Practice 2 on Domains and Ranges

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These are too, for familiarity with functions, particularly, domains and ranges. And these are for your algebra skill, too. And the algebra is on inequalities, which are sometimes not quite straightforward. So let's find now, the range of each function as follows.

0. $y = f(x) = x$ for x real.

1. $y = f(x) = x$ for $0 \le x \le 1$.

2. $y = f(x) = 2x$ for x real.

3. $y = f(x) = 2x$ for $0 \le x \le 1$.

4. $y = f(x) = 2x - 3$ for $0 \le x \le 1$.

5. $y = f(x) = x^2$ for x real.

6. $y = f(x) = x^2$ for $x \ge 0$.

7. $y = f(x) = x^2$ for $x \le 0$.

8. $y = f(x) = x^2$ for $0 \le x \le 1$.

9. $y = f(x) = x^2$ for $x \ge 1$.

A. $y = f(x) = x^2$ for $x \ge -1$.

B. $y = f(x) = x^2$ for $x \le 1$.

C. $y = f(x) = x^2$ for $1 \le x < 2$.

D. $y = f(x) = x^2$ for $1 \le x \le 2$ or $-2 \le x \le -1$.

Suggestions or Solutions
To the Problems in Practice 2

0. Find the range of a function where $y = f(x) = x$ for x real.

First, what is the domain of a function?

The domain is the set of all the inputs, which are all the numbers that the input variable can get, so a function can hold (be defined) for only the numbers in the domain.

Next, we have $y = f(x) = x$ for x real.

And '<u>for x real</u>' means that every real number can be an input of the function.
That is, the domain is a set of all real numbers.

So in the function f, x is the input variable, and can get every real number.
Therefore, the function f can hold for all real numbers.

Next, *the range is the set of all the outputs*. And the function f is an identity function.

So after every execution of the function f, the output is the same as the input.
That is, whenever f produces an output, the output is the same as the input.

So the range is the same as the domain, and thus, is a set of all real numbers, too.

And such a function as f is said to be *one-to-one*, too, so *all the outputs* are *different*.
What then, about a many-to-one function?

By a function many-to-one, at least one output gets produced more than once.
So if a function is one-to-one, it produces at least two identical outputs.
That is, some outputs have to be the same.
And as an extreme example, we can have a constant function. What then, is constant?

Every output is the same, so the output is constant, and thus, the value of the function is constant. In such a function, the domain has many numbers, that is, many inputs, yet the range has only one number, that is, every output is always the same.

So for instance, we can have $\boldsymbol{y = g(x) = 3}$, and every output is 3 for all the inputs.

Thus, we get $\boldsymbol{g(1) = g(-5) = g(7) = g(0.3) = \ldots = 3}$.

1. Find the range of a function $y = f(x) = x$ for $0 \le x \le 1$.

In this case, too, the range is the same as the domain. How?

The function $\boldsymbol{f}$ is an identity function, so after each and every execution of $\boldsymbol{f}$, the output is the same as the input. So the range is the same as the domain.
What is the domain though, of the function $\boldsymbol{f}$?

The domain is the set of all the inputs. And the input variable gets every input.
And the input variable is $\boldsymbol{x}$, and we have this: $\boldsymbol{0 \le x \le 1}$.
So the domain is the set of all the real numbers from 0 to 1.
Specifying thus, the domain of $\boldsymbol{f}$, we can put it this way: $\boldsymbol{0 \le x \le 1}$.
And we say that the function $\boldsymbol{f}$ is defined for all the real numbers from 0 to 1.

Next, the range is the same as the domain, so the range, too, is a set of all the real numbers from 0 to 1.
And the range is the set of all the outputs. Of a function then, what gets every output?

The output variable does. And of the function $\boldsymbol{y = f(x)}$, the output variable is $\boldsymbol{y}$.
So $\boldsymbol{y}$ gets each and every output, that is, all the numbers in the range.
And we know the range is a set of all the real numbers from 0 to 1.
So specifying the range, we can put it this way: $\boldsymbol{0 \le y \le 1}$.

And thus, specifying the range, we often do it specifying the extent of the input variable, and, specifying the domain, too, we do it specifying the extent of the output variable.

2. Find the range of a function $y = f(x) = 2x$ for x real.

First, that $\boldsymbol{x}$ is real means that every real number can be an input of the function.
So the domain is a set of all real numbers.
Thus, the function $\boldsymbol{f}$ can hold for all real numbers.

Next, the expression of the function $\boldsymbol{f}$ is $\mathbf{2}\boldsymbol{x}$.
That is, after every execution of $\boldsymbol{f}$, the output is twice the input, which is a real number.
So each of all the outputs is twice each of all the inputs, and thus, is a real number.

So every real number can be an output.
So the set of all the outputs is a set of all real numbers, too.
Therefore, the range is a set of all real numbers, too.

So is the same true, too, for any function whose domain is a set of all real numbers?

In other words, if a function can hold for all real numbers, is the range a set of all real numbers, too?

Not necessarily, and we will see why not shortly.

3. Find the range of a function $y = f(x) = 2x$ for $0 \le x \le 1$.

First, the domain is $\mathbf{0} \le \boldsymbol{x} \le \mathbf{1}$, so the function $\boldsymbol{f}$ is defined for $\mathbf{0} \le \boldsymbol{x} \le \mathbf{1}$.

And next, the expression of $\boldsymbol{f}$ is $\mathbf{2}\boldsymbol{x}$. So each output is twice each input.

The output variable $\boldsymbol{y}$ gets each output, and we have $\boldsymbol{y} = \mathbf{2}\boldsymbol{x}$, where $\boldsymbol{x}$ gets each input.

And we have $\mathbf{0} \le \boldsymbol{x} \le \mathbf{1}$, too, so we get $\mathbf{0} \le \boldsymbol{x} \le \mathbf{1} \Rightarrow \mathbf{0} \le \mathbf{2}\boldsymbol{x} \le \mathbf{2} \Rightarrow \mathbf{0} \le \boldsymbol{y} \le \mathbf{2}$.

We know the range is the set of all the outputs, and $\boldsymbol{y}$ gets each and every output.

So the range is $\mathbf{0} \le \boldsymbol{y} \le \mathbf{2}$.

4. Find the range of a function $y = f(x) = 2x - 3$ for $0 \le x \le 1$.

To begin with, what is the domain of the function f?

The domain is $\mathbf{0 \le x \le 1}$, So the function f is defined for all the real numbers from 0 to 1.

In the expression $\mathbf{2x - 3}$, each input is multiplied by 2, and 3 is subtracted from the product, then the difference is the output, which is put into y, the output variable.

And we have $\mathbf{y = 2x - 3}$.

So we get $\mathbf{0 \le x \le 1 \Rightarrow 0 \le 2x \le 2 \Rightarrow 0 - 3 \le 2x - 3 \le 2 - 3 \Rightarrow -3 \le y \le -1}$.

And the range is the set of all the outputs, that is, the set of all the y-values.

Therefore, the range is $\mathbf{-3 \le y \le -1}$.

5. Find the range of a function $y = f(x) = x^2$ for x real.

The domain is a set of all real numbers, so f can hold for all real numbers.
And the expression is $\mathbf{x^2}$. So each output is the square of each input. So?

No matter what a real number may be, the square of it is ≥ 0. So?

Each output is ≥ 0. And y gets each output. Thus, the range is $\mathbf{y \ge 0}$.

6. Find the range of a function $y = f(x) = x^2$ for $x \ge 0$.

First, the domain is $\mathbf{x \ge 0}$, so the function f can hold for all the real numbers $\ge \mathbf{0}$.

Next, the expression is $\mathbf{x^2}$, and the square of a real number $\ge \mathbf{0}$ is positive or 0, too. So?

Each output is ≥ 0. And $\boldsymbol{y}$ gets each output. Thus, the range is $\boldsymbol{y \geq 0}$.

7. Find the range of a function $y = f(x) = x^2$ for $x \leq 0$.

First, the domain is $\boldsymbol{x \leq 0}$, so the function $\boldsymbol{f}$ is defined for all the real numbers $\leq \mathbf{0}$.
Next, the expression is $\boldsymbol{x^2}$, and the square of a real number $\leq \mathbf{0}$ is $\geq \mathbf{0}$. How?

The square of any number negative is positive, and $0^2 = 0$.
So every output is ≥ 0. And $\boldsymbol{y}$ gets each output. Thus, the range is $\boldsymbol{y \geq 0}$.

8. Find the range of a function $y = f(x) = x^2$ for $0 \leq x \leq 1$.

First, the domain is $\boldsymbol{0 \leq x \leq 1}$, so the function $\boldsymbol{f}$ holds for all the real numbers from 0 to 1.

Next, we know the range is the set of all the outputs, and the output variable gets each and every output. So the range is the set of all the numbers the output variable gets.

The output variable is $\boldsymbol{y}$, and we have $\boldsymbol{y = x^2}$.

And we have $\boldsymbol{0 \leq x \leq 1}$, too.

So we get $\boldsymbol{0 \leq x \leq 1 \Rightarrow 0 \leq x^2 \leq 1 \Rightarrow 0 \leq y \leq 1}$. Thus, the range is $\boldsymbol{0 \leq y \leq 1}$.

9. Find the range of a function $y = f(x) = x^2$ for $x \geq 1$.

The domain is $\boldsymbol{x \geq 1}$, so the function $\boldsymbol{f}$ is defined for $\boldsymbol{x \geq 1}$. And the expression is $\boldsymbol{x^2}$.

So we get $\boldsymbol{x \geq 1 \Rightarrow x^2 \geq 1 \Rightarrow y \geq 1}$, since $\boldsymbol{y = f(x) = x^2}$. And thus, the range is $\boldsymbol{y \geq 1}$.

A. Find the range of a function $y = f(x) = x^2$ for $x \geq -1$.

The domain is $\boldsymbol{x \geq -1}$, so the function $\boldsymbol{f}$ is defined for $\boldsymbol{x \geq -1}$. And the expression is $\boldsymbol{x^2}$.

So is the square of such a real number greater than or equal to 1?
In other words, is it the case where $\boldsymbol{x \geq -1 \Rightarrow x^2 \geq 1}$?

No, it is not the case. The square of a real number ≥ **-1** is ≥ **0**. How?

First, the square of a real number is ≥ 0, no matter what real number it may be.

Next, in $\boldsymbol{f}$, $\boldsymbol{x}$ gets each input, which can be 0, and the square of 0 is 0. So $\boldsymbol{x^2}$ can be 0.

So it the case where $\boldsymbol{x \geq -1 \Rightarrow x^2 \geq 0}$. Therefore, the range is $\boldsymbol{y \geq 0}$.

It is in fact, <u>not a good idea</u> to apply inequalities to find the range if the function is not linear as $\boldsymbol{2x}$ or $\boldsymbol{5x - 3}$, etc. What then, is a good idea?

Usually, using the graph of a function, we can get the range readily and quickly.
And more importantly, it is safe to get the range graphically.
So it's not only easy and fast but safe, too, to find the range using a graph.

So graphing matters.

The examples though, in this practice, are quite straightforward even though they are not linear. So you can find the ranges without much difficulty even if not using the graphs.

And it's in fact, a good idea to do some examples without using graphs, too, if the examples are not quite complicated, of course. And doing those examples, you can see better how algebra works with inequalities, so you get to improve your algebra, of course, and get used to tracking outputs changing as inputs change. So you are going to do some more examples with no graphs used in this practice set and in the next set, too.

B. Find the range of a function $y = f(x) = x^2$ for $x \leq 1$.

The domain is $\boldsymbol{x \leq 1}$, so the function $\boldsymbol{f}$ is defined for $\boldsymbol{x \leq 1}$. And the expression is $\boldsymbol{x^2}$.

The square of a real number $\leq \mathbf{1}$ is greater than or equal to not 1 but 0. How?

First, the square of a real number is ≥ 0.

Next, in $\boldsymbol{f}$, $\boldsymbol{x}$ gets each input, which can be 0, and the square of 0 is 0. So $\boldsymbol{x^2}$ can be 0.

So it the case where $\boldsymbol{x \leq 1 \Rightarrow x^2 \geq 0}$. Therefore, the range is $\boldsymbol{y \geq 0}$.

C. Find the range of a function $y = f(x) = x^2$ for $1 \leq x < 2$.

The domain is $\boldsymbol{1 \leq x < 2}$, and we need to note that the domain does not include 2.
So the function $\boldsymbol{f}$ is defined for $\boldsymbol{1 \leq x < 2}$, and not for $\boldsymbol{x = 2}$.

The output variable is $\boldsymbol{y}$, and we have $\boldsymbol{y = x^2}$.

So we get $\boldsymbol{1 \leq x < 2 \Rightarrow 1 \leq x^2 < 4 \Rightarrow 1 \leq y < 4}$. Therefore, the range is $\boldsymbol{1 \leq y < 4}$.

D. Find the range of a function $y = f(x) = x^2$ for $1 \leq x \leq 2$ or $-2 \leq x \leq -1$.

The domain is $\boldsymbol{1 \leq x \leq 2}$ or $\boldsymbol{-2 \leq x \leq -1}$. So $\boldsymbol{f}$ is defined for $\boldsymbol{1 \leq x \leq 2}$ or $\boldsymbol{-2 \leq x \leq -1}$.
The output variable is $\boldsymbol{y}$, and we have $\boldsymbol{y = x^2}$. So we get

$\boldsymbol{1 \leq x \leq 2 \Rightarrow 1 \leq x^2 \leq 4 \Rightarrow 1 \leq y \leq 4}$.
$\boldsymbol{-2 \leq x \leq -1 \Rightarrow 1 \leq x^2 \leq 4 \Rightarrow 1 \leq y \leq 4}$, which happens to be the same as the one above.

Thus, we get $\boldsymbol{1 \leq y \leq 4}$, which is the range.