Examples B in Irrationals in Arithmetic 3

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Examples B in Irrationals

0. Assuming $\boldsymbol{a} = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$, and $\boldsymbol{F(x) = x^3 - 6x^2}$, find $\boldsymbol{F(a)}$.

1. Assuming $\boldsymbol{p = 1 + \sqrt[3]{2} + \sqrt[3]{4}}$, and $\boldsymbol{F(x) = x^3 - 3x^2 - 3x - 1}$, find $\boldsymbol{F(p)}$.

2. Assuming $\boldsymbol{q = 2 + \sqrt{3}}$, and $\boldsymbol{F(p)} = \dfrac{p^4 - p^3 - 9p^2 - 5p + 2}{p^2 - 4p + 3}$, find $\boldsymbol{F(q)}$.

Suggestions or Solutions
To the Examples B in Irrationals

0. Assuming $a = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, and $F(x) = x^3 - 6x^2$, find $F(a)$.

Simplifying a first, we can get $a = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{2-2\sqrt{2}+1}{2-1} = 3-2\sqrt{2}$.

So we get $2\sqrt{2} = 3 - a \Rightarrow 8 = 9 - 6a + a^2 \Rightarrow a^2 - 6a + 1 = 0$.

Next, we have $F(x) = x^3 - 6x^2$, so we get $F(a) = a^3 - 6a^2$.

We have $a^2 - 6a + 1 = 0$. So we get $a^2 - 6a = -1 \Rightarrow a^3 - 6a^2 = -a = 2\sqrt{2} - 3$.

Thus, we get $F(a) = 2\sqrt{2} - 3$.

And of course, since $f(a) = a^3 - 6a^2 = -a$, and $a = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, we can put it this way, too:

$F(a) = \frac{1-\sqrt{2}}{\sqrt{2}+1}$.

1. Assuming $p = 1 + \sqrt[3]{2} + \sqrt[3]{4}$, and $F(x) = x^3 - 3x^2 - 3x - 1$, find $F(p)$.

To begin with, we can get $p - 1 = \sqrt[3]{2} + \sqrt[3]{4}$, so we get $(p-1)^3 = (\sqrt[3]{2} + \sqrt[3]{4})^3$.

Meanwhile, $(p-1)^3 = p^3 - 3p^2 + 3p - 1$, and

$$(\sqrt[3]{2} + \sqrt[3]{4})^3 = 2 + 3\sqrt[3]{2}\sqrt[3]{4}(\sqrt[3]{2} + \sqrt[3]{4}) + 4 = 6 + 3\sqrt[3]{8}(\sqrt[3]{2} + \sqrt[3]{4}) = 6 + 6(\sqrt[3]{2} + \sqrt[3]{4})$$

$\Rightarrow (\sqrt[3]{2} + \sqrt[3]{4})^3 = 6(1 + \sqrt[3]{2} + \sqrt[3]{4}) = 6p$, since $p = 1 + \sqrt[3]{2} + \sqrt[3]{4}$.

So we get $p^3 - 3p^2 + 3p - 1 = 6p \Rightarrow p^3 - 3p^2 - 3p - 1 = 0 \Rightarrow F(p) = 0$.

2. Assuming $\boldsymbol{q = 2 + \sqrt{3}}$, and $\boldsymbol{F(p) = \dfrac{p^4 - p^3 - 9p^2 - 5p + 2}{p^2 - 4p + 3}}$, find $\boldsymbol{F(q)}$.

To begin with, we have $\boldsymbol{q = 2 + \sqrt{3}}$.
So we can get $\boldsymbol{(q - 2)^2 = 3 \Rightarrow q^2 - 4q + 4 = 3 \Rightarrow q^2 - 4q + 1 = 0}$.

Next, we have $\boldsymbol{F(p) = \dfrac{p^4 - p^3 - 9p^2 - 5p + 2}{p^2 - 4p + 3}}$.

So we can get $\boldsymbol{F(q) = \dfrac{q^4 - q^3 - 9q^2 - 5q + 2}{q^2 - 4q + 3}}$.

And doing a sythetic division of the numerator by $\boldsymbol{q^2 - 4q + 1}$, we get

$$
\begin{array}{r|l}
 & q^2 + 3q + 2 \\
\hline
q^2 - 4q + 1 & q^4 - q^3 - 9q^2 - 5q + 2 \\
 & \underline{q^4 - 4q^3 + q^2} \\
 & \quad 3q^3 - 10q^2 - 5q + 2 \\
 & \quad \underline{3q^3 - 12q^2 + 3q} \\
 & \qquad\quad 2q^2 - 8q + 2 \\
 & \qquad\quad \underline{2q^2 - 8q + 2} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

And thus, we get $\boldsymbol{F(q) = \dfrac{(q^2 + 3q + 2)(q^2 - 4q + 1)}{q^2 - 4q + 3}}$.

And we know $\boldsymbol{q^2 - 4q + 1 = 0}$.

So we get $\boldsymbol{F(q) = 0}$.