac{C}{B} = 0 \Rightarrow b = c = 0}$ from the example 1, because $\boldsymbol{\frac{C}{B}}$ is an irrational and $\boldsymbol{b}$ and $\boldsymbol{c}$ are rationals.

So we get $\boldsymbol{a = 0 \Rightarrow b = c = 0 \Rightarrow a = b = c = 0}$.

Then, next, we get

$\boldsymbol{b = 0 \Rightarrow a^2A^2 - c^2C^2 = 0 \Rightarrow a^2A^2 - c^2C^2 = 0 \Rightarrow (aA - cC)(aA + cC)}$

$\boldsymbol{\Rightarrow aA = cC}$ or $\boldsymbol{aA = -cC}$. So we get

$\boldsymbol{aA = cC \Rightarrow a = c\frac{C}{A} \Rightarrow a - c\frac{C}{A} = 0 \Rightarrow a = c = 0}$ from the example 1, because $\boldsymbol{\frac{C}{A}}$ is an irrational and $\boldsymbol{a}$ and $\boldsymbol{c}$ are rationals.

$\boldsymbol{aA = -cC \Rightarrow a = -c\frac{C}{A} \Rightarrow a + c\frac{C}{A} = 0 \Rightarrow a = c = 0}$ from the example 1, because $\boldsymbol{\frac{C}{A}}$ is an irrational and $\boldsymbol{a}$ and $\boldsymbol{c}$ are rationals.

So we get $\boldsymbol{b = 0 \Rightarrow a = c = 0 \Rightarrow a = b = c = 0}$.

And thus, we get $\boldsymbol{a = b = c = 0}$.