

Sense of Arithmetic 5

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Click this to start.

Read the expressions below filling in the blanks by calculation by heart. If however, you want to put down the solutions, use a separate sheet of paper. And just keep reading them at your pace. And of course, you don't have to read them all at once.

Each reading can take 5, 10, or 15 minutes at a time, or as much as you can concentrate. It can help increase sense of arithmetic, and mental math.

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$$0 + _ = 0 = _ \times 2$$

$$0 - 0 = _$$

$$_ + 1 = 1$$

$$_ - 1 = -1$$

$$1 - _ = 1$$

$$1 + _ = 2 = _ \times 2$$

$$1 - _ = 0$$

$$2 + 1 = _ = 1 \times _ + 1 = 1 + 1 + _ = 1 \times _ = 3$$

$$2 - 1 = 1 + _ - 1 = 1 + _ = 1$$

$$1 - _ = 1 - (1 + 1) = 1 - 1 - _ = 0 - 1 = -1$$

$$3 + _ = 5 = 1 \times 3 + 1 \times _ = 1 \times (3 + _) = 1 \times _ = 5$$

$$3 - 2 = 1 + _ - 2 = 1 + _ = 1$$

$$2 - _ = 2 - (2 + 1) = 2 - _ - 1 = _ - 1 = -1$$

$$5 + _ = 8, 5 - _ = 2, \text{ and } 3 - _ = -2$$

$$\begin{aligned} 8 + _ \\ &= 8 + (2 + 2) = (8 + _) + 2 = 10 + _ = 12 \\ &= (2 + _) + 4 = 2 + (_ + 4) = 2 + 10 = 12 \end{aligned}$$

$$8 - _ = (4 + _) - 4 = 4 + (_ - 4) = 4 + 0 = 4$$

$$\begin{aligned} 10 + _ \\ &= 10 \times (1 + _) + 1 \times (0 + _) \\ &= 10 \times _ = 10 \end{aligned}$$

$$10 - 0 = 10 = 10 \times (1 - _) + 1 \times (0 - _) = 10 + _ = 10$$

$$10 + _ = 11$$

$$\begin{aligned} 10 - 1 &= (0 + 1 \times _) - 1 = 0 + (1 \times _ - 1) \\ &= 0 + (_ - 1) = 0 + (9 + _ - 1) = 0 + _ \end{aligned}$$

$$\begin{aligned} 1 - 10 &= 1 - (0 + 1 \times _) = 1 - (0 + 1 + 1 \times _) = 1 - 1 - (0 + 1 \times _) \\ &= 0 - (0 + _) = 0 - _ \end{aligned}$$

$$10 + _ = 19$$

$$10 - 9 = (0 + 1 \times _) - 9 = (0 + (1 + _)) - 9 = 1 + _ - 9 = _ + 0$$

$$10 + _ = 20 = 10 \times _ = 20$$

$$10 - 10 = 10 \times (1 - _) = 10 \times _ = _$$

$$10 + __ = 21$$

$$10 - 11 = 10 - (\underline{\quad} + 1) = 10 - 10 + \underline{\quad} = 0 + \underline{\quad} = -1$$

$$\begin{aligned} 11 - 10 &= (10 + \underline{\quad}) - (10 + \underline{\quad}) \\ &= 10 \times (1 - \underline{\quad}) + 1 \times (1 - \underline{\quad}) = 0 + \underline{\quad} = \underline{\quad} \end{aligned}$$

$$10 + \underline{\quad} = 29$$

$$\begin{aligned} 10 - 19 &= 10 - (10 + \underline{\quad}) \\ &= (10 - \underline{\quad}) + (0 - \underline{\quad}) = 0 + \underline{\quad} = -9 \end{aligned}$$

$$19 - 10 = (10 + \underline{\quad}) - 10 = (10 - \underline{\quad}) + (\underline{\quad} - 0) = 0 + \underline{\quad} = 9$$

$$14 + \underline{\quad} = 18$$

$$14 - 4 = \underline{\quad} \times (1 - 0) + 1 \times (4 - 4) = 10 \times 1 + 1 \times \underline{\quad} = 10 + \underline{\quad} = 10$$

$$\begin{aligned} 4 - 14 &= 10 \times (0 - \underline{\quad}) + 1 \times (4 - 4) \\ &= 10 \times (-1) + 1 \times \underline{\quad} = 10 \times (-1) + \underline{\quad} = \underline{\quad} \end{aligned}$$

$$\begin{aligned} 16 + 9 &= 16 + (\underline{\quad} + 5) = (16 + \underline{\quad}) + 5 = ((10 + \underline{\quad}) + 4) + 5 \\ &= (10 + (\underline{\quad} + 4)) + 5 = (10 + \underline{\quad}) + 5 = 10 \times \underline{\quad} + 5 = 20 + \underline{\quad} \end{aligned}$$

$$\begin{aligned} 16 + 9 &= (10 + \underline{\quad}) + 9 = (10 + (5 + \underline{\quad})) + 9 = ((10 + 5) + \underline{\quad}) + 9 \\ &= (10 + \underline{\quad}) + (\underline{\quad} + 9) = (10 + \underline{\quad}) + 10 = 10 + \underline{\quad} + 10 \\ &= 10 + \underline{\quad} + 5 = 10 \times \underline{\quad} + 5 = 20 + 5 = 25 \end{aligned}$$

$$16 - 9 = 10 \times (\underline{\quad} - 0) + 1 \times (6 - \underline{\quad}) = 10 \times 1 + \underline{\quad} = 10 + \underline{\quad} = 7$$

$$\begin{aligned} 9 - 16 &= 10 \times (0 - \underline{\quad}) + 1 \times (9 - \underline{\quad}) = 10 \times \underline{\quad} + 1 \times \underline{\quad} = -10 + 3 \\ &= 3 + \underline{\quad} = 3 - \underline{\quad} = 1 \times (3 - \underline{\quad}) = 1 \times \underline{\quad} = -7 \end{aligned}$$

$$\begin{aligned}
 -10 + 3 &= -(7 + _) + 3 = 3 + (- (7 + _)) \\
 &= 3 + (- (_ + 7)) = 3 - (_ + 7) = _ - 3 - 7 = 0 - 7 = -7
 \end{aligned}$$

$$\begin{aligned}
 39 + 58 &= (30 + _) + (_ + 8) = 30 + _ + 9 + _ \\
 &= 30 + _ + 7 + (_ + 8) = 30 + _ + 7 + _ = _ + 7 = _
 \end{aligned}$$

$$\begin{aligned}
 &30 + _ + 7 + _ \\
 &= 10 \times (3 + _ + 1) + 7 \\
 &= 10 \times (_ + 1) + 7 \\
 &= 10 \times _ + 7 \\
 &= 90 + _ \\
 &= _
 \end{aligned}$$

$$\begin{aligned}
 39 - 58 &= 10 \times (_ - 5) + 1 \times (_ - 8) \\
 &= 10 \times _ + 1 \times _ \\
 &= 10 \times _ + ((-1) \times _ + 1) \\
 &= -10 + (1 + _ \times 10) \\
 &= -10 + (1 + _ \times (1 + 9)) \\
 &= -10 + (1 + ((-1) + _)) \\
 &= -10 + (1 + _) + (-9) \\
 &= -10 + (_ - 1) + (-9) \\
 &= -10 + _ + (-9) \\
 &= -10 + _ \\
 &= (-10) \times _ + (-1) \times _ = _
 \end{aligned}$$

$$78 + 85 + 69$$

$$= (70 + 80 + 60) + (8 + 5 + 9)$$

$$= (70 + 80 + 60) + (8 + (2 + _) + 9)$$

$$= (70 + 80 + 60) + ((_ + 3) + 9)$$

$$= (70 + 80 + 60) + (_ + (3 + 9))$$

$$= (70 + 80 + 60) + (_ + ((2 + 1) + 9))$$

$$= (70 + 80 + 60) + (_ + (2 + 10))$$

$$= (70 + 80 + 60) + (_ \times 2 + 2)$$

$$= 10 \times (7 + 8 + 6 + 2) + 1 \times _$$

$$= 10 \times (15 + 6 + 2) + 1 \times _$$

$$= 10 \times (21 + 2) + _ \times 2$$

$$= 10 \times 23 + 1 \times _$$

$$= 10 \times (10 \times _ + 3) + 1 \times 2$$

$$= 100 \times _ + 10 \times _ + 1 \times 2$$

$$= 200 + 30 + 2 = 232$$

$$98 - 24 - 58 = (98 - 24) - 58 = 74 - 58 = 16$$

$$(98 - 24) - 58 = 98 - (_ + 58) = 98 - _ = 16$$

$$\begin{aligned}
& 34 - 45 - 76 + 27 \\
&= 34 - (45 + _) + 27 \\
&= 34 + 27 - (45 + _) \\
&= 61 - _ \\
&= (60 + 1) + ((-100) + (-20) + (-1)) \\
&= -100 + (_ + (-20)) + (1 + (-1)) \\
&= -100 + _ + 0 \\
&= -100 \times 1 + _ \times 4 + 1 \times 0 \\
&= -100 \times 0 + (_ \times 4 + (-10) \times 10) + 0 \\
&= 0 + 10 \times 4 + 10 \times _ + 0 \\
&= 10 \times (4 + _) \\
&= 10 \times _ \\
&= _ \times 10 \\
&= -60 \\
&= 34 + 27 + _ - 76 \\
&= 61 + (- (_ + 76)) \\
&= 61 + _ \\
&= 61 - (_ + 60) \\
&= 61 + _ - 60 \\
&= _
\end{aligned}$$

$$0 \times 0 = _, \quad 0 \times 1 = _, \quad 0 \times (-1) = _, \quad 1 \times 0 = _, \quad \text{and } (-1) \times 0 = _$$

$$3 \times 2 = 3 + _ = 6, \text{ and } (-3) \times _ = -3 + (-3) = -6$$

$$3 \times (-2) = 3 \times (-1) \times _ = _ \times 3 \times 2 = -1 \times _ = _$$

$$-3 \times (-2) = -1 \times 3 \times _ \times 2 = -1 \times _ \times 3 \times 2 = _$$

$$12 \times _ = 12 + 12 + 12 = 36 = (10 + _) \times 3$$

$$= 10 \times 3 + _ \times 3 = 30 + _ = _$$

$$15 \times 12 = (10 + _) \times 12 = 10 \times 12 + _ \times 12$$

$$= 10 \times (10 + _) + 5 \times (10 + _)$$

$$= 10 \times _ + 10 \times 2 + 5 \times _ + 5 \times 2 = _ + 20 + 50 + 10 = _$$

$$0.1 + _ = 0.2$$

$$0.1 - _ = 0$$

$$4.4 + 32.7$$

$$= (1 \times 4 + 0.1 \times _) + (10 \times _ + 1 \times 2 + 0.1 \times 7)$$

$$= 10 \times (0 + _) + 1 \times (4 + _) + 0.1 \times (_ + 7)$$

$$= 10 \times _ + 1 \times (4 + _) + 0.1 \times (_ + 1)$$

$$= 10 \times _ + 1 \times (4 + _ + 1) + 0.1 \times 1$$

$$= 30 + _ + 0.1$$

$$= _$$

$$4.4 - 32.7$$

$$\begin{aligned}
 &= (1 \times _ + 0.1 \times 4) - (10 \times _ + 1 \times 2 + 0.1 \times _) \\
 &= 10 \times (0 + _) + 1 \times (4 + (-2)) + 0.1 \times (_ + (-7)) \\
 &= _ \times (-3) + 1 \times 2 + 0.1 \times (-3) \\
 &= 10 \times ((-2) + (-1)) + 1 \times _ + 0.1 \times (-3) \\
 &= 10 \times (-2) + _ \times (-1) + 1 \times _ + 0.1 \times (-3) \\
 &= 10 \times (-2) + (-10) + 1 \times _ + 0.1 \times (-3) \\
 &= 10 \times (-2) + 1 \times _ + 1 \times _ + 0.1 \times (-3) \\
 &= 10 \times (-2) + 1 \times (_ + 2) + 0.1 \times (-3) \\
 &= 10 \times _ + 1 \times (-8) + 0.1 \times (-3) = _
 \end{aligned}$$

$$\begin{aligned}
 1.8 \times _ &= 1.8 + 1.8 = 1 \times (1 + _) + 0.1 \times (_ + 8) \\
 &= 1 \times (1 + _) + 0.1 \times (_ + 6) = 1 \times (1 + _ + 1) + 0.1 \times _ \\
 &= 1 \times _ + 0.1 \times _ = _
 \end{aligned}$$

$$\begin{aligned}
 1.8 + 1.8 &= (1 + 0.8) \times _ = 1 \times 2 + 0.8 \times 2 \\
 &= 1 \times 2 + 0.1 \times _ \times 2 = 1 \times 2 + 0.1 \times _ \\
 &= 1 \times 2 + 0.1 \times (_ + 6) = 1 \times 2 + 0.1 \times 10 + 0.1 \times _ \\
 &= 1 \times (_ + 1) + 0.1 \times 6 = 1 \times _ + 0.1 \times _ = _
 \end{aligned}$$

$$0.3 \times 0.2 = 0.1 \times 3 \times 0.1 \times _ = 0.1 \times 0.1 \times 3 \times _ = 0.01 \times _ = _$$

$$0.3 \times 0.8 = 0.1 \times _ \times 0.1 \times 8$$

$$= 0.1 \times 0.1 \times _ \times 8$$

$$= 0.01 \times _$$

$$= 0.01 \times (_ + 4)$$

$$= 0.01 \times 20 + 0.01 \times _$$

$$= 0.01 \times _ \times 10 + 0.01 \times _$$

$$= 0.01 \times 10 \times _ + 0.01 \times 4$$

$$= 0.1 \times _ + 0.01 \times 4$$

$$= _ + 0.04$$

$$= _$$

$$1.9 \times 0.7$$

$$= (1 + 0.9) \times 0.7 = 1 \times _ + 0.9 \times 0.7$$

$$= 0.7 + 0.1 \times _ \times 9 \times 7 = 0.7 + 0.01 \times (60 + _)$$

$$= 0.7 + 0.01 \times (10 \times _ + 3) = 0.7 + (0.01 \times 10) \times _ + 0.03$$

$$= 0.7 + 0.1 \times _ + 0.03 = 0.7 + _ + 0.03 = 0.1 \times (_ + 6) + 0.03$$

$$= 0.1 \times (_ + 3) + 0.03 = 0.1 \times 10 + _ + 0.03 = _ + 0.3 + 0.03 = _$$

$$\begin{aligned}
5.9 \times 9.8 &= (5 + _) \times (_ + 0.8) = 5 \times (_ + 0.8) + 0.9 \times (_ + 0.8) \\
&= 5 \times 9 + _ \times 0.8 + 0.9 \times 9 + 0.9 \times 0.8 \\
&= 45 + 5 \times (0.1 \times _) + 0.1 \times (9 \times _) + 0.1 \times 0.1 \times (9 \times _) \\
&= 45 + 5 \times (8 \times 0.1) + 0.1 \times 81 + 0.01 \times _ \\
&= 45 + (5 \times _) \times 0.1 + 0.1 \times 81 + 0.01 \times _ \\
&= 45 + 40 \times 0.1 + 0.1 \times 81 + 0.01 \times _ \\
&= 45 + 0.1 \times 40 + 0.1 \times 81 + 0.01 \times _ \\
&= 45 + 0.1 \times (40 + _) + 0.01 \times _ = 45 + 0.1 \times 121 + 0.01 \times 72 \\
&= 40 + 5 + 0.1 \times (100 + _ + 1) + 0.01 \times (70 + 2) \\
&= 40 + 5 + 0.1 \times 100 + 0.1 \times _ + 0.1 + 0.01 \times _ + 0.01 \times 2 \\
&= 40 + 5 + 0.1 \times 10 \times _ + 0.1 \times 10 \times 2 + 0.1 + 0.01 \times 10 \times _ + 0.01 \times 2 \\
&= 40 + 5 + _ + 2 + 0.1 + 0.1 \times 7 + 0.01 \times 2 \\
&= 50 + 7 + 0.1 \times (1 + _) + 0.01 \times 2 = 50 + 7 + 0.1 \times _ + 0.01 \times 2 = _
\end{aligned}$$

$$\begin{aligned}
0.04 \times 7.9 &= 0.04 \times (_ + 0.9) = 0.04 \times 7 + _ \times 0.9 \\
&= (0.01 \times _) \times 7 + (0.01 \times _) \times 0.9 = 0.01 \times (_ \times 7) + (0.01 \times 4) \times (_ \times 9) \\
&= 0.01 \times (_ \times 7) + 0.01 \times _ \times 4 \times 9 = 0.01 \times _ + 0.001 \times _ \\
&= 0.01 \times _ + 0.001 \times (30 + _) = 0.01 \times _ + 0.001 \times (10 \times _ + _) \\
&= 0.01 \times _ + 0.01 \times _ + 0.001 \times _ = 0.01 \times (28 + _) + 0.001 \times _ \\
&= 0.01 \times _ + 0.006 = 0.01 \times (30 + _) + 0.006 \\
&= 0.01 \times (10 \times _) + 0.01 \times _ + 0.006 = _ \times 3 + 0.01 \times _ + 0.006 \\
&= _ + 0.01 + 0.006 = _
\end{aligned}$$

$1 / _ = 1$ since $1 = 1 \times 1$, and $-1 / 1 = _$ since $-1 = 1 \times (-1) = (-1) \times 1$

$1 / (-1) = _$ since $1 = (-1) \times (-1) = 1$

$-1 / (-1) = _$ since $-1 = (-1) \times 1 = -1$, and $2 / 1 = 2$ since $2 = 1 \times 2$

$-2 / 1 = _$ since $-2 = 1 \times (-2) = (-2) \times 1$

$2 / (-1) = _$ since $2 = (-1) \times (-2) = (-1) \times (_ \times 2)$
 $= ((-1) \times _) \times 2 = 1 \times 2 = 2$

$2 / 2 = 1$ since $2 = 2 \times 1$, and $-2 / 2 = _$,
 since $-2 = 2 \times (-1) = (-1) \times 2 = -2$

$2 / -2 = _$, since $2 = (-2) \times (-1) = (-1) \times 2 \times (-1) = (-1) \times (-1) \times 2 = 1 \times 2$

$4 / 2 = _$, since $4 = 2 \times _$, and $6 / _ = 2$, since $6 = _ \times 2$

$10 / _ = 10$, since $10 = 1 \times _$, and $12 / _ = 4$, since $12 = _ \times 4$

$28 / 2 = (20 + _) / 2 = (20 / 2) + (_ / 2) = 10 + _ = _$

$= (_ - 2) / 2 = (_ / 2) - (2 / 2) = _ - 1 = _$

$$1248 / 4$$

$$= (1200 + 40 + 8) / 4$$

$$= (1200 / _) + (40 / _) + (8 / _)$$

$$= ((100 \times _) / 4) + ((10 \times _) / 4) + ((1 \times _) / 4)$$

$$= (100 \times (_ / 4)) + (10 \times (_ / 4)) + (1 \times (_ / 4))$$

$$= 100 \times _ + 10 \times _ + 1 \times _$$

$$= 300 + 10 + 2$$

$$= 312$$

$$1248 / 4 / 2 = (1248 / _) / 2 = 1248 / (_ \times 2) = 1248 / _ = 156$$

$$1248 / 4 / 2 = 312 / _ = (100 \times _ + 10 \times _ + 1 \times _) / _$$

$$= 100 \times (3 / 2) + 10 \times (1 / 2) + 1 \times (2 / 2)$$

$$= 100 \times (_ + 1 / 2) + 10 \times (1 / _) + 1 \times (2 / _)$$

$$= 100 \times 1 + 100 / _ + 10 / _ + 1 \times _$$

$$= 100 \times 1 + (10 \times _) / 2 + (1 \times _) / 2 + 1 \times 1$$

$$= 100 \times 1 + 10 \times (_ / 2) + 1 \times (_ / 2) + 1 \times 1$$

$$= 100 \times 1 + 10 \times _ + 1 \times _ + 1 \times 1$$

$$= 100 \times 1 + 10 \times _ + 1 \times (_ + 1)$$

$$= 100 \times 1 + 10 \times _ + 1 \times _ = _$$

$$1 / 2$$

$$= (0.1 \times _) / 2 = 0.1 \times (_ / 2) = 0.1 \times _ = _$$

$$= 0.5, \text{ since } 0.5 \times _ = 1$$

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$$0.1 / 2$$

$$= (0.01 \times _) / 2 = 0.01 \times (_ / 2) = 0.01 \times _ = _$$

$$= 0.05, \text{ since } 0.05 \times _ = 0.1$$

$$3 / 2 = (2 + _) / 2 = 2 / 2 + _ / 2 = 1 + _ = _$$

$$15 / 6$$

$$= (12 + _) / 6 = 12 / 6 + _ / 6 = 2 + (1 \times _) / 6 = 2 + ((0.1 \times _) \times 3) / 6$$

$$= 2 + (0.1 \times (_ \times 3)) / 6 = 2 + (0.1 \times _) / 6 = 2 + 0.1 \times (_ / 6)$$

$$= 2 + 0.1 \times _ = 2 + _ = _$$

$$3.9 / 2$$

$$= (3 + _) / 2 = (1 \times 3 + 0.1 \times _) / 2 = (1 \times 3) / 2 + (0.1 \times _) / 2$$

$$= 1 \times (_ / 2) + 0.1 \times (_ / 2) = 1 \times ((_ + 1) / 2) + 0.1 \times ((_ + 1) / 2)$$

$$= 1 \times ((2 / 2) + (1 / 2)) + 0.1 \times ((_ / 2) + (1 / 2))$$

$$= 1 \times (1 + _) + 0.1 \times (_ + 0.5) = 1 \times 1 + 0.5 + 0.4 + 0.05$$

$$= 1 \times 1 + 0.1 \times (_ + 4) + 0.01 \times _ = 1 \times 1 + 0.1 \times _ + 0.01 \times _ = _$$

$$2 / 4 = (1 \times _) / 4 = ((0.1 \times _) \times 2) / 4 = (0.1 \times (_ \times 2)) / 4 = (0.1 \times _) / 4$$

$$= 0.1 \times (_ / 4) = 0.1 \times _ = _$$

$$\begin{aligned}
 1 / 4 &= (0.1 \times _) / 4 = 0.1 \times (_ / 4) = 0.1 \times ((_ + 2) / 4) \\
 &= 0.1 \times ((_ / 4) + (2 / 4)) = 0.1 \times (2 + _) = 0.1 \times 2 + _ = 0.1 \times 2 + _ \times 5 \\
 &= _
 \end{aligned}$$

$$9.7 / 4$$

$$\begin{aligned}
 &= (9 + _) / 4 = 9 / 4 + _ / 4 \\
 &= (8 + 1) / 4 + (0.1 \times _) / 4 = 2 + 1 / 4 + 0.1 \times (_ / 4) \\
 &= 2 + 0.25 + 0.1 \times ((_ + 3) / 4) = 2 + 0.25 + 0.1 \times (_ + 3 / 4) \\
 &= 2 + 0.25 + 0.1 \times (1 + (0.1 \times _) / 4) \\
 &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (_ / 4)) \\
 &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times ((_ + 2) / 4)) \\
 &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (_ + 2 / 4)) \\
 &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (_ + 0.5)) \\
 &= 2 + 0.25 + 0.1 \times (1 + _ + 0.05) = 2 + 0.25 + 0.1 + _ + 0.005 \\
 &= 2 + 0.2 + _ + 0.1 + 0.07 + 0.005 \\
 &= 2 + 0.1 \times (_ + 1) + 0.01 \times (5 + 7) + 0.005 \\
 &= 2 + 0.1 \times _ + 0.01 \times (10 + 2) + 0.005 \\
 &= 2 + 0.1 \times _ + 0.01 \times 10 + 0.01 \times 2 + 0.005 \\
 &= 2 + 0.1 \times _ + 0.1 \times 1 + 0.01 \times 2 + 0.005 \\
 &= 2 + 0.1 \times _ + 0.01 \times 2 + 0.005 = _
 \end{aligned}$$

$$1 / 0.1 = (0.1 \times _) / 0.1 = (10 \times _) / 0.1 = 10 \times (_ / 0.1) = 10 \times 1$$

$$1 / 0.2$$

$$= (0.1 \times _) / 0.2 = (10 \times _) / 0.2$$

$$= 10 \times (_ / 0.2) = 10 \times ((0.01 \times _) / 0.2)$$

$$= 10 \times (0.01 \times (_ / 0.2)) = 10 \times (0.01 \times ((0.1 \times _) / 0.2))$$

$$= 10 \times (0.01 \times ((0.1 \times (2 \times _)) / 0.2))$$

$$= 10 \times (0.01 \times (((0.1 \times 2) \times _) / 0.2))$$

$$= 10 \times (0.01 \times ((0.2 \times _) / 0.2))$$

$$= 10 \times (0.01 \times (_ \times 0.2) / 0.2))$$

$$= 10 \times (0.01 \times (_ \times (0.2 / 0.2)))$$

$$= 10 \times (0.01 \times (_ \times 1))$$

$$= 10 \times (0.01 \times (10 \times _))$$

$$= 10 \times (0.1 \times _)$$

$$= (10 \times 0.1) \times _$$

$$= 1 \times _$$

$$= _$$

$$2 / 0.4 = (0.1 \times _) / 0.4 = (0.1 \times (4 \times _)) / 0.4 = ((0.1 \times 4) \times _) / 0.4$$

$$= (0.4 \times _) / 0.4 = (_ \times 0.4) / 0.4 = _ \times (0.4 / 0.4) = _ \times 1 = _$$

$$\begin{aligned}
& 7.9 / 0.4 \\
&= (7 + 0.9) / 0.4 \\
&= 7 / _ + 0.9 / _ \\
&= (4 + _) / 0.4 + (0.1 \times _) / 0.4 \\
&= 4 / 0.4 + 3 / 0.4 + 0.1 \times (9 / 0.4) \\
&= 10 + (1 \times _) / 0.4 + 0.1 \times (1 \times _) / 0.4 \\
&= 10 + ((0.1 \times _) \times 3) / 0.4 + 0.1 \times 0.1 \times _ \times 9 / 0.4 \\
&= 10 + (0.1 \times _) / 0.4 + 0.1 \times (0.1 \times _) / 0.4 \\
&= 10 + 0.1 \times ((_ + 2) / 0.4) + 0.1 \times 0.1 \times (_ + 2) / 0.4 \\
&= 10 + 0.1 \times (_ / 0.4 + 5) + 0.1 \times 0.1 \times (_ / 0.4 + 5) \\
&= 10 + 0.1 \times ((_ \times 4) / 0.4 + 5) + 0.1 \times 0.1 \times (22 \times 4 / 0.4 + 5) \\
&= 10 + 0.1 \times (_ \times (4 / 0.4) + 5) + 0.1 \times 0.1 \times (22 \times (4 / 0.4) + 5) \\
&= 10 + 0.1 \times (_ \times 10 + 5) + 0.1 \times 0.1 \times (22 \times 10 + 5) \\
&= 10 + 0.1 \times _ \times 10 + 0.1 \times _ + 0.1 \times (0.1 \times (22 \times 10) + 0.1 \times 5) \\
&= 10 + 7 + _ + 0.1 \times (22 + 0.5) \\
&= 10 + 7 + _ + 0.1 \times (20 + _) + 0.05 \\
&= 10 + 7 + _ + 0.1 \times (2 \times 10) + 0.2 + 0.05 \\
&= 10 + 7 + _ + 2 + 0.2 + 0.05 \\
&= 10 + 9 + _ + 0.05 \\
&= _
\end{aligned}$$

The numbers in bold below are the solutions, of course. Solutions alone however, don't mean much. So you may want to just keep reading the calculations below, if you want to, of course.

Note however, you don't have to do calculations the way below.

Each calculation just shows that **there are many ways we can calculate**.

$$0 + \mathbf{0} = 0 = \mathbf{0} \times 2, \text{ and } 0 - 0 = \mathbf{0}$$

$$\mathbf{0} + 1 = 1, \text{ and } \mathbf{0} - 1 = -1$$

$$1 - \mathbf{0} = 1$$

$$1 + \mathbf{1} = 2 = \mathbf{1} \times 2$$

$$1 - \mathbf{1} = 0$$

$$2 + 1 = \mathbf{3} = 1 \times \mathbf{2} + 1 = 1 + 1 + \mathbf{1} = 1 \times \mathbf{3} = 3$$

$$2 - 1 = 1 + \mathbf{1} - 1 = 1 + \mathbf{0} = 1$$

$$1 - \mathbf{2} = 1 - (1 + 1) = 1 - 1 - \mathbf{1} = 0 - 1 = -1$$

$$3 + \mathbf{2} = 5 = 1 \times 3 + 1 \times \mathbf{2} = 1 \times (3 + \mathbf{2}) = 1 \times \mathbf{5} = 5$$

$$3 - 2 = 1 + \mathbf{2} - 2 = 1 + \mathbf{0} = 1$$

$$2 - \mathbf{3} = 2 - (2 + 1) = 2 - \mathbf{2} - 1 = \mathbf{0} - 1 = -1$$

$$5 + 3 = 8, 5 - \mathbf{3} = 2, \text{ and } 3 - \mathbf{5} = -2$$

$$8 + \mathbf{4}$$

$$= 8 + (2 + 2) = (8 + \mathbf{2}) + 2 = 10 + \mathbf{2} = 12$$

$$= (2 + \mathbf{6}) + 4 = 2 + (\mathbf{6} + 4) = 2 + 10 = 12$$

$$8 - \mathbf{4} = (4 + \mathbf{4}) - 4 = 4 + (\mathbf{4} - 4) = 4 + 0 = 4$$

$$10 + \mathbf{0} = 10 = 10 \times (1 + \mathbf{0}) + 1 \times (0 + \mathbf{0}) = 10 \times \mathbf{1} = 10$$

$$10 - 0 = 10 = 10 \times (1 - \mathbf{0}) + 1 \times (0 - \mathbf{0}) = 10 + \mathbf{0} = 10$$

$$10 + \mathbf{1} = 11$$

$$10 - 1 = (0 + 1 \times \mathbf{10}) - 1 = 0 + (1 \times \mathbf{10} - 1) = 0 + (\mathbf{10} - 1)$$

$$= 0 + (9 + \mathbf{1} - 1) = 0 + \mathbf{9}$$

$$1 - 10 = 1 - (0 + 1 \times \mathbf{10}) = 1 - (0 + 1 + 1 \times \mathbf{9}) = 1 - 1 - (0 + 1 \times \mathbf{9})$$

$$= 0 - (0 + \mathbf{9}) = 0 - \mathbf{9}$$

$$10 + \mathbf{9} = 19$$

$$10 - 9 = (0 + 1 \times \mathbf{10}) - 9 = (0 + (1 + \mathbf{9})) - 9 = 1 + \mathbf{9} - 9 = \mathbf{9} + 0$$

$$10 + \mathbf{10} = 20 = 10 \times \mathbf{2} = 20$$

$$10 - 10 = 10 \times (1 - \mathbf{1}) = 10 \times \mathbf{0} = \mathbf{0}$$

$$10 + \mathbf{11} = 21$$

$$10 - 11 = 10 - (\mathbf{10} + 1) = 10 - 10 + \mathbf{1} = 0 + \mathbf{(-1)} = -1$$

$$11 - 10 = (10 + \mathbf{1}) - (10 + \mathbf{0}) = 10 \times (1 - \mathbf{1}) + 1 \times (1 - \mathbf{0}) = 0 + \mathbf{1} = \mathbf{1}$$

$$10 + \mathbf{19} = 29$$

$$10 - 19 = 10 - (10 + \mathbf{9}) = (10 - \mathbf{0}) + (0 - \mathbf{19}) = 0 + \mathbf{(-9)} = -9$$

$$19 - 10 = (10 + \mathbf{9}) - 10 = (10 - \mathbf{10}) + (\mathbf{9} - 0) = 0 + \mathbf{9} = 9$$

$$14 + \mathbf{4} = 18$$

$$14 - 4 = \mathbf{10} \times (1 - 0) + 1 \times (4 - 4) = 10 \times 1 + 1 \times \mathbf{0} = 10 + \mathbf{0} = 10$$

$$4 - 14 = 10 \times (0 - \mathbf{1}) + 1 \times (4 - 4)$$

$$= 10 \times (-1) + 1 \times \mathbf{0} = 10 \times (-1) + \mathbf{0} = \mathbf{-10}$$

$$16 + 9 = 16 + (\mathbf{4} + 5) = (16 + \mathbf{4}) + 5 = ((10 + \mathbf{6}) + 4) + 5$$

$$= (10 + (\mathbf{6} + 4)) + 5 = (10 + \mathbf{10}) + 5 = 10 \times \mathbf{2} + 5 = 20 + \mathbf{5}$$

$$16 + 9 = (10 + \mathbf{6}) + 9 = (10 + (5 + \mathbf{1})) + 9$$

$$= ((10 + 5) + \mathbf{1}) + 9 = (10 + \mathbf{5}) + (\mathbf{1} + 9)$$

$$= (10 + \mathbf{5}) + 10 = 10 + \mathbf{5} + 10 = 10 + \mathbf{10} + 5 = 10 \times \mathbf{2} + 5 = 20 + 5 = 25$$

$$16 - 9 = 10 \times (\mathbf{1} - 0) + 1 \times (6 - \mathbf{9}) = 10 \times 1 + \mathbf{(-3)} = 10 + \mathbf{(-3)} = 7$$

$$9 - 16$$

$$= 10 \times (0 - \mathbf{1}) + 1 \times (9 - \mathbf{6}) = 10 \times (\mathbf{-1}) + 1 \times \mathbf{3}$$

$$= \mathbf{-10} + \mathbf{3} = 3 + (\mathbf{-10}) = 3 - \mathbf{10} = 1 \times (3 - \mathbf{10}) = 1 \times (\mathbf{-7}) = -7$$

$$\mathbf{-10} + \mathbf{3}$$

$$= -(7 + \mathbf{3}) + 3 = 3 + (-(7 + \mathbf{3})) = 3 + (-(\mathbf{3} + 7)) = 3 - (\mathbf{3} + 7)$$

$$= \mathbf{3} - 3 - 7 = 0 - 7 = -7$$

$$39 + 58$$

$$= (30 + \mathbf{9}) + (\mathbf{50} + 8) = 30 + \mathbf{50} + 9 + \mathbf{8}$$

$$= 30 + \mathbf{50} + 7 + (\mathbf{2} + 8) = \mathbf{30} + \mathbf{50} + 7 + \mathbf{10}$$

$$= \mathbf{90} + 7 = \mathbf{97}$$

$$\mathbf{30} + \mathbf{50} + 7 + \mathbf{10} = 10 \times (3 + \mathbf{5} + 1) + 7 = 10 \times (\mathbf{8} + 1) + 7$$

$$= 10 \times \mathbf{9} + 7 = 90 + \mathbf{7} = \mathbf{97}$$

$$39 - 58 = 10 \times (\mathbf{3} - 5) + 1 \times (\mathbf{9} - 8) = 10 \times (\mathbf{-2}) + 1 \times \mathbf{1}$$

$$= 10 \times (\mathbf{-1}) + ((-1) \times \mathbf{10} + 1) = -10 + (1 + (\mathbf{-1}) \times 10)$$

$$= -10 + (1 + (\mathbf{-1}) \times (1 + 9)) = -10 + (1 + ((-1) + (\mathbf{-9})))$$

$$= -10 + (1 + (\mathbf{-1})) + (-9) = -10 + (\mathbf{1} - 1) + (-9) = -10 + \mathbf{0} + (-9)$$

$$= -10 + (\mathbf{-9}) = (-10) \times \mathbf{1} + (-1) \times \mathbf{9} = \mathbf{-19}$$

$$\begin{aligned}
78 + 85 + 69 &= (70 + 80 + 60) + (8 + 5 + 9) \\
&= (70 + 80 + 60) + (8 + (2 + 3) + 9) = (70 + 80 + 60) + ((10 + 3) + 9) \\
&= (70 + 80 + 60) + (10 + (3 + 9)) \\
&= (70 + 80 + 60) + (10 + ((2 + 1) + 9)) \\
&= (70 + 80 + 60) + (10 + (2 + 10)) = (70 + 80 + 60) + (10 \times 2 + 2) \\
&= 10 \times (7 + 8 + 6 + 2) + 1 \times 2 = 10 \times (15 + 6 + 2) + 1 \times 2 \\
&= 10 \times (21 + 2) + 1 \times 2 = 10 \times 23 + 1 \times 2 = 10 \times (10 \times 2 + 3) + 1 \times 2 \\
&= 100 \times \mathbf{2} + 10 \times \mathbf{3} + 1 \times 2 = 200 + 30 + 2 = 232
\end{aligned}$$

$$98 - 24 - 58 = (98 - 24) - 58 = 74 - 58 = 16$$

$$\mathbf{98 - 24 - 58} = 98 - (\mathbf{24} + 58) = 98 - \mathbf{82} = 16$$

$$\begin{aligned}
34 - 45 - 76 + 27 &= 34 - (45 + \mathbf{76}) + 27 \\
&= 34 + 27 - (45 + \mathbf{76}) = 61 - \mathbf{121} = (60 + 1) + ((-100) + (-20) + (-1)) \\
&= -100 + (60 + (-20)) + (1 + (-1)) = -100 + 40 + 0 \\
&= -100 \times 1 + 10 \times 4 + 1 \times 0 = -100 \times 0 + (10 \times 4 + (-10) \times 10) + 0 \\
&= 0 + 10 \times 4 + 10 \times (-10) + 0 = 10 \times (4 + (-10)) = 10 \times (-6) \\
&= (-6) \times 10 = -60 = 34 + 27 + (-45) - 76 = 61 + (-(45 + 76)) \\
&= 61 + (-121) = 61 - (61 + 60) = 61 + (\mathbf{-61}) - 60 = \mathbf{-60}
\end{aligned}$$

$$0 \times 0 = \mathbf{0}, 0 \times 1 = \mathbf{0}, 0 \times (-1) = \mathbf{0}, 1 \times 0 = \mathbf{0}, \text{ and } (-1) \times 0 = \mathbf{0}$$

$$3 \times 2 = 3 + \mathbf{3} = 6, \text{ and } (-3) \times \mathbf{2} = -3 + (-3) = -6$$

$$3 \times (-2) = 3 \times (-1) \times \mathbf{2} = \mathbf{(-1)} \times 3 \times 2 = -1 \times \mathbf{6} = \mathbf{-6}$$

$$-3 \times (-2) = -1 \times 3 \times \mathbf{(-1)} \times 2 = -1 \times \mathbf{(-1)} \times 3 \times 2 = \mathbf{6}$$

$$\begin{aligned} 12 \times \mathbf{3} &= 12 + 12 + 12 = 36 = (10 + \mathbf{2}) \times 3 \\ &= 10 \times 3 + \mathbf{2} \times 3 = 30 + \mathbf{6} = \mathbf{36} \end{aligned}$$

$$15 \times 12$$

$$\begin{aligned} &= (10 + \mathbf{5}) \times 12 = 10 \times 12 + \mathbf{5} \times 12 = 10 \times (10 + \mathbf{2}) + 5 \times (10 + \mathbf{2}) \\ &= 10 \times \mathbf{10} + 10 \times 2 + 5 \times \mathbf{10} + 5 \times 2 = \mathbf{100} + 20 + 50 + 10 = \mathbf{180} \end{aligned}$$

$$0.1 + \mathbf{0.1} = 0.2$$

$$0.1 - \mathbf{0.1} = 0$$

$$4.4 + 32.7$$

$$= (1 \times 4 + 0.1 \times \mathbf{4}) + (10 \times \mathbf{3} + 1 \times 2 + 0.1 \times 7)$$

$$= 10 \times (0 + \mathbf{3}) + 1 \times (4 + \mathbf{2}) + 0.1 \times (\mathbf{4} + 7)$$

$$= 10 \times \mathbf{3} + 1 \times (4 + \mathbf{2}) + 0.1 \times (\mathbf{10} + 1)$$

$$= 10 \times \mathbf{3} + 1 \times (4 + \mathbf{2} + 1) + 0.1 \times 1 = 30 + \mathbf{7} + 0.1 = \mathbf{37.1}$$

$$4.4 - 32.7$$

$$= (1 \times \mathbf{4} + 0.1 \times 4) - (10 \times \mathbf{3} + 1 \times 2 + 0.1 \times \mathbf{7})$$

$$= 10 \times (0 + \mathbf{(-3)}) + 1 \times (4 + \mathbf{(-2)}) + 0.1 \times (\mathbf{4} + \mathbf{(-7)})$$

$$= \mathbf{10} \times \mathbf{(-3)} + 1 \times 2 + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times ((-2) + \mathbf{(-1)}) + 1 \times \mathbf{2} + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times \mathbf{(-2)} + \mathbf{10} \times \mathbf{(-1)} + 1 \times \mathbf{2} + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times \mathbf{(-2)} + \mathbf{(-10)} + 1 \times \mathbf{2} + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times \mathbf{(-2)} + 1 \times \mathbf{(-10)} + 1 \times \mathbf{2} + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times \mathbf{(-2)} + 1 \times (\mathbf{(-10)} + 2) + 0.1 \times \mathbf{(-3)}$$

$$= 10 \times \mathbf{(-2)} + 1 \times \mathbf{(-8)} + 0.1 \times \mathbf{(-3)} = \mathbf{-28.3}$$

$$1.8 \times 2$$

$$= \mathbf{1.8} + \mathbf{1.8} = 1 \times (1 + \mathbf{1}) + 0.1 \times (\mathbf{8} + 8) = 1 \times (1 + \mathbf{1}) + 0.1 \times (\mathbf{10} + 6)$$

$$= 1 \times (1 + \mathbf{1} + 1) + 0.1 \times \mathbf{6} = 1 \times \mathbf{3} + 0.1 \times \mathbf{6} = \mathbf{3.6}$$

$$\mathbf{1.8} + \mathbf{1.8}$$

$$= (1 + 0.8) \times \mathbf{2} = 1 \times 2 + 0.8 \times 2 = 1 \times 2 + 0.1 \times \mathbf{8} \times 2$$

$$= 1 \times 2 + 0.1 \times \mathbf{16} = 1 \times 2 + 0.1 \times (\mathbf{10} + 6)$$

$$= 1 \times 2 + 0.1 \times 10 + 0.1 \times \mathbf{6} = 1 \times (\mathbf{2} + 1) + 0.1 \times 6$$

$$= 1 \times \mathbf{3} + 0.1 \times \mathbf{6} = \mathbf{3.6}$$

$$0.3 \times 0.2 = 0.1 \times 3 \times 0.1 \times \mathbf{2} = 0.1 \times 0.1 \times 3 \times \mathbf{2} = 0.01 \times \mathbf{6} = \mathbf{0.06}$$

$$0.3 \times 0.8 = 0.1 \times \mathbf{3} \times 0.1 \times 8$$

$$= 0.1 \times 0.1 \times \mathbf{3} \times 8$$

$$= 0.01 \times \mathbf{24} = 0.01 \times (\mathbf{20} + 4)$$

$$= 0.01 \times 20 + 0.01 \times \mathbf{4}$$

$$= 0.01 \times \mathbf{2} \times 10 + 0.01 \times \mathbf{4}$$

$$= 0.01 \times 10 \times \mathbf{2} + 0.01 \times 4$$

$$= 0.1 \times \mathbf{2} + 0.01 \times 4 = \mathbf{0.2} + 0.04 = \mathbf{0.24}$$

$$1.9 \times 0.7 = (1 + 0.9) \times 0.7$$

$$= 1 \times \mathbf{0.7} + 0.9 \times 0.7$$

$$= 0.7 + 0.1 \times \mathbf{0.1} \times 9 \times 7$$

$$= 0.7 + 0.01 \times (60 + \mathbf{3})$$

$$= 0.7 + 0.01 \times (10 \times \mathbf{6} + 3)$$

$$= 0.7 + (0.01 \times 10) \times \mathbf{6} + 0.03$$

$$= 0.7 + 0.1 \times \mathbf{6} + 0.03 = 0.7 + \mathbf{0.6} + 0.03$$

$$= 0.1 \times (\mathbf{7} + 6) + 0.03$$

$$= 0.1 \times (\mathbf{10} + 3) + 0.03 = 0.1 \times 10 + \mathbf{0.3} + 0.03$$

$$= \mathbf{1} + 0.3 + 0.03 = \mathbf{1.33}$$

$$5.9 \times 9.8$$

$$= (5 + 0.9) \times (9 + 0.8) = 5 \times (9 + 0.8) + 0.9 \times (9 + 0.8)$$

$$= 5 \times 9 + 5 \times 0.8 + 0.9 \times 9 + 0.9 \times 0.8$$

$$= 45 + 5 \times (0.1 \times 8) + 0.1 \times (9 \times 9) + 0.1 \times 0.1 \times (9 \times 8)$$

$$= 45 + 5 \times (8 \times 0.1) + 0.1 \times 81 + 0.01 \times 72$$

$$= 45 + (5 \times 8) \times 0.1 + 0.1 \times 81 + 0.01 \times 72$$

$$= 45 + 40 \times 0.1 + 0.1 \times 81 + 0.01 \times 72$$

$$= 45 + 0.1 \times 40 + 0.1 \times 81 + 0.01 \times 72$$

$$= 45 + 0.1 \times (40 + 81) + 0.01 \times 72 = 45 + 0.1 \times 121 + 0.01 \times 72$$

$$= 40 + 5 + 0.1 \times (100 + 20 + 1) + 0.01 \times (70 + 2)$$

$$= 40 + 5 + 0.1 \times 100 + 0.1 \times 20 + 0.1 + 0.01 \times 70 + 0.01 \times 2$$

$$= 40 + 5 + 0.1 \times 10 \times 10 + 0.1 \times 10 \times 2 + 0.1$$

$$+ 0.01 \times 10 \times 7 + 0.01 \times 2 = 40 + 5 + 10 + 2 + 0.1 + 0.1 \times 7 + 0.01 \times 2$$

$$= 50 + 7 + 0.1 \times (1 + 7) + 0.01 \times 2 = 50 + 7 + 0.1 \times 8 + 0.01 \times 2$$

$$= \mathbf{57.82}$$

$$0.04 \times 7.9$$

$$= 0.04 \times (7 + 0.9)$$

$$= 0.04 \times 7 + 0.04 \times 0.9$$

$$= (0.01 \times 4) \times 7 + (0.01 \times 4) \times 0.9$$

$$= 0.01 \times (4 \times 7) + (0.01 \times 4) \times (0.1 \times 9)$$

$$= 0.01 \times (4 \times 7) + 0.01 \times 0.1 \times 4 \times 9$$

$$= 0.01 \times 28 + 0.001 \times 36$$

$$= 0.01 \times 28 + 0.001 \times (30 + 6)$$

$$= 0.01 \times 28 + 0.001 \times (10 \times 3 + 6)$$

$$= 0.01 \times 28 + 0.01 \times 3 + 0.001 \times 6$$

$$= 0.01 \times (28 + 3) + 0.001 \times 6$$

$$= 0.01 \times 31 + 0.006 = 0.01 \times (30 + 1) + 0.006$$

$$= 0.01 \times (10 \times 3) + 0.01 \times 1 + 0.006$$

$$= 0.1 \times 3 + 0.01 \times 1 + 0.006$$

$$= \mathbf{0.3} + 0.01 + 0.006 = \mathbf{0.316}$$

$$1 / \mathbf{1} = 1 \text{ since } 1 = 1 \times 1, \text{ and } -1 / 1 = \mathbf{-1} \text{ since } -1 = 1 \times (-1) = (-1) \times 1$$

$$1 / (-1) = \mathbf{-1} \text{ since } 1 = (-1) \times (-1) = 1$$

$-1 / (-1) = \mathbf{1}$ since $-1 = (-1) \times 1 = -1$, and $2 / 1 = 2$ since $2 = 1 \times 2$

$-2 / 1 = \mathbf{-2}$ since $-2 = 1 \times (-2) = (-2) \times 1$

$2 / (-1) = \mathbf{-2}$ since $2 = (-1) \times (-2) = (-1) \times ((\mathbf{-1}) \times 2)$
 $= ((-1) \times (\mathbf{-1})) \times 2 = 1 \times 2 = 2$

$2 / 2 = 1$, since $2 = 2 \times 1$, and $-2 / 2 = \mathbf{-1}$,
 since $-2 = 2 \times (-1) = (-1) \times 2 = -2$

$2 / -2 = \mathbf{-1}$ since $2 = (-2) \times (-1) = (-1) \times 2 \times (-1) = (-1) \times (-1) \times 2 = 1 \times 2$

$4 / 2 = \mathbf{2}$, since $4 = 2 \times \mathbf{2}$, and $6 / \mathbf{3} = 2$, since $6 = \mathbf{3} \times 2$

$10 / \mathbf{1} = 10$, since $10 = 1 \times \mathbf{10}$, and $12 / \mathbf{3} = 4$, since $12 = \mathbf{3} \times 4$

$28 / 2 = (20 + \mathbf{8}) / 2 = (20 / 2) + (\mathbf{8} / 2) = 10 + \mathbf{4} = \mathbf{14}$
 $= (\mathbf{30} - 2) / 2 = (\mathbf{30} / 2) - (2 / 2) = \mathbf{15} - 1 = \mathbf{14}$

$1248 / 4 = (1200 + 40 + 8) / 4 = (1200 / 4) + (40 / 4) + (8 / 4)$
 $= ((100 \times 12) / 4) + ((10 \times 4) / 4) + ((1 \times 8) / 4)$
 $= (100 \times (12 / 4)) + (10 \times (4 / 4)) + (1 \times (8 / 4))$
 $= 100 \times 3 + 10 \times 1 + 1 \times 2 = 300 + 10 + 2 = 312$

$$\begin{aligned}
 &1248 / 4 / 2 \\
 &= (1248 / 4) / 2 \\
 &= 1248 / (4 \times 2) = 1248/8 = 156
 \end{aligned}$$

$$\begin{aligned}
 &1248 / 4 / 2 \\
 &= 312 / 2 \\
 &= (100 \times 3 + 10 \times 1 + 1 \times 2) / 2 \\
 &= 100 \times (3 / 2) + 10 \times (1 / 2) + 1 \times (2 / 2) \\
 &= 100 \times (1 + 1 / 2) + 10 \times (1 / 2) + 1 \times (2 / 2) \\
 &= 100 \times 1 + 100 / 2 + 10 / 2 + 1 \times 1 \\
 &= 100 \times 1 + (10 \times 10) / 2 + (1 \times 10) / 2 + 1 \times 1 \\
 &= 100 \times 1 + 10 \times (10 / 2) + 1 \times (10 / 2) + 1 \times 1 \\
 &= 100 \times 1 + 10 \times 5 + 1 \times 5 + 1 \times 1 \\
 &= 100 \times 1 + 10 \times 5 + 1 \times (5 + 1) \\
 &= 100 \times 1 + 10 \times 5 + 1 \times 6 \\
 &= 156
 \end{aligned}$$

$$\begin{aligned}
 1 / 2 &= (0.1 \times 10) / 2 = 0.1 \times (10 / 2) = 0.1 \times 5 = 0.5 \\
 &= 0.5 \text{ since } 0.5 \times 2 = 1
 \end{aligned}$$

$$0.1 / 2 = (0.01 \times 10) / 2 = 0.01 \times (10 / 2) = 0.01 \times 5 = 0.05$$

$$3 / 2 = (2 + \mathbf{1}) / 2 = 2 / 2 + \mathbf{1} / 2 = 1 + \mathbf{0.5} = \mathbf{1.5}$$

$$15 / 6$$

$$= (12 + \mathbf{3}) / 6 = 12 / 6 + \mathbf{3} / 6 = 2 + (1 \times \mathbf{3}) / 6 = 2 + ((0.1 \times \mathbf{10}) \times 3) / 6$$

$$= 2 + (0.1 \times (\mathbf{10} \times 3)) / 6 = 2 + (0.1 \times \mathbf{30}) / 6$$

$$= 2 + 0.1 \times (\mathbf{30} / 6) = 2 + 0.1 \times \mathbf{5} = 2 + \mathbf{0.5} = \mathbf{2.5}$$

$$3.9 / 2 = (3 + \mathbf{0.9}) / 2 = (1 \times 3 + 0.1 \times \mathbf{9}) / 2$$

$$= (1 \times 3) / 2 + (0.1 \times \mathbf{9}) / 2 = 1 \times (\mathbf{3} / 2) + 0.1 \times (\mathbf{9} / 2)$$

$$= 1 \times ((\mathbf{2} + 1) / 2) + 0.1 \times ((\mathbf{8} + 1) / 2)$$

$$= 1 \times ((2 / 2) + (1 / 2)) + 0.1 \times ((\mathbf{8} / 2) + (1 / 2))$$

$$= 1 \times (1 + \mathbf{0.5}) + 0.1 \times (\mathbf{4} + 0.5) = 1 \times 1 + 0.5 + 0.4 + 0.05$$

$$= 1 \times 1 + 0.1 \times (\mathbf{5} + 4) + 0.01 \times \mathbf{5} = 1 \times 1 + 0.1 \times \mathbf{9} + 0.01 \times \mathbf{5}$$

$$= \mathbf{1.95}$$

$$2 / 4 = (1 \times \mathbf{2}) / 4 = ((0.1 \times \mathbf{10}) \times 2) / 4 = (0.1 \times (\mathbf{10} \times 2)) / 4$$

$$= (0.1 \times \mathbf{20}) / 4 = 0.1 \times (\mathbf{20} / 4) = 0.1 \times \mathbf{5} = \mathbf{0.5}$$

$$1 / 4 = (0.1 \times \mathbf{10}) / 4 = 0.1 \times (\mathbf{10} / 4) = 0.1 \times ((\mathbf{8} + 2) / 4)$$

$$= 0.1 \times ((\mathbf{8} / 4) + (2 / 4)) = 0.1 \times (2 + \mathbf{0.5}) = 0.1 \times 2 + \mathbf{0.05}$$

$$= 0.1 \times 2 + \mathbf{0.01} \times 5 = \mathbf{0.25}$$

$$\begin{aligned} 9.7 / 4 &= (9 + 0.7) / 4 \\ &= 9 / 4 + 0.7 / 4 \\ &= (8 + 1) / 4 + (0.1 \times 7) / 4 \\ &= 2 + 1 / 4 + 0.1 \times (7 / 4) \\ &= 2 + 0.25 + 0.1 \times ((4 + 3) / 4) \\ &= 2 + 0.25 + 0.1 \times (1 + 3 / 4) \\ &= 2 + 0.25 + 0.1 \times (1 + (0.1 \times 30) / 4) \\ &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (30 / 4)) \\ &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times ((28 + 2) / 4)) \\ &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (7 + 2 / 4)) \\ &= 2 + 0.25 + 0.1 \times (1 + 0.1 \times (7 + 0.5)) \\ &= 2 + 0.25 + 0.1 \times (1 + 0.7 + 0.05) \\ &= 2 + 0.25 + 0.1 + 0.07 + 0.005 \\ &= 2 + 0.2 + 0.05 + 0.1 + 0.07 + 0.005 \\ &= 2 + 0.1 \times (2 + 1) + 0.01 \times (5 + 7) + 0.005 \\ &= 2 + 0.1 \times 3 + 0.01 \times (10 + 2) + 0.005 \\ &= 2 + 0.1 \times 3 + 0.01 \times 10 + 0.01 \times 2 + 0.005 \\ &= 2 + 0.1 \times 3 + 0.1 \times 1 + 0.01 \times 2 + 0.005 \\ &= 2 + 0.1 \times 4 + 0.01 \times 2 + 0.005 \\ &= \mathbf{2.425} \end{aligned}$$

$$\begin{aligned}
 1 / 0.1 &= (0.1 \times \mathbf{10}) / 0.1 \\
 &= (10 \times \mathbf{0.1}) / 0.1 = 10 \times (\mathbf{0.1} / 0.1) = 10 \times 1
 \end{aligned}$$

$$\begin{aligned}
 1 / 0.2 & \\
 &= (0.1 \times \mathbf{10}) / 0.2 \\
 &= (10 \times \mathbf{0.1}) / 0.2 \\
 &= 10 \times (\mathbf{0.1} / 0.2) \\
 &= 10 \times ((0.01 \times \mathbf{10}) / 0.2) \\
 &= 10 \times (0.01 \times (\mathbf{10} / 0.2)) \\
 &= 10 \times (0.01 \times ((0.1 \times \mathbf{100}) / 0.2)) \\
 &= 10 \times (0.01 \times ((0.1 \times (2 \times \mathbf{50})) / 0.2)) \\
 &= 10 \times (0.01 \times (((0.1 \times 2) \times \mathbf{50}) / 0.2)) \\
 &= 10 \times (0.01 \times ((0.2 \times \mathbf{50}) / 0.2)) \\
 &= 10 \times (0.01 \times ((\mathbf{50} \times 0.2) / 0.2)) \\
 &= 10 \times (0.01 \times (\mathbf{50} \times (0.2 / 0.2))) \\
 &= 10 \times (0.01 \times (\mathbf{50} \times 1)) \\
 &= 10 \times (0.01 \times (10 \times \mathbf{5})) \\
 &= 10 \times (0.1 \times \mathbf{5}) \\
 &= (10 \times 0.1) \times \mathbf{5} \\
 &= 1 \times \mathbf{5} = \mathbf{5}
 \end{aligned}$$

$$2 / 0.4 = (0.1 \times 20) / 0.4 = (0.1 \times (4 \times 5)) / 0.4$$

$$= ((0.1 \times 4) \times 5) / 0.4 = (0.4 \times 5) / 0.4$$

$$= (5 \times 0.4) / 0.4 = \mathbf{5} \times (0.4 / 0.4) = \mathbf{5} \times 1 = \mathbf{5}$$

$$7.9 / 0.4$$

$$= (7 + 0.9) / 0.4 = 7 / 0.4 + 0.9 / 0.4$$

$$= (4 + 3) / 0.4 + (0.1 \times 9) / 0.4 = 4 / 0.4 + 3 / 0.4 + 0.1 \times (9 / 0.4)$$

$$= 10 + (1 \times 3) / 0.4 + 0.1 \times (1 \times 9) / 0.4$$

$$= 10 + ((0.1 \times 10) \times 3) / 0.4 + 0.1 \times 0.1 \times 10 \times 9 / 0.4$$

$$= 10 + (0.1 \times 30) / 0.4 + 0.1 \times (0.1 \times 90) / 0.4$$

$$= 10 + 0.1 \times ((28 + 2) / 0.4) + 0.1 \times 0.1 \times (88 + 2) / 0.4$$

$$= 10 + 0.1 \times (28 / 0.4 + 5) + 0.1 \times 0.1 \times (88 / 0.4 + 5)$$

$$= 10 + 0.1 \times ((7 \times 4) / 0.4 + 5) + 0.1 \times 0.1 \times (22 \times 4 / 0.4 + 5)$$

$$= 10 + 0.1 \times (7 \times (4 / 0.4) + 5) + 0.1 \times 0.1 \times (22 \times (4 / 0.4) + 5)$$

$$= 10 + 0.1 \times (7 \times 10 + 5) + 0.1 \times 0.1 \times (22 \times 10 + 5)$$

$$= 10 + 0.1 \times 7 \times 10 + 0.1 \times 5 + 0.1 \times (0.1 \times (22 \times 10) + 0.1 \times 5)$$

$$= 10 + 7 + 0.5 + 0.1 \times (22 + 0.5)$$

$$= 10 + 7 + 0.5 + 0.1 \times (20 + 2) + 0.05$$

$$= 10 + 7 + 0.5 + 0.1 \times (2 \times 10) + 0.2 + 0.05$$

$$= 10 + 7 + 0.5 + 2 + 0.2 + 0.05 = 10 + 9 + 0.7 + 0.05 = \mathbf{19.75}$$

Note again, you don't have to do calculations the way above. Each calculation just shows that **there are many ways we can calculate**.

And there can be many ways you can break a particular value into many other values, and put them together to get the same particular value as many ways as you can think of.

And the same is true for expressions of many kinds, too. For instance, we can get these:

$$2a + b = a + a + b = 2a + 2b - b = 2(a + b) - b = \dots$$

$$a^2 + 2a = a^2 + 2a + 1 - 1 = (a + 1)^2 - 1 = \dots$$

$$a^2 + b^2 = a^2 + b^2 + 2ab - 2ab = (a + b)^2 - 2ab = \dots$$