Examples 4 in Inverse Functions

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0. Assuming $\boldsymbol{f(x) = x^2 - 3x + 2}$ for $\boldsymbol{x \geq 2}$, find $\boldsymbol{g(x)}$ so that we get $\boldsymbol{g\{f(x)\} = x}$.

1. Assuming $\boldsymbol{(g \bullet f)(x) = 2x + 1}$, $\boldsymbol{f(x) = 3x + 4}$, $\boldsymbol{(u \bullet v)(x) = 2x + 1}$, and $\boldsymbol{u(x) = 3x + 4}$, find $\boldsymbol{g(x)}$ and $\boldsymbol{v(x)}$.

Suggestions or Solutions

To the Problem in the Example 0

Assuming $\boldsymbol{f(x) = x^2 - 3x + 2}$ for $\boldsymbol{x \geq 2}$, find $\boldsymbol{g(x)}$ so that we get $\boldsymbol{g\{f(x)\} = x}$.

Assuming $\boldsymbol{I}$ is an identity function, we can set $\boldsymbol{g\{f(x)\} = (g \bullet f)(x) = I(x) = x}$.

So if $\boldsymbol{f}$ is invertible, and thus, is one-to-one, we get $\boldsymbol{g = f^{-1}}$.

Assuming $\boldsymbol{a}$ and $\boldsymbol{b}$ are constant, and $\boldsymbol{a \neq b}$, we can get

$\boldsymbol{f(a) - f(b) = a^2 - 3a + 2 - (b^2 - 3b + 2) = a^2 - b^2 - 3(a - b) = (a - b)(a + b) - 3(a - b)}$

$\boldsymbol{= (a - b)(a + b - 3)}$. So if $\boldsymbol{a + b - 3}$ cannot be 0, we get $\boldsymbol{f(a) \neq f(b)}$.

Suppose now, $\boldsymbol{a \geq 2}$, and $\boldsymbol{b > 2}$, since $\boldsymbol{a \neq b}$. Suppose also, $\boldsymbol{a + b - 3 = 0}$.

Then, we get $\boldsymbol{a = 3 - b}$. Thus, we get $\boldsymbol{a \geq 2 \Rightarrow 3 - b \geq 2 \Rightarrow b \leq 1}$, but we have $\boldsymbol{b > 2}$.

So $\boldsymbol{a + b - 3 \neq 0}$, and thus, $\boldsymbol{f}$ is one-to-one. So finding $\boldsymbol{f^{-1}}$, we can begin this way:

$\boldsymbol{y = x^2 - 3x + 2 = x^2 - 3x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (x - \tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (x - \tfrac{3}{2})^2 - \tfrac{1}{4}}$.

So we get $\boldsymbol{y + \tfrac{1}{4} = (x - \tfrac{3}{2})^2 \Rightarrow x - \tfrac{3}{2} = \pm\sqrt{y + \tfrac{1}{4}} \Rightarrow x = \pm\sqrt{y + \tfrac{1}{4}} + \tfrac{3}{2}}$.

We know the domain in $\boldsymbol{f}$ is $\boldsymbol{x \geq 2}$, so finding the range in $\boldsymbol{f}$, we get

$\boldsymbol{x \geq 2 \Rightarrow x - \tfrac{3}{2} \geq \tfrac{1}{2} \Rightarrow (x - \tfrac{3}{2})^2 \geq \tfrac{1}{4} \Rightarrow (x - \tfrac{3}{2})^2 - \tfrac{1}{4} \geq 0}$. So the range is $\boldsymbol{y \geq 0}$.

And thus, finding the domain back, we get

$\boldsymbol{y \geq 0 \Rightarrow y + \tfrac{1}{4} \geq \tfrac{1}{4} \Rightarrow \sqrt{y + \tfrac{1}{4}} \geq \tfrac{1}{2} \Rightarrow \sqrt{y + \tfrac{1}{4}} + \tfrac{3}{2} \geq 2 \Rightarrow x \geq 2}$, which is the domain.

So we get $\boldsymbol{x = \sqrt{y + \tfrac{1}{4}} + \tfrac{3}{2}}$. And next, swapping the variables, we get $\boldsymbol{y = \sqrt{x + \tfrac{1}{4}} + \tfrac{3}{2}}$.

And thus, the inverse is $\boldsymbol{y = f^{-1}(x) = \sqrt{x + \tfrac{1}{4}} + \tfrac{3}{2}}$ for $\boldsymbol{x \geq 0}$.

We know $\boldsymbol{g = f^{-1}}$. So we get $\boldsymbol{y = g(x) = \sqrt{x + \tfrac{1}{4}} + \tfrac{3}{2}}$ for $\boldsymbol{x \geq 0}$.

If not quite sure of the idea behind the processes above, follow the steps below.

What do we mean by $g\{f(x)\} = x$?

It is a composite function where the function f blends into the other function g.

So we can set $g\{f(x)\} = (g \bullet f)(x) = x$, which is thus, an identity function as $y = I(x) = x$.

And thus, assuming I is an identity function, we can just set it this way, too: $g \bullet f = I$.

And we have a theorem where $f^{-1} \bullet f = I$ if f is invertible.

So if f is invertible, we can see this: $g = f^{-1}$. What then, do we mean by f is invertible?

If f is one-to-one, f is invertible, and has its inverse.

So let's check to see if f is one-to-one.

Assuming a and b are constant, and $a \neq b$, we can check to see if $f(a) - f(b) \neq 0$, that is, $f(a) \neq f(b)$. If it is the case, f is one-to-one.

And doing the check, we need to set $a \geq 2$, and $b > 2$, or equivalently, $a > 2$, and $b \geq 2$.

That's because the domain is $x \geq 2$, and $a \neq b$. And thus, doing the check, we get

$f(a) - f(b) = a^2 - 3a + 2 - (b^2 - 3b + 2) = a^2 - b^2 - 3(a - b) = (a - b)(a + b) - 3(a - b)$

$= (a - b)(a + b - 3)$.

And we know $a - b \neq 0$ since $a \neq b$.
So if $a + b - 3$ cannot be 0, either, we get $f(a) \neq f(b)$.

Suppose $\boldsymbol{a} + \boldsymbol{b} - \mathbf{3} = \mathbf{0}$. Then, we get $\boldsymbol{a} = \mathbf{3} - \boldsymbol{b}$. We have $\boldsymbol{a} \geq \mathbf{2}$, and $\boldsymbol{b} > \mathbf{2}$, too.

So we get $\boldsymbol{a} \geq \mathbf{2} \Rightarrow \mathbf{3} - \boldsymbol{b} \geq \mathbf{2} \Rightarrow \boldsymbol{b} \leq \mathbf{1}$, which however, contradicts the fact that $\boldsymbol{b} > \mathbf{2}$.

So we get $\boldsymbol{a} + \boldsymbol{b} - \mathbf{3} \neq \mathbf{0}$ if $\boldsymbol{a} \geq \mathbf{2}$, and $\boldsymbol{b} > \mathbf{2}$.

And the same is true, too, for the case where $\boldsymbol{a} > \mathbf{2}$, and $\boldsymbol{b} \geq \mathbf{2}$. That's because in this case, the case where $\boldsymbol{a} \geq \mathbf{2}$, and $\boldsymbol{b} > \mathbf{2}$ is equivalent to the case where $\boldsymbol{a} > \mathbf{2}$, and $\boldsymbol{b} \geq \mathbf{2}$.

So $\boldsymbol{f}$ is one-to-one, and thus, is invertible. And finding first, the expression of the inverse, we solve for $\boldsymbol{x}$, the equation made of the expression of $\boldsymbol{f}$, then swap the variables.

Without using the quadratic formula, we can begin this way:

$$\boldsymbol{y} = \boldsymbol{x}^2 - 3\boldsymbol{x} + 2 = \boldsymbol{x}^2 - 3\boldsymbol{x} + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (\boldsymbol{x} - \tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (\boldsymbol{x} - \tfrac{3}{2})^2 - \tfrac{1}{4}.$$

So we get $\boldsymbol{y} + \frac{1}{4} = (\boldsymbol{x} - \frac{3}{2})^2 \Rightarrow \boldsymbol{x} - \frac{3}{2} = \pm\sqrt{\boldsymbol{y} + \frac{1}{4}} \Rightarrow \boldsymbol{x} = \pm\sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2}$, which means two expressions.

Thus, we need to choose one from $\boldsymbol{x} = \sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2}$, and $\boldsymbol{x} = -\sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2}$.

We know the domain of $\boldsymbol{f}$ is $\boldsymbol{x} \geq \mathbf{2}$, so finding the range of $\boldsymbol{f}$, we get

$\boldsymbol{x} \geq 2 \Rightarrow \boldsymbol{x} - \frac{3}{2} \geq \frac{1}{2} \Rightarrow (\boldsymbol{x} - \frac{3}{2})^2 \geq \frac{1}{4} \Rightarrow (\boldsymbol{x} - \frac{3}{2})^2 - \frac{1}{4} \geq 0$. So the range is $\boldsymbol{y} \geq \mathbf{0}$.

Thus, finding the domain back, which is $\boldsymbol{x} \geq \mathbf{2}$, we get

$\boldsymbol{y} \geq 0 \Rightarrow \boldsymbol{y} + \frac{1}{4} \geq \frac{1}{4} \Rightarrow \sqrt{\boldsymbol{y} + \frac{1}{4}} \geq \frac{1}{2} \Rightarrow -\sqrt{\boldsymbol{y} + \frac{1}{4}} \leq -\frac{1}{2} \Rightarrow -\sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2} \leq 1 \Rightarrow \boldsymbol{x} \leq 1$, which is not the domain. So next, using the other expression, we can get

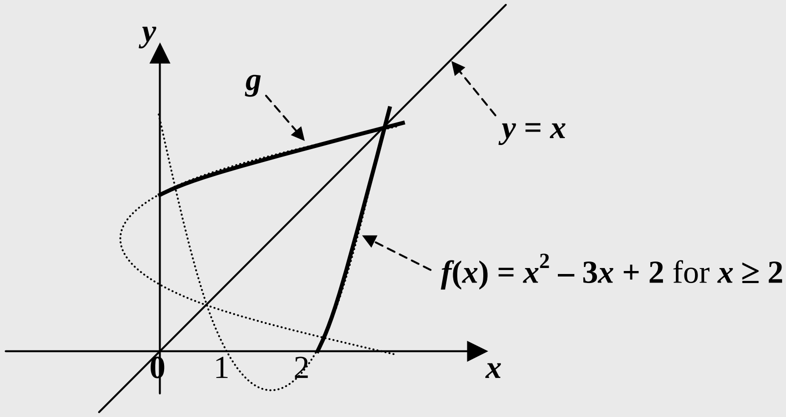
$\boldsymbol{y} \geq 0 \Rightarrow \boldsymbol{y} + \frac{1}{4} \geq \frac{1}{4} \Rightarrow \sqrt{\boldsymbol{y} + \frac{1}{4}} \geq \frac{1}{2} \Rightarrow \sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2} \geq 2 \Rightarrow \boldsymbol{x} \geq 2$, which is the domain.

So we get $\boldsymbol{x} = \sqrt{\boldsymbol{y} + \frac{1}{4}} + \frac{3}{2}$. And next, swapping the variables, we get $\boldsymbol{y} = \sqrt{\boldsymbol{x} + \frac{1}{4}} + \frac{3}{2}$.

And thus, the inverse is $\boldsymbol{y} = \boldsymbol{f}^{-1}(\boldsymbol{x}) = \sqrt{\boldsymbol{x} + \frac{1}{4}} + \frac{3}{2}$ for $\boldsymbol{x} \geq \mathbf{0}$.

And we know $g = f^{-1}$. So we get $y = g(x) = \sqrt{x+\frac{1}{4}}+\frac{3}{2}$ for $x \geq 0$.

Fig. 3.0



• And we can get the same, too, using the idea on composite functions.

Assuming $h = g \bullet f$, we can set $h(x) = (g \bullet f)(x) = x$.

In other words, $h(x) = g(f(x)) = x$.

That is, putting the expression of f into the input variable in the expression of g, we get x, which is the expression of the composite function h.

So suppose next, $t = g(s)$, where s is just the input variable, and t is merely the output variable. Then, putting the expression of f into the input variable s in the expression of g, we get x.

That is, putting $x^2 - 3x + 2$ into the input variable s in the expression of g, we get x.

So we can set $s = x^2 - 3x + 2$. And thus, finding the expression of g, we get first,

$$s = x^2 - 3x + 2 = x^2 - 3x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (x - \tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + 2 = (x - \tfrac{3}{2})^2 - \tfrac{1}{4}.$$

So next, we get $s + \frac{1}{4} = (x - \frac{3}{2})^2 \Rightarrow x - \frac{3}{2} = \pm\sqrt{s+\frac{1}{4}} \Rightarrow x = \pm\sqrt{s+\frac{1}{4}} + \frac{3}{2}$, which means two expressions. We need thus, to choose either of the two.   How though?

We know $\boldsymbol{x} \geq \mathbf{2}$, and we have $\boldsymbol{s} = \boldsymbol{x}^2 - 3\boldsymbol{x} + 2 = (\boldsymbol{x} - \frac{3}{2})^2 - \frac{1}{4}$. So we get first,

$\boldsymbol{x} \geq 2 \Rightarrow \boldsymbol{x} - \frac{3}{2} \geq \frac{1}{2} \Rightarrow (\boldsymbol{x} - \frac{3}{2})^2 \geq \frac{1}{4} \Rightarrow (\boldsymbol{x} - \frac{3}{2})^2 - \frac{1}{4} \geq 0$. So we get $\boldsymbol{s} \geq \mathbf{0}$.

And we have $\boldsymbol{x} = \pm\sqrt{\boldsymbol{s} + \frac{1}{4}} + \frac{3}{2}$. So using $\boldsymbol{s} \geq \mathbf{0}$, we should be able to get this back: $\boldsymbol{x} \geq \mathbf{2}$.

$\boldsymbol{s} \geq 0 \Rightarrow \boldsymbol{s} + \frac{1}{4} \geq \frac{1}{4} \Rightarrow \sqrt{\boldsymbol{s} + \frac{1}{4}} \geq \frac{1}{2} \Rightarrow -\sqrt{\boldsymbol{s} + \frac{1}{4}} \leq -\frac{1}{2} \Rightarrow -\sqrt{\boldsymbol{s} + \frac{1}{4}} + \frac{3}{2} \leq 1 \Rightarrow \boldsymbol{x} \leq 1$.

$\boldsymbol{s} \geq 0 \Rightarrow \boldsymbol{s} + \frac{1}{4} \geq \frac{1}{4} \Rightarrow \sqrt{\boldsymbol{s} + \frac{1}{4}} \geq \frac{1}{2} \Rightarrow \sqrt{\boldsymbol{s} + \frac{1}{4}} + \frac{3}{2} \geq 2 \Rightarrow \boldsymbol{x} \geq 2$.

So we get $\boldsymbol{x} = \sqrt{\boldsymbol{s} + \frac{1}{4}} + \frac{3}{2}$, which is the expression of $\boldsymbol{g}$. And we have set $\boldsymbol{t} = \boldsymbol{g}(\boldsymbol{s})$.

So we get $\boldsymbol{t} = \boldsymbol{g}(\boldsymbol{s}) = \sqrt{\boldsymbol{s} + \frac{1}{4}} + \frac{3}{2}$.

We know $\boldsymbol{s}$ and $\boldsymbol{t}$ are just the input variable and the output variable in the function $\boldsymbol{g}$.

That is, $\boldsymbol{g}$ is simply in the $\boldsymbol{s}$-$\boldsymbol{t}$ system, which is just another name of a perpendicular coordinate system as the $\boldsymbol{x}$-$\boldsymbol{y}$ system. In a perpendicular coordinate system where we use two variables, two coordinate axes are perpendicular to each other.

So putting $\boldsymbol{g}$ in the $\boldsymbol{x}$-$\boldsymbol{y}$ system, we just use $\boldsymbol{x}$ and $\boldsymbol{y}$ in place of $\boldsymbol{s}$ and $\boldsymbol{t}$, and thus, we can just set $\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}) = \sqrt{\boldsymbol{x} + \frac{1}{4}} + \frac{3}{2}$ for $\boldsymbol{x} \geq \mathbf{0}$.

Suggestions or Solutions
To the **Problem** in the Example **1**

Assuming $(g \bullet f)(x) = 2x + 1$, $f(x) = 3x + 4$, $(u \bullet v)(x) = 2x + 1$, and $u(x) = 3x + 4$, find $g(x)$ and $v(x)$.

Finding f^{-1} first, we get first, $y = 3x + 4 \Rightarrow x = \frac{y-4}{3}$.

So we get $y = f^{-1}(x) = \frac{x-4}{3}$ for x real.

Thus next, assuming $h = g \bullet f$, we get $h(x) = 2x + 1$, and also, $h \bullet f^{-1} = g \bullet f \bullet f^{-1} = g$.

So we get $h \bullet f^{-1} = h(f^{-1}) = 2f^{-1} + 1 = 2 \cdot \frac{x-4}{3} + 1 = \frac{2x-8+3}{3} = \frac{2x-5}{3}$.

Therefore, we get $y = g(x) = \frac{2x-5}{3}$ for x real.

We have $(g \bullet f)(x) = g(f(x)) = 2x + 1$. So assuming $t = g(s)$, we can set $s = 3x + 4$.

And we know that x in $2x + 1$ is the same as the x in $3x + 4$.

And thus, finding the expression of the function g, we get

$s = 3x + 4 \Rightarrow x = \frac{s-4}{3} \Rightarrow 2x + 1 = 2 \cdot \frac{s-4}{3} + 1 = \frac{2s-8+3}{3} = \frac{2s-5}{3}$.

So we get $t = g(s) = \frac{2s-5}{3}$.

The two functions u and f are the same. So we have $y = u^{-1}(x) = \frac{x-4}{3}$ for x real.

Thus, setting $h = u \bullet v$, we get $h(x) = 2x + 1$, and also, $u^{-1} \bullet h = u^{-1} \bullet u \bullet v = v$.

That is, $u^{-1} \bullet h = v$. We have $u^{-1}(x) = \frac{x-4}{3}$, and $h(x) = 2x + 1$.

So we get $u^{-1} \bullet h = u^{-1}(h) = \frac{h-4}{3} = \frac{2x+1-4}{3} = \frac{2x-3}{3}$. And thus, $y = v(x) = \frac{2x-3}{3}$ for x real.

We have $u(x) = 3x + 1$, and $(u \bullet v)(x) = 2x + 1$.

So we can just get $(u \bullet v)(x) = u(v(x)) = 3 \cdot v(x) + 4 = 2x + 1$.

And thus, we get $3 \cdot v(x) = 2x + 1 - 4 = 2x - 3 \Rightarrow v(x) = \frac{2x-3}{3}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Let's begin with the function g.

We are given two functions, one is a composite function $(g \bullet f)(x) = 2x + 1$, and the other is a function $f(x) = 3x + 4$.

In this case, we can find the function g using either of two ideas. One is the idea where composite function gets made, and the other is using theorems on inverse functions.

Let's begin with the theorems first. We can use two theorem as follows.

One is that $f \bullet f^{-1}$ and $f^{-1} \bullet f$ are identity functions as $y = I(x) = x$.

And the other is that $p \bullet q \bullet r = (p \bullet q) \bullet r = p \bullet (q \bullet r)$, where p, q, and r are functions, of course.

So putting the two theorems together, we can see these:

$$g \bullet f \bullet f^{-1} = g \bullet I = g, \quad g \bullet f^{-1} \bullet f = g \bullet I = g, \quad f \bullet f^{-1} \bullet g = I \bullet g = g, \quad \text{and } f^{-1} \bullet f \bullet g = I \bullet g = g.$$

And thus, of the four equalities above, we use this: $g \bullet f \bullet f^{-1} = g$.

So we want to find f^{-1}.

Finding the expression first, we solve for x the equation $y = 3x + 4$, then swap the variables. So solving the equation, we get

$y = 3x + 4 \Rightarrow x = \frac{y-4}{3}$. And next, swapping the variables, we get $y = \frac{x-4}{3}$.

So we get $y = f^{-1}(x) = \frac{x-4}{3}$ for x real.
Thus next, assuming $h = g \bullet f$, we can set $h(x) = (g \bullet f)(x) = 2x + 1$.

So we get $h(x) = 2x + 1$. And also, we can get $h \bullet f^{-1} = g \bullet f \bullet f^{-1}$, which is g, of course.

That is, $h \bullet f^{-1} = g$.

Now, we get $\boldsymbol{h \bullet f^{-1} = h(f^{-1}) = 2f^{-1} + 1 = 2 \cdot \frac{x-4}{3} + 1 = \frac{2x-8+3}{3} = \frac{2x-5}{3}}$.

Therefore, we get $\boldsymbol{y = g(x) = \frac{2x-5}{3}}$ for $\boldsymbol{x}$ real.

• And let's next, use the idea on composite functions.

Suppose again, $\boldsymbol{h = g \bullet f}$. Then, we can set again $\boldsymbol{h(x) = (g \bullet f)(x) = 2x + 1}$.

In other words, $\boldsymbol{h(x) = g(f(x)) = 2x + 1}$.

That is, putting the expression of $\boldsymbol{f}$ into the input variable in the expression of $\boldsymbol{g}$, we get $\boldsymbol{2x + 1}$, which is the expression of the composite function $\boldsymbol{h}$.

So suppose next, $\boldsymbol{t = g(s)}$, where $\boldsymbol{s}$ is just the input variable, and $\boldsymbol{t}$ is merely the output variable.

Then, putting the expression of $\boldsymbol{f}$ into the input variable $\boldsymbol{s}$ in the expression of $\boldsymbol{g}$, we get $\boldsymbol{2x + 1}$.

That is, putting $\boldsymbol{3x + 4}$ into the input variable $\boldsymbol{s}$ in the expression of $\boldsymbol{g}$, we get $\boldsymbol{2x + 1}$.

Then, we can set $\boldsymbol{s = 3x + 4}$. Now, from where is $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$?

The $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$ is from the expression of $\boldsymbol{f}$, which is $\boldsymbol{3x + 4}$.

That is, $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$ is the same as the $\boldsymbol{x}$ in $\boldsymbol{3x + 4}$. And we have $\boldsymbol{s = 3x + 4}$.

So putting $\boldsymbol{x}$ in terms of $\boldsymbol{s}$, we get an expression in terms of $\boldsymbol{s}$, and then, we can put the expression into $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$.

That is, solving $\boldsymbol{s = 3x + 4}$ for $\boldsymbol{x}$, we get the solution which is an expression in terms of $\boldsymbol{s}$, and then, we can put the solution into $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$, because $\boldsymbol{x}$ in $\boldsymbol{2x + 1}$ is the same as the $\boldsymbol{x}$ in $\boldsymbol{3x + 4}$. What then, do we get?

We get an expression in terms of ***s***, which is the expression of the function ***g***.

So putting $s = 3x + 4$ back into ***s*** in the expression of ***g***, we get an expression in terms of ***x***, and the expression is $2x + 1$, which is the very expression of the composite function ***h***.

And thus, finding the expression of ***g***, we get

$s = 3x + 4 \Rightarrow x = \frac{s-4}{3} \Rightarrow 2x + 1 = 2 \cdot \frac{s-4}{3} + 1 = \frac{2s-8+3}{3} = \frac{2s-5}{3}$.

So we get $t = g(s) = \frac{2s-5}{3}$.

We know ***s*** and ***t*** are just the input variable and the output variable in the function ***g***.

That is, ***g*** is simply in the ***s-t*** system, which is just another name of a perpendicular coordinate system as the ***x-y*** system.

So putting ***g*** in the ***x-y*** system, we just use ***x*** and ***y*** in place of ***s*** and ***t***, and thus, we can just set $y = g(x) = \frac{2x-5}{3}$.

- And let's next, move on to the other function $v(x)$.

As in the case above, using the two theorems, we can find the function $v(x)$.

We have $(u \bullet v)(x) = 2x + 1$, and $u(x) = 3x + 4$.

So using the two theorems above, we can get $u^{-1} \bullet u \bullet v = v$.

And thus, we want to find first, u^{-1}. However, we have already got the inverse because the two functions ***u*** and ***f*** are the same. So we have $y = u^{-1}(x) = \frac{x-4}{3}$ for ***x*** real.

Thus, setting again $h = u \bullet v$, we can set $h(x) = (u \bullet v)(x) = 2x + 1$.

So we get again $h(x) = 2x + 1$. And we can get $u^{-1} \bullet h = u^{-1} \bullet u \bullet v$, which is ***v***, of course.

That is, $u^{-1} \bullet h = v$. We have $u^{-1}(x) = \frac{x-4}{3}$, and $h(x) = 2x + 1$.

So we get $\boldsymbol{u^{-1}\bullet h = u^{-1}(h)} = \frac{h-4}{3} = \frac{2x+1-4}{3} = \frac{2x-3}{3}$. And thus, $\boldsymbol{y = v(x)} = \frac{2x-3}{3}$ for $\boldsymbol{x}$ real.

• And let's next, find the function $\boldsymbol{v}$ using the idea on composite functions.

Suppose again, $\boldsymbol{h = u\bullet v}$. Then, we can set again $\boldsymbol{h(x) = (u\bullet v)(x) = 2x + 1}$.

In other words, $\boldsymbol{h(x) = u(v(x)) = 2x + 1}$.

That is, putting the expression of $\boldsymbol{v}$ into the input variable in the expression of $\boldsymbol{u}$, we get $\boldsymbol{2x + 1}$, which is the expression of the composite function $\boldsymbol{h}$.

In this case though, unlike the case of $\boldsymbol{g}$ above, we have the expression of the function $\boldsymbol{u}$.

We have $\boldsymbol{u(x) = 3x + 1}$.

So we can just get $\boldsymbol{(u\bullet v)(x) = u(v(x)) = 3\cdot v(x) + 4 = 2x + 1}$.

And thus, we get $\boldsymbol{3\cdot v(x) = 2x + 1 - 4 = 2x - 3 \Rightarrow v(x)} = \frac{2x-3}{3}$.