Practice 3 on Domains and Ranges

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These are also, for familiarity with functions, particularly, domains and ranges. And also, these are for your algebra on inequalities, which are sometimes not quite straightforward.

Find the range of each function as follows.

0. $y = f(x) = x^2$ for $0 \leq x < 1$ or $-2 \leq x \leq -1$.

1. $y = f(x) = x^2$ for $0 \leq x < 0.1$ or $-2 \leq x < -1$.

2. $y = f(x) = -x^2$ for x real.

3. $y = f(x) = -x^2$ for $x \geq 0$.

4. $y = f(x) = -x^2$ for $x \leq 0$.

5. $y = f(x) = -x^2$ for $0 \leq x \leq 1$.

6. $y = f(x) = -x^2$ for $x \geq 1$.

7. $y = f(x) = -x^2$ for $x \geq -1$.

8. $y = f(x) = -x^2$ for $x \leq 1$.

9. $y = f(x) = -x^2$ for $1 \leq x < 2$.

A. $y = f(x) = -x^2$ for $1 \le x \le 2$ or $-2 \le x \le -1$.

B. $y = f(x) = -x^2$ for $0 \le x < 1$ or $-2 \le x \le -1$.

C. $y = f(x) = -x^2$ for $0 \le x < 0.1$ or $-2 \le x < -1$.

D. $y = f(x) = 2x^2$ for x real.

E. $y = f(x) = 2x^2$ for $x \ge 0$.

F. $y = f(x) = 2x^2$ for $x \le 0$.

G. $y = f(x) = 2x^2$ for $0 \le x \le 1$.

H. $y = f(x) = 2x^2$ for $x \ge 1$.

I. $y = f(x) = -2x^2$ for $x \ge -1$.

J. $y = f(x) = -2x^2$ for $x \le 1$.

K. $y = f(x) = -2x^2$ for $1 < x \le 2$.

L. $y = f(x) = 2x^2 + x$ for $1 \le x \le 2$ or $-2 \le x \le -1$.

Suggestions or Solutions
To the Problems in Practice 3

To begin with, when finding ranges of functions nonlinear as the ones here, we may not want to apply inequalities only.
Usually, using the graph of a function, you can get the range readily and quickly.
And more importantly, you can get it safely.
The examples though, in this practice, are quite straightforward even though they are not linear. So you can find the ranges without much difficulty even if using no graphs.
And it's in fact, a good idea to do some examples without using graphs, too, if the examples are not quite complicated, of course.
Doing most of these examples with no graphs, you can improve your algebra on inequalities, and improve your skill of tracking outputs changing as inputs change.

0. Find the range of a function $y = f(x) = x^2$ for $0 \le x < 1$ or $-2 \le x \le -1$.

The domain is $0 \le x < 1$ or $-2 \le x \le -1$. So f holds for $0 \le x < 1$ or $-2 \le x \le -1$.
The output variable is y, and we have $y = x^2$. So we get

$0 \le x < 1 \Rightarrow 0 \le x^2 < 1 \Rightarrow 0 \le y \le 1$.
$-2 \le x \le -1 \Rightarrow 1 \le x^2 \le 4 \Rightarrow 0 \le y \le 4$.

Thus, we get $0 \le y \le 1$ or $0 \le y \le 4$, so we get $0 \le y \le 4$, which is the range.

1. Find the range of a function $y = f(x) = x^2$ for $0 \le x < 0.1$ or $-2 \le x < -1$.

The domain is $0 \le x < 0.1$ or $-2 \le x < -1$. So f can hold for $0 \le x < 0.1$ or $-2 \le x < -1$.
The output variable is y, and we have $y = x^2$. So we get

$0 \le x < 0.1 \Rightarrow 0 \le x^2 < 0.01 \Rightarrow 0 \le y < 0.01$.
$-2 \le x < -1 \Rightarrow 1 < x^2 \le 4 \Rightarrow 1 < y \le 4$.

So we get $0 \le y < 0.01$ or $1 < y \le 4$. And we know y gets each and every output of f.

Therefore, range is $0 \le y < 0.01$ or $1 < y \le 4$.

2. Find the range of a function where $y = f(x) = -x^2$ for x real.

That x is real means that any real number can be an input. That is, the domain is a set of all real numbers. So the function f holds for all real numbers.

- Whatever a real number it may be, the square of it is ≥ 0.

However, the expression is $-x^2$, so each output is the negative of the square of each input. Thus, each output is less and or equal to 0.
And we know y gets each and every output of the function f. So the range is $y \le 0$.

3. Find the range of a function $y = f(x) = -x^2$ for $x \ge 0$.

The domain is $x \ge 0$. So the function f holds for $x \ge 0$. And the expression is $-x^2$.

And we have $y = -x^2$, too.

So we get $x \ge 0 \Rightarrow x^2 \ge 0 \Rightarrow -x^2 \le 0 \Rightarrow y \le 0$. Therefore, the range is $y \le 0$.

4. Find the range of a function $y = f(x) = -x^2$ for $x \le 0$.

The domain is $x \le 0$. So the function f holds for $x \le 0$. And the expression is $-x^2$.

And we have $y = -x^2$, too.

So we get $x \le 0 \Rightarrow x^2 \ge 0 \Rightarrow -x^2 \le 0 \Rightarrow y \le 0$. Therefore, the range is $y \le 0$.

5. Find the range of a function $y = f(x) = -x^2$ for $0 \le x \le 1$.

The domain is $0 \le x \le 1$. So the function f holds for $0 \le x \le 1$. And the expression is $-x^2$.

And we have $y = -x^2$, too.

So we get $0 \le x \le 1 \Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0 \Rightarrow -1 \le y \le 0$, which is the range.

6. Find the range of a function $y = f(x) = -x^2$ for $x \geq 1$.

The domain is $x \geq 1$. So the function f holds for $x \geq 1$. And the expression is $-x^2$.
And we have $y = -x^2$, too.
So we get $x \geq 1 \Rightarrow x^2 \geq 1 \Rightarrow -x^2 \leq -1 \Rightarrow y \leq -1$, which is the range.

7. Find the range of a function $y = f(x) = -x^2$ for $x \geq -1$.

The domain is $x \geq -1$. So the function f holds for $x \geq -1$. And the expression is $-x^2$.

The square of a real number ≥ -1 is greater than or equal to not 1 but 0.

In f, x gets each input, which can be 0, and the square of 0 is 0.

However, the expression is $-x^2$, so each output is the negative of each square.
Thus, each output is less than or equal to 0.

In other words, it the case where $x \geq -1 \Rightarrow x^2 \geq 0 \Rightarrow -x^2 \leq 0$.

And we have $y = -x^2$, too. Therefore, the range is $y \leq 0$.

8. Find the range of a function $y = f(x) = -x^2$ for $x \leq 1$.

The domain is $x \leq 1$. So f holds for $x \leq 1$. And the expression is $-x^2$.

The square of a real number ≤ 1 is greater than or equal to not 1 but 0.
That's because such a real number can be 0, and the square of 0 is 0.

However, the expression is $-x^2$, so each output is the negative of each square.
In other words, it the case where $x \leq 1 \Rightarrow x^2 \geq 0 \Rightarrow -x^2 \leq 0$.

And we have $y = -x^2$, too. Therefore, the range is $y \leq 0$.

9. Find the range of a function $y = f(x) = -x^2$ for $1 \le x < 2$.

The domain is $\mathbf{1 \le x < 2}$, and we need to note that the domain does not include 2.

So f holds for $\mathbf{1 \le x < 2}$, and not for $\mathbf{x = 2}$.

The output variable is y, and we have $\mathbf{y = -x^2}$.

So we get $\mathbf{1 \le x < 2 \Rightarrow 1 \le x^2 < 4 \Rightarrow -4 < -x^2 \le -1 \Rightarrow -4 < y \le -1}$, which is the range.

A. Find the range of a function $y = f(x) = -x^2$ for $1 \le x \le 2$ or $-2 \le x \le -1$.

The domain is $\mathbf{1 \le x \le 2}$ or $\mathbf{-2 \le x \le -1}$. So f holds for $\mathbf{1 \le x \le 2}$ or $\mathbf{-2 \le x \le -1}$.

The output variable is y, and $\mathbf{y = -x^2}$. So we get

$\mathbf{1 \le x \le 2 \Rightarrow 1 \le x^2 \le 4 \Rightarrow -4 \le -x^2 \le -1 \Rightarrow -4 \le y \le -1}$.

$\mathbf{-2 \le x \le -1 \Rightarrow 1 \le x^2 \le 4 \Rightarrow -4 \le -x^2 \le -1 \Rightarrow -4 \le y \le -1}$, the same the one above.

Thus, we get $\mathbf{-4 \le y \le -1}$, which is the range.

B. Find the range of a function $y = f(x) = -x^2$ for $0 \le x < 1$ or $-2 \le x \le -1$.

The domain is $\mathbf{0 \le x < 1}$ or $\mathbf{-2 \le x \le -1}$. So f holds for $\mathbf{0 \le x < 1}$ or $\mathbf{-2 \le x \le -1}$.

The output variable is y, and $\mathbf{y = -x^2}$. So we get

$\mathbf{0 \le x < 1 \Rightarrow 0 \le x^2 < 1 \Rightarrow -1 \le -x^2 \le 0 \Rightarrow -1 \le y \le 0}$.

$\mathbf{-2 \le x \le -1 \Rightarrow 1 \le x^2 \le 4 \Rightarrow -4 \le -x^2 \le 0 \Rightarrow -4 \le y \le 0}$.

Thus, we get $\mathbf{-1 \le y \le 0}$ or $\mathbf{-4 \le y \le 0}$, so we get $\mathbf{-4 \le y \le 0}$, which is the range.

C. Find the range of a function $y = f(x) = -x^2$ for $0 \le x < 0.1$ or $-2 \le x < -1$.

The domain is $\mathbf{0 \le x < 0.1}$ or $\mathbf{-2 \le x < -1}$. So f can hold for $\mathbf{0 \le x < 0.1}$ or $\mathbf{-2 \le x < -1}$.

The output variable is y, and we have $y = -x^2$. So we get

$\mathbf{0 \le x < 0.1 \Rightarrow 0 \le x^2 < 0.01 \Rightarrow -0.01 < -x^2 \le 0 \Rightarrow -0.01 < y \le 0}$.

$\mathbf{-2 \le x < -1 \Rightarrow 1 < x^2 \le 4 \Rightarrow -4 \le -x^2 < -1 \Rightarrow -4 \le y < -1}$.

Thus, the range is $\mathbf{-0.01 < y \le 0}$ or $\mathbf{-4 \le y < -1}$.

D. Find the range of a function $y = f(x) = 2x^2$ for x real.

That x is real means that any real number can be the input.
That is, the domain is a set of all real numbers. So f holds for all real numbers.

Whatever a real number it may be, the square of it is ≥ 0.
And the expression is $\mathbf{2x^2}$, so each output is twice the square of each input.

Nevertheless, every output is still greater than or equal to 0.
Therefore, the range is $\mathbf{y \ge 0}$.

E. Find the range of a function $y = f(x) = 2x^2$ for $x \ge 0$.

The domain is $\mathbf{x \ge 0}$. So f holds for $\mathbf{x \ge 0}$. And we have $\mathbf{y = f(x) = 2x^2}$.

So we get $\mathbf{x \ge 0 \Rightarrow 2x^2 \ge 0 \Rightarrow y \ge 0}$. Therefore, the range is $\mathbf{y \ge 0}$.

F. Find the range of a function $y = f(x) = 2x^2$ for $x \le 0$.

The domain is $\mathbf{x \le 0}$. So f holds for $\mathbf{x \le 0}$. And we have $\mathbf{y = f(x) = 2x^2}$.

So we get $\mathbf{x \le 0 \Rightarrow x^2 \ge 0 \Rightarrow 2x^2 \ge 0 \Rightarrow y \ge 0}$. Therefore, the range is $\mathbf{y \ge 0}$.

G. Find the range of a function $y = f(x) = 2x^2$ for $0 \le x \le 1$.

The domain is $\mathbf{0 \le x \le 1}$. Consequently, f holds for $\mathbf{0 \le x \le 1}$. And we have $\mathbf{y = 2x^2}$.

So we get $\mathbf{0 \le x \le 1 \Rightarrow 0 \le x^2 \le 1 \Rightarrow 0 \le 2x^2 \le 2 \Rightarrow 0 \le y \le 2}$, which is the range.

H. Find the range of a function $y = f(x) = 2x^2$ for $x \ge 1$.

The domain is $\mathbf{x \le 0}$. So f holds for $\mathbf{x \le 0}$. And we have $\mathbf{y = f(x) = 2x^2}$.

So we get $\mathbf{1 \le x \Rightarrow 1 \le x^2 \Rightarrow 2 \le 2x^2 \Rightarrow y \ge 2}$, which is the range.

I. Find the range of a function $y = f(x) = -2x^2$ for $x \ge -1$.

The domain is $\mathbf{x \ge -1}$. So f holds for $\mathbf{x \ge -1}$. And we have $\mathbf{y = f(x) = -2x^2}$.

So we get $\mathbf{-1 \le x \Rightarrow 0 \le x^2}$ since x can be 0, and thus, we get

$\mathbf{0 \le 2x^2 \Rightarrow 0 \ge -2x^2 \Rightarrow y \le 0}$, which is the range.

J. Find the range of a function $y = f(x) = -2x^2$ for $x \le 1$.

The domain is $\mathbf{x \le 1}$. So f holds for $\mathbf{x \le 1}$. And we have $\mathbf{y = f(x) = -2x^2}$.

So we get $\mathbf{1 \ge x \Rightarrow 0 \le x^2}$ since x can be 0, and thus, we get

$\mathbf{0 \le 2x^2 \Rightarrow 0 \ge -2x^2 \Rightarrow y \le 0}$, which is the range.

K. Find the range of a function $y = f(x) = -2x^2$ for $1 < x \le 2$.

The domain is $\mathbf{1 < x \le 2}$. So f holds for $\mathbf{1 < x \le 2}$. And we have $\mathbf{y = f(x) = -2x^2}$.

So we get $\mathbf{1 < x \le 2 \Rightarrow 1 < x^2 \le 4 \Rightarrow -4 \le -x^2 < -1 \Rightarrow -8 \le -2x^2 < -2 \Rightarrow -8 \le y < -2}$.

Therefore, the range is $\mathbf{-8 \le y < -2}$.

L. Find the range of a function $y = f(x) = 2x^2 + x$ for $1 \le x \le 2$ or $-2 \le x \le -1$.

The domain is $\mathbf{1 \le x \le 2}$ or $\mathbf{-2 \le x \le -1}$. So f holds for $\mathbf{1 \le x \le 2}$ or $\mathbf{-2 \le x \le -1}$.

And we have $\mathbf{y = f(x) = 2x^2 + x}$, which is not simple quadratic as $\mathbf{x^2}$ or $\mathbf{3x^2}$.
So it may not be simple enough to apply inequalities only to find the range.

That's because it's not quite simple to keep track of the output, because the output does not usually change constant manner. That is to say that it is often the case, it does not keep increasing or decreasing. So it's not a good idea to apply inequalities only to track the outputs changing as the inputs change. What then, is a good idea?

In such a case in fact, we can get the range easily and fast using the graph of f.
More importantly though, it is safe to get the range graphically.
So it's not only easy and fast but safe, too, to find the range using a graph.

So graphing matters.

Let's one more time though, find the range applying inequalities.
That is, we are going to get the range tacking the outputs changing as the inputs change.

To begin with, the domain is made of two intervals.

One is $\mathbf{1 \le x \le 2}$, and the other is $\mathbf{-2 \le x \le -1}$. Thus, we have two cases.

First, $\mathbf{1 \le x \le 2 \Rightarrow 1 \le x^2 \le 4 \Rightarrow 2 \le 2x^2 \le 8}$, so one case is $\mathbf{1 \le x \le 2}$ and $\mathbf{2 \le 2x^2 \le 8}$.

Next, $\mathbf{-2 \le x \le -1 \Rightarrow 1 \le x^2 \le 4 \Rightarrow 2 \le 2x^2 \le 8}$, so the other is $\mathbf{-2 \le x \le -1}$ and $\mathbf{2 \le 2x^2 \le 8}$.

And we have $\mathbf{y = 2x^2 + x}$, which is the sum of $\mathbf{2x^2}$ and $\mathbf{x}$.

So beginning with the case where $\mathbf{1 \le x \le 2}$ and $\mathbf{2 \le 2x^2 \le 8}$, we can see these:

- First, when $\mathbf{x}$ is 1, $\mathbf{2x^2}$ is 2, and when $\mathbf{x}$ is 2, $\mathbf{2x^2}$ is 8.

- Next, if the value of $\mathbf{x}$ increases from 1 to 2, the value of $\mathbf{2x^2}$ increases from 2 to 8.

So the value of $(\boldsymbol{2x^2 + x})$ keeps increasing as the value of $\boldsymbol{x}$ changes from 1 to 2.

Thus, the smallest value of $(\boldsymbol{2x^2 + x})$ is the sum of the minimums of $\boldsymbol{2x^2}$ and $\boldsymbol{x}$, and the greatest is the sum of the maximums of the two.

So next, let's get the maximums and minimums.

To begin with, we have $\boldsymbol{1 \le x \le 2}$, so the minimum of $\boldsymbol{x}$ is 1, and the maximum is 2.

And next, we have $\boldsymbol{2 \le 2x^2 \le 8}$, so the minimum of $\boldsymbol{2x^2}$ is 2, and the maximum is 8.

Thus, we get $\boldsymbol{2 + 1 \le 2x^2 + x \le 8 + 2 \Rightarrow 3 \le 2x^2 + x \le 10}$.

So we get $\boldsymbol{3 \le 2x^2 + x \le 10}$ for $\boldsymbol{1 \le x \le 2}$. Thus, we get $\boldsymbol{3 \le y \le 10}$ for $\boldsymbol{1 \le x \le 2}$.

And we can put the idea above the way below, too.

We know that the smallest value of $(\boldsymbol{2x^2 + x})$ is the sum of the minimums of $\boldsymbol{2x^2}$ and $\boldsymbol{x}$, and that the biggest value of $(\boldsymbol{2x^2 + x})$ is the sum of the maximums of $\boldsymbol{2x^2}$ and $\boldsymbol{x}$.

And we have $\boldsymbol{y = f(x) = 2x^2 + x}$ for $\boldsymbol{1 \le x \le 2}$.

So we can say that when $\boldsymbol{x}$ is 1, the value of $\boldsymbol{f(x)}$ is the smallest, and that when $\boldsymbol{x}$ is 2, the value of $\boldsymbol{f(x)}$ is the biggest.

That is to say that the smallest value of $\boldsymbol{f(x)}$ is $\boldsymbol{f(1)}$, and that the biggest value of $\boldsymbol{f(x)}$ is $\boldsymbol{f(2)}$. So we can put it this way: $\boldsymbol{f(1) \le f(x) \le f(2)}$.

And we have $\boldsymbol{f(1) = 2 \cdot 1^2 + 1 = 3}$, and $\boldsymbol{f(2) = 2 \cdot 2^2 + 2 = 10}$.

Thus, we get $\boldsymbol{3 \le f(x) \le 10}$. And we have $\boldsymbol{y = f(x)}$. So we get $\boldsymbol{3 \le y \le 10}$ for $\boldsymbol{1 \le x \le 2}$.

Next, moving on to the second case where $\boldsymbol{-2 \le x \le -1}$ and $\boldsymbol{2 \le 2x^2 \le 8}$, we can see these:

- First, when $\boldsymbol{x}$ is -2, $\boldsymbol{2x^2}$ is 8, and when $\boldsymbol{x}$ is -1, $\boldsymbol{2x^2}$ is 2.
- Next, if the value of $\boldsymbol{x}$ increases from -2 to -1, the value of $\boldsymbol{2x^2}$ decreases from 8 to 2.

And when one thing decreases, if we keep adding a negative to it, it keeps decreasing.

So the value of ($\mathbf{2}\boldsymbol{x}^2 + \boldsymbol{x}$) decreases constantly as $\boldsymbol{x}$ changes from -2 to -1.

And we have $\boldsymbol{y = f(x) = 2x^2 + x}$ for $\mathbf{-2 \le \boldsymbol{x} \le -1}$.

So we can say that when $\boldsymbol{x}$ is -2, the value of $\boldsymbol{f(x)}$ is the greatest, and that when $\boldsymbol{x}$ is -1, the value of $\boldsymbol{f(x)}$ is the smallest.

That is to say that the biggest value of $\boldsymbol{f(x)}$ is $\boldsymbol{f(-2)}$, and that the smallest value of $\boldsymbol{f(x)}$ is $\boldsymbol{f(-1)}$. So we can put it this way: $\boldsymbol{f(-2) \ge f(x) \ge f(-1)}$.

And we have $\boldsymbol{f(-1) = 2\cdot(-1)^2 + (-1) = 1}$, and $\boldsymbol{f(-2) = 2\cdot(-2)^2 + (-2) = 6}$.

Thus, we get $\boldsymbol{1 \le f(x) \le 6}$. And we have $\boldsymbol{y = f(x)}$. So we get $\boldsymbol{1 \le y \le 6}$ for $\boldsymbol{-2 \le x \le -1}$.

Now, putting threads together, we have

$\boldsymbol{3 \le y \le 10}$ for $\boldsymbol{1 \le x \le 2}$, and $\boldsymbol{1 \le y \le 6}$ for $\boldsymbol{-2 \le x \le -1}$.

That is, when $\boldsymbol{1 \le x \le 2}$ or $\boldsymbol{-2 \le x \le -1}$, we get $\boldsymbol{3 \le y \le 10}$ or $\boldsymbol{1 \le y \le 6}$.

And notice that an interval where $\boldsymbol{3 \le y \le 6}$ is common to $\boldsymbol{3 \le y \le 10}$ and $\boldsymbol{1 \le y \le 6}$.

So we get $\boldsymbol{1 \le y \le 10}$ for $\boldsymbol{1 \le x \le 2}$ or $\boldsymbol{-2 \le x \le -1}$.

Therefore, the range is $\boldsymbol{1 \le y \le 10}$.

As stated earlier though, finding ranges of functions nonlinear, we may not want to approach the ranges the way above. We may not want to just apply inequalities only.

Usually, using the graph of a function, you can get the range easily and fast.
More importantly though, you can get it safely. You don’t want to get the wrong solution readily and quickly, do you?

So graphing matters. And it does a lot.

(Graphing is covered in the book **Conics**. And the operations with graphs or curves are covered in the book **Graph Operations**.)

So let’s now try getting the range considering the nature of the curve of $\boldsymbol{f}$.

Examining the expression where $\mathbf{2}\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{x}$, we can see it's quadratic, and the coefficient of the quadratic term is 2, which is *positive*, so the curve is a parabola, and is *concave-up*.

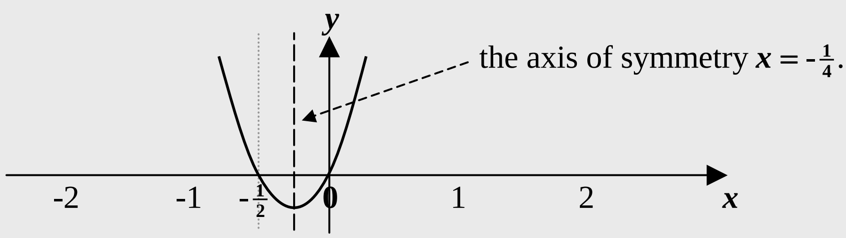
So since the parabola is concave-up, as the value of $\boldsymbol{x}$ increases on the left of the *axis of symmetry*, the value of $\boldsymbol{f(x)}$, that is, the value of $\boldsymbol{y}$ keeps decreasing, and as the value of $\boldsymbol{x}$ increases on the right of the axis of symmetry, the value of $\boldsymbol{f(x)}$ keeps increasing.

(If not sure of such a coefficient and such an axis of symmetry, refer to **CONICS 2**.)

And we have $\boldsymbol{y = f(x) = 2x^2 + x}$ for $\mathbf{1 \le \boldsymbol{x} \le 2}$ or $\mathbf{-2 \le \boldsymbol{x} \le -1}$.

Of this parabola, the axis of symmetry is $\boldsymbol{x} = -\frac{1}{4}$.
The axis of symmetry of a parabola $\boldsymbol{y = ax^2 + bx + c}$ is a line $\boldsymbol{x} = -\frac{b}{2a}$.

Fig. L



So the interval where $\mathbf{-2 \le \boldsymbol{x} \le -1}$ is on the left of the axis of symmetry, which is $\boldsymbol{x} = -\frac{1}{4}$.

Thus, we can simply get $\mathbf{-2 \le \boldsymbol{x} \le -1 \Rightarrow \boldsymbol{f}(-2) \ge \boldsymbol{f(x)} \ge \boldsymbol{f}(-1)}$.

And the other interval where $\mathbf{1 \le \boldsymbol{x} \le 2}$ is on the right of the axis of symmetry.

Thus, we can simply get $\mathbf{1 \le \boldsymbol{x} \le 2 \Rightarrow \boldsymbol{f}(1) \le \boldsymbol{f(x)} \le \boldsymbol{f}(2)}$.

So next, we want to get the values of $\boldsymbol{f}\mathbf{(1)}$, $\boldsymbol{f}\mathbf{(2)}$, $\boldsymbol{f}\mathbf{(-2)}$, and $\boldsymbol{f}\mathbf{(-1)}$.

Then, we get $\boldsymbol{f}\mathbf{(1) = 2\boldsymbol{x}^2 + \boldsymbol{x} = 2 + 1 = 3}$, and by the same token, we get
$\boldsymbol{f}\mathbf{(2) = 8 + 2 = 10}$, $\boldsymbol{f}\mathbf{(-2) = 8 - 2 = 6}$, and $\boldsymbol{f}\mathbf{(-1) = 2 - 1 = 1}$.

So we get $\mathbf{\boldsymbol{f}(1) \le \boldsymbol{f(x)} \le \boldsymbol{f}(2) \Rightarrow 3 \le \boldsymbol{f(x)} \le 10}$, and $\mathbf{\boldsymbol{f}(-1) \le \boldsymbol{f(x)} \le \boldsymbol{f}(-2) \Rightarrow 1 \le \boldsymbol{f(x)} \le 6}$.

And we have $\boldsymbol{y = f(x)}$. So we get $\mathbf{3 \le \boldsymbol{y} \le 10}$ or $\mathbf{1 \le \boldsymbol{y} \le 6}$.

And $\mathbf{3 \le \boldsymbol{y} \le 10}$ is common to $\mathbf{3 \le \boldsymbol{y} \le 10}$ and $\mathbf{1 \le \boldsymbol{y} \le 6}$. Therefore, the range is $\mathbf{1 \le \boldsymbol{y} \le 10}$.