Practice 1 on Domains and Ranges

Click this to start.

Practice 1 on Domains and Ranges

These are for familiarity with functions, particularly, domains and ranges. Some of these can be however, too easy to some students. Try them all, though.

0. Assuming $\boldsymbol{y = f(x) = x}$ for $\boldsymbol{x}$ real, find the output for each input as follows.

0. $\boldsymbol{x = 0}$.   1. $\boldsymbol{x = 1}$.   2. $\boldsymbol{x = 3}$.   3. $\boldsymbol{x = 0.2}$.   4. $\boldsymbol{x = -0.7}$.

1. Assuming $\boldsymbol{y = f(x) = 2x + 3}$ for $\boldsymbol{x}$ real, find the output for each input as follows.

0. $\boldsymbol{x = 0}$.   1. $\boldsymbol{x = 1}$.   2. $\boldsymbol{x = 2}$.   3. $\boldsymbol{x = 3}$.   4. $\boldsymbol{x = -0.7}$.   5. $\boldsymbol{x = -0.6}$.   6. $\boldsymbol{x = -0.5}$.

2. Assuming $\boldsymbol{y = f(x) = x^2 - 1}$ for $\boldsymbol{-1 \le x \le 2}$, find the output for each input as follows.

0. $\boldsymbol{x = 0}$.   1. $\boldsymbol{x = 1}$.   2. $\boldsymbol{x = -1}$.   3. $\boldsymbol{x = 2}$.   4. $\boldsymbol{x = -0.2}$.   5. $\boldsymbol{x = 3}$.   6. $\boldsymbol{x = -3}$.

7. $\boldsymbol{x = -0.5}$.   8. $\boldsymbol{x = 0.5}$.   9. $\boldsymbol{x = -0.6}$.   A. $\boldsymbol{x = -1.5}$.   B. $\boldsymbol{x = 1.5}$.

Suggestions or Solutions
To the Problems in the Example 0

Assuming $\boldsymbol{y = f(x) = x}$ for $\boldsymbol{x}$ real, find the output for each input as follows.

0. $\boldsymbol{x = 0}$. 1. $\boldsymbol{x = 1}$. 2. $\boldsymbol{x = 3}$. 3. $\boldsymbol{x = 0.2}$. 4. $\boldsymbol{x = -0.7}$.

To begin with, what is the domain of the function $\boldsymbol{f}$?

The function $\boldsymbol{f}$ is defined for all real numbers.
So '<u>for $\boldsymbol{x}$ real</u>' means that every real number can be an input of the function $\boldsymbol{f}$.
Thus, the domain of $\boldsymbol{f}$ is a set of all real numbers.

0. $\boldsymbol{x = 0 \Rightarrow y = f(0) = 0}$, which is the output for the input $\boldsymbol{0}$.

1. $\boldsymbol{x = 1 \Rightarrow y = f(1) = 1}$, which is the output for $\boldsymbol{x = 1}$.

2. $\boldsymbol{x = 3 \Rightarrow y = f(3) = 3}$.

3. $\boldsymbol{x = 0.2 \Rightarrow y = f(0.2) = 0.2}$.

4. $\boldsymbol{x = -0.7 \Rightarrow y = f(-0.7) = -0.7}$.

So for each and every input, the output is the same as the input. In other words, every time the function $\boldsymbol{f}$ produces an output, the output is the same as the input. That is, in every pair of the input and output, both are *identical*.

So such a function is called an *identity function*.
And each of all the functions below is an identity function, too. So each is no other than each of all the others, and the same functions can have different names.

$\boldsymbol{y = f(x) = x}$ for $\boldsymbol{x}$ real. $\boldsymbol{t = g(s) = s}$ for $\boldsymbol{s}$ real. $\boldsymbol{v = h(u) = u}$ for $\boldsymbol{u}$ real.

$\boldsymbol{q = k(p) = p}$ for $\boldsymbol{p}$ real. $\boldsymbol{n = r(m) = m}$ for $\boldsymbol{m}$ real.

Suggestions or Solutions
To the Problems in the Example 1

Assuming $\boldsymbol{y = f(x) = 2x + 3}$ for $\boldsymbol{x}$ real, find the output for each input as follows.

0. $\boldsymbol{x = 0}$. 1. $\boldsymbol{x = 1}$. 2. $\boldsymbol{x = 2}$. 3. $\boldsymbol{x = 3}$. 4. $\boldsymbol{x = -0.7}$. 5. $\boldsymbol{x = -0.6}$. 6. $\boldsymbol{x = -0.5}$.

0. $\boldsymbol{x = 0 \Rightarrow y = f(0) = 2 \cdot 0 + 3 = 2 \times 0 = 0 + 3 = 3}$, which is the output for $\boldsymbol{x = 0}$.

1. $\boldsymbol{x = 1 \Rightarrow y = f(1) = 2 \cdot 1 + 3 = 2 + 3 = 5}$, which is the output for the input **1**.

2. $\boldsymbol{x = 2 \Rightarrow y = f(2) = 2 \cdot 2 + 3 = 4 + 3 = 7}$.

3. $\boldsymbol{x = 3 \Rightarrow y = f(3) = 2 \cdot 3 + 3 = 6 + 3 = 9}$.

4. $\boldsymbol{x = -0.7 \Rightarrow y = f(-0.7) = 2 \cdot (-0.7) + 3 = -1.4 + 3 = 1.6}$.

5. $\boldsymbol{x = -0.6 \Rightarrow y = f(-0.6) = 2 \cdot (-0.6) + 3 = -1.2 + 3 = 1.8}$.

6. $\boldsymbol{x = -0.5 \Rightarrow y = f(-0.5) = 2 \cdot (-0.5) + 3 = -1 + 3 = 2}$.

Suggestions or Solutions
To the Problems in the Example 2

Assuming $y = f(x) = x^2 - 1$ for $-1 \le x \le 2$, find the output for each input as follows.

0. $x = 0$. 1. $x = 1$. 2. $x = -1$. 3. $x = 2$. 4. $x = -0.2$. 5. $x = 3$. 6. $x = -3$.

7. $x = -0.5$. 8. $x = 0.5$. 9. $x = -0.6$. A. $x = -1.5$. B. $x = 1.5$.

0. $x = 0 \Rightarrow y = f(0) = 0^2 - 1 = -1$, which is the output for $x = 0$.

1. $x = 1 \Rightarrow y = f(1) = 1^2 - 1 = 0$, which is the output for the input **1**.

2. $x = -1 \Rightarrow y = f(1) = (-1)^2 - 1 = 0$.

3. $x = 2 \Rightarrow y = f(2) = 2^2 - 1 = 3$.

4. $x = -0.2 \Rightarrow y = f(-0.2) = (-0.2)^2 - 1 = 0.04 - 1 = 0.96$.

5. **3** is not in the domain, so the function f does not hold for $x = 3$.
So the output for $x = 3$, that is, $f(3)$ does not exist. That is, f is not defined for $x = 3$.

6. **-3** is not in the domain, so the function f cannot be defined for $x = -3$.
In other words, -3 cannot be an input, so $f(-3)$ does not exist.

7. $x = -0.5 \Rightarrow y = f(-0.5) = (-0.5)^2 - 1 = 0.25 - 1 = -0.75$.

8. $x = 0.5 \Rightarrow y = f(0.5) = 0.5^2 - 1 = 0.25 - 1 = -0.75$.

9. $x = -0.6 \Rightarrow y = f(-0.6) = (-0.6)^2 - 1 = 0.36 - 1 = -0.64$.

A. **-1.5** is not included in the domain, so the function f cannot be defined for $x = -1.5$.
Therefore, $f(-1.5)$ does not exist.

B. $x = 1.5 \Rightarrow y = f(1.5) = 1.5^2 - 1 = 2.25 - 1 = 1.25$.