Examples 1 in Exponential & Log Functions

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Examples 1 in Exponential & Log Functions

In this set of examples, we construct the graph of a function, which is exponential. Constructing it, we put in the graph, the curve of the function. That is, we get the curve of the function.
And a function is said to make or produce its curve, too, because it generates each and every point in the curve.

We are going to cover though, not just one graph, but many graphs.
So we will get to work with many different functions, which are quite similar though.

And equations have their curves, too.
So we will get to see how to make and work with curves of some functions similar, and curves of some equations, too. And thus, this example will take quite a few pages.

That's because graphing matters. And so does algebra, of course.

Graphing, you get the curve of a function or equation, and can actually see what the function or equation is doing, and thus, can see better what you can do about it.
So what?

So you can get to the solution easily and quickly, together with, of course, your good algebra, because it actually connects the problem to the solution.

And the example is as follows.

Construct the graph of a function as follows. $\boldsymbol{y = f(x) = 2^{|x|}}$ for $\boldsymbol{x}$ real.

Suggestions or Solutions
To the Problem in the Example Given

Construct the graph of $y = f(x) = 2^{|x|}$ for x real.

Putting a function in a graph, we get the curve of it, and can actually see how it behaves, so doing problems with functions, we can see better the solutions' whereabouts.

Now, the function given is an exponential function, of which the domain is a set of all real numbers. In its expression however, absolute sign is applied to the input variable, which is $\boldsymbol{x}$, of course. So what does the curve look like?

The curve is symmetric about the $\boldsymbol{y}$-axis. Why?

If the domain is symmetric about the origin, and absolute sign is applied to every input variable, the curve is symmetric about the $\boldsymbol{y}$-axis.
What do we mean by the domain symmetric about the origin, though?

Suppose for instance, $\boldsymbol{a}$ is constant, a function is defined for $|\boldsymbol{x}| \le \boldsymbol{a}$, that is, $-\boldsymbol{a} \le \boldsymbol{x} \le \boldsymbol{a}$, another is defined for $|\boldsymbol{x}| < \boldsymbol{a}$, that is, $-\boldsymbol{a} < \boldsymbol{x} < \boldsymbol{a}$, and another is defined for $\boldsymbol{x}$ real, that is, a set of all real numbers. Then, each of all the domains is symmetric about the origin.

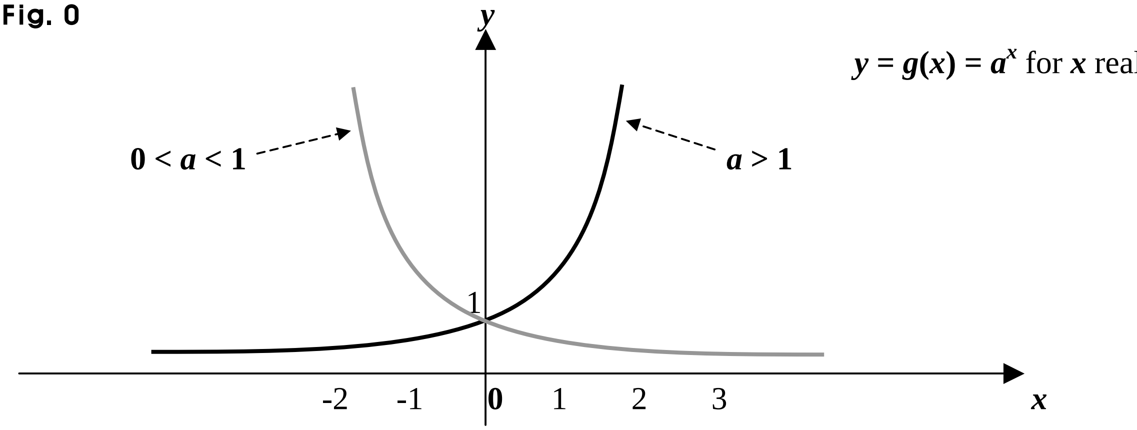
And if absolute sign is applied to every input variable, and the input is negative, the output is the same as the one for an input positive and equal in magnitude. So for instance, we get $\boldsymbol{f(-1) = f(1)}$, and the same is true, too, for all the other inputs negative.

And thus, the curve is symmetric about the $\boldsymbol{y}$-axis.

So let see now, how the curve is symmetric about the $\boldsymbol{y}$-axis.

Assuming first, $\boldsymbol{y = g(x) = a^x}$ for $\boldsymbol{x}$ real, and putting it in a graph, we will get

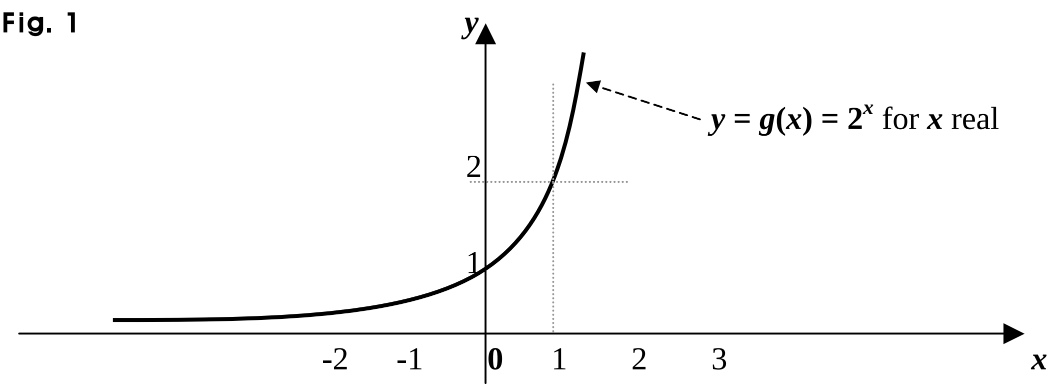
Fig. 0



Both curves gray and black pass through the point **(0, 1)**, of course.

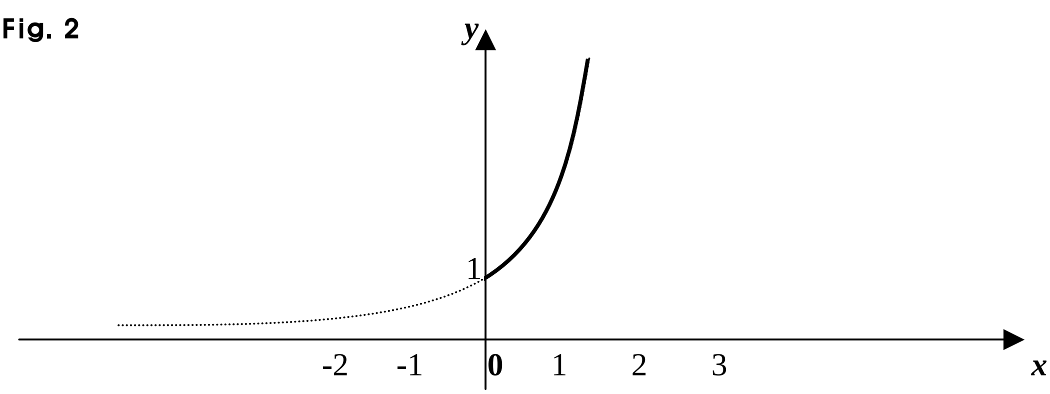
And for instance, if the base $\boldsymbol{a}$ is 2, the curve of the function $\boldsymbol{g}$ will be similar to the curve in black above, and thus, will be as follows.

Fig. 1



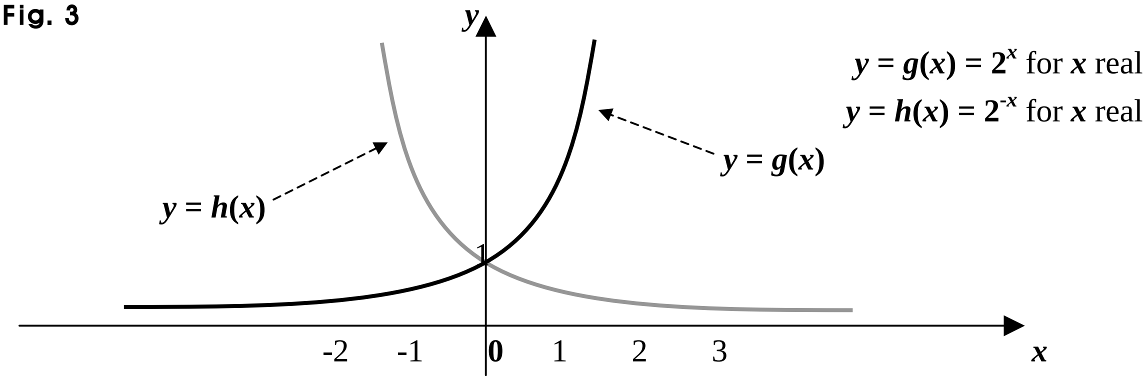
So in the graph above, when $\boldsymbol{x \geq 0}$, the amount of the curve of $\boldsymbol{g}$ is as below.

Fig. 2



And next, putting in a graph, another function $\boldsymbol{y = h(x) = 2^{-x}}$ for $\boldsymbol{x}$ real, we get

Fig. 3



Why?

We have $\mathbf{2^{-x} = (2^{-1})^x = (\tfrac{1}{2})^x}$.

So the graph of $\boldsymbol{h}$ is like the graph of the function $\boldsymbol{y = g(x) = a^x}$ where $\mathbf{0 < \boldsymbol{a} < 1}$.
And in fact, the curve of $\boldsymbol{h}$ is symmetric about the $\boldsymbol{y}$-axis to the curve of $\boldsymbol{y = g(x) = 2^x}$.
Why?

Suppose in $\boldsymbol{h}$, the $\boldsymbol{x}$-value changes from 0 to -2, for instance.

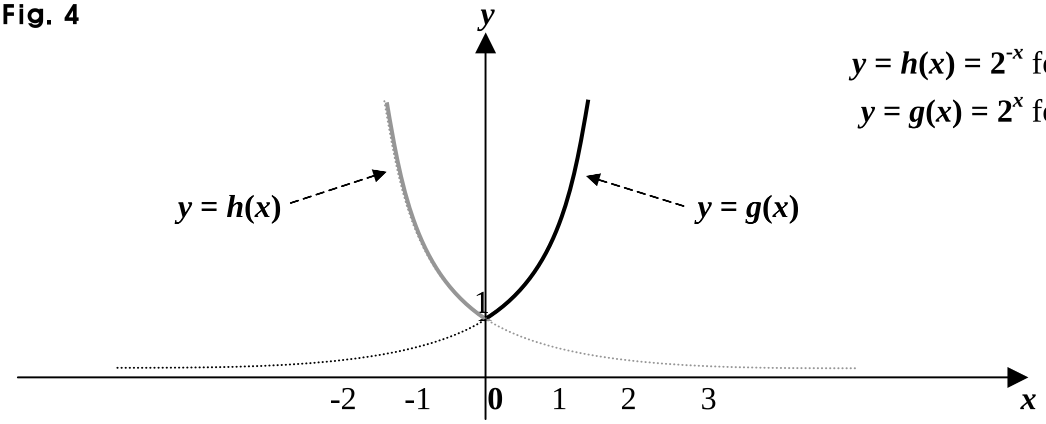
How then, does the value of $(-\boldsymbol{x})$ change?

It changes from 0 to 2, of course. And we have $\boldsymbol{y = h(x) = 2^{-x}}$. So when $\boldsymbol{x}$ changes from 0 to -2, the value of $\boldsymbol{h(x)}$ is the same as the value of $\boldsymbol{g(x)}$ when $\boldsymbol{x}$ change from 0 to 2.

Thus for instance, $\boldsymbol{h}\mathbf{(0)} = \boldsymbol{g}\mathbf{(0)}$, $\boldsymbol{h}\mathbf{(-1)} = \boldsymbol{g}\mathbf{(1)}$, and $\boldsymbol{h}\mathbf{(-2)} = \boldsymbol{g}\mathbf{(2)}$,

So when $\boldsymbol{x}$ changes from 0 to -2, the curve itself made by $\boldsymbol{h(x)}$ is exactly the same as the curve made by $\boldsymbol{g(x)}$ when $\boldsymbol{x}$ changes from 0 to 2, but is proceeding in the opposite direction. And thus, both curves are symmetric about the $\boldsymbol{y}$-axis.

Fig. 4



Now, in this problem, we have $\boldsymbol{y = f(x) = 2^{|x|}}$ for $\boldsymbol{x}$ real.

To begin with, we get $\boldsymbol{f(x) = f(-x)}$, because $\boldsymbol{2^{|x|} = 2^{|-x|}}$. What then, is the value of $\boldsymbol{f(x)}$ equal to when the $\boldsymbol{x}$-value changes from 0 to -2, for instance?

It is equal to the value of $\boldsymbol{f(x)}$ when the $\boldsymbol{x}$-value changes from 0 to 2.

Thus, for instance, we have $\boldsymbol{f(-1) = 2^{|-1|} = 2^{1} = 2}$, and $\boldsymbol{f(1) = 2^{|1|} = 2}$, so we get

$\boldsymbol{f(-1) = f(1)}$. And the same is true, too, for all the other values of $\boldsymbol{x}$.

So when $\boldsymbol{x}$ changes from 0 to -2, the curve itself made by $\boldsymbol{f}$ is exactly the same as the curve made by $\boldsymbol{f}$ when $\boldsymbol{x}$ changes from 0 to 2, but is proceeding in the opposite direction.

Thus, both curves are symmetric about the $\boldsymbol{y}$-axis. And we can put it the way below, too:

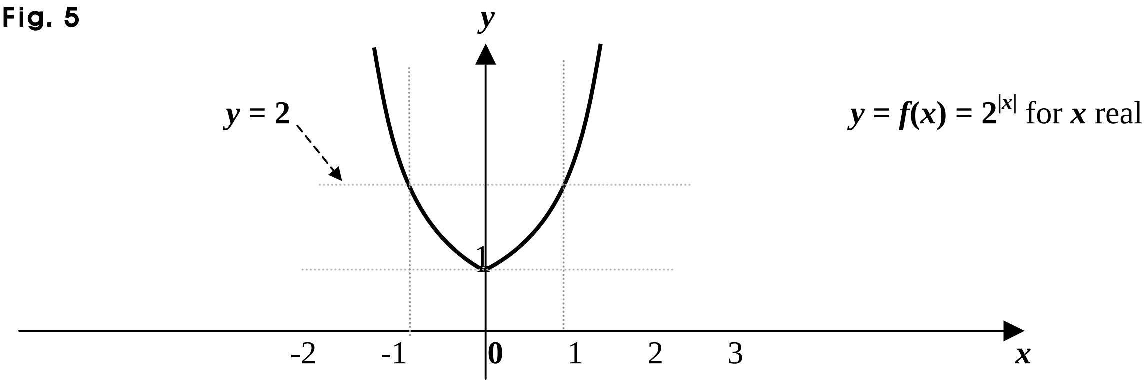
When $\boldsymbol{x}$ changes from -2 to 0, the curve itself made by $\boldsymbol{f}$ is exactly the same as the curve made by $\boldsymbol{f}$ when $\boldsymbol{x}$ changes from 0 to 2, but is going downward. Thus again, both curves are symmetric about the $\boldsymbol{y}$-axis.

So when $\boldsymbol{x}$ changes from negative infinity to 0, the curve itself made by $\boldsymbol{f}$ is exactly the same as the curve made by $\boldsymbol{f}$ when $\boldsymbol{x}$ change from 0 to infinity, but is going downward.

And thus, both curves are symmetric about the $\boldsymbol{y}$-axis.

So the entire curve of $\boldsymbol{f}$ is symmetric about the $\boldsymbol{y}$-axis, and is as follows.

Fig. 5



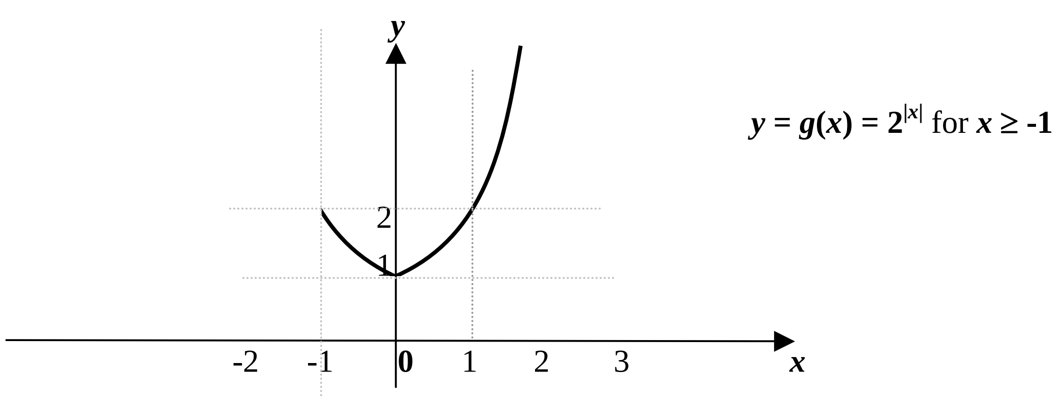
And quite often, we put the function above this way, too: $\boldsymbol{y = f(|x|) = 2^{|x|}}$ for $\boldsymbol{x}$ real.

So for instance, setting $\boldsymbol{y = p(|x|)}$, we apply absolute sign to every $\boldsymbol{x}$ in the expression of $\boldsymbol{p}$, and thus, if $\boldsymbol{y = p(x) = 2^{x+1} + 2x + 1}$, we get $\boldsymbol{y = p(|x|) = 2^{|x|+1} + 2|x| + 1}$.

Suppose next, $\boldsymbol{y = g(x) = 2^{|x|}}$ for $\boldsymbol{x \geq -1}$.

Then, the domain is not symmetric about the origin, so the curve of $\boldsymbol{g}$ is not symmetric about the $\boldsymbol{y}$-axis, and we get

Fig. 6



How then, about the curve of $\boldsymbol{y = g(x) = 2^{x-1}}$?

We can readily get it shifting by 1 the curve of a function $\boldsymbol{y = f(x) = 2^x}$ in the direction of the $\boldsymbol{x}$-axis. Why?

Suppose in $\boldsymbol{g}$, the $\boldsymbol{x}$-value changes from 1 to 3, for instance.

How then, does the value of $(\boldsymbol{x - 1})$ change?

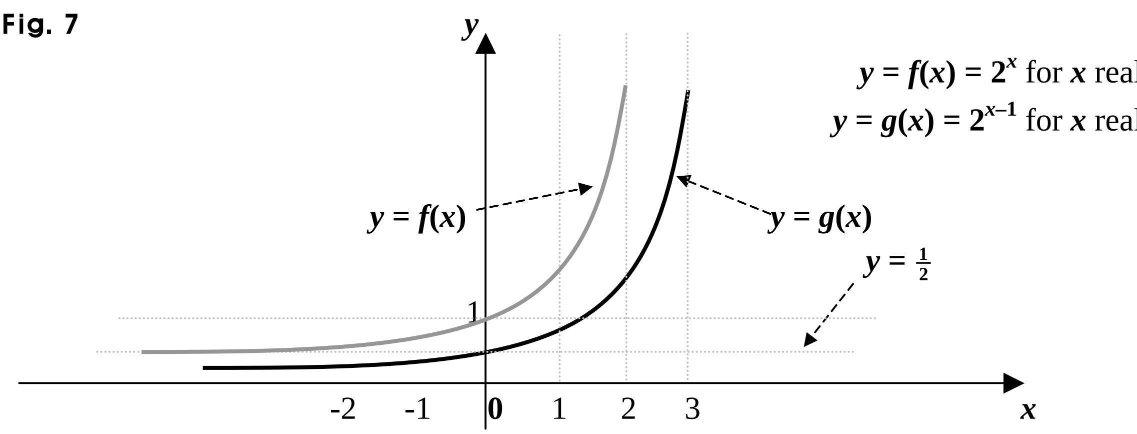
It changes from 0 to 2, of course. And we have $\boldsymbol{y = f(x) = 2^x}$. So when $\boldsymbol{x}$ changes from 0 to 2, the value of $\boldsymbol{f(x)}$ is the same as the value of $\boldsymbol{g(x)}$ when $\boldsymbol{x}$ changes from 1 to 3.

Thus, for instance, $\boldsymbol{f(0) = g(1)}$, $\boldsymbol{f(1) = g(2)}$, and $\boldsymbol{f(2) = g(3)}$.

So when $\boldsymbol{x}$ changes from 1 to 3, the curve made by $\boldsymbol{g(x)}$ is exactly the same as the curve made by $\boldsymbol{f(x)}$ when $\boldsymbol{x}$ changes from 0 to 2, and will behave in the same manner.

And thus, shifting by 1 the entire curve of $\boldsymbol{f}$ in the direction of the $\boldsymbol{x}$-axis, we can get the entire curve of $\boldsymbol{g}$.

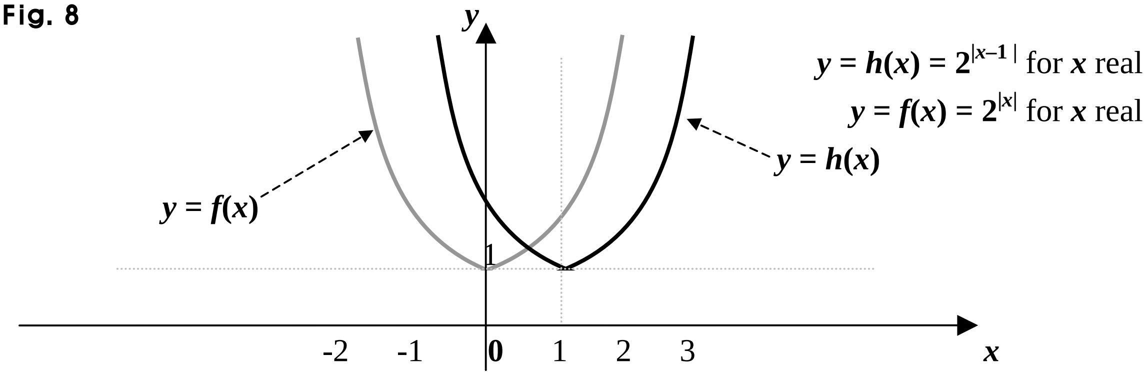
Fig. 7



And the same is true, too, for the curve of a function $\boldsymbol{y = h(x) = 2^{|x-1|}}$.

That is, shifting by 1 the curve of a function $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{x}$-axis, we can readily get the curve of $\boldsymbol{y = h(x) = 2^{|x-1|}}$.

Fig. 8



How then, about the curve of this function: $y = g(x) = 2^{x+1}$?

We can readily get it shifting by -1 the curve of a function $y = f(x) = 2^x$ in the direction of the x-axis. Why?

Suppose in g, the x-value changes from -1 to 2, for instance.

How then, does the value of $(x + 1)$ change?

It changes from 0 to 3, of course. And we have $y = f(x) = 2^x$.

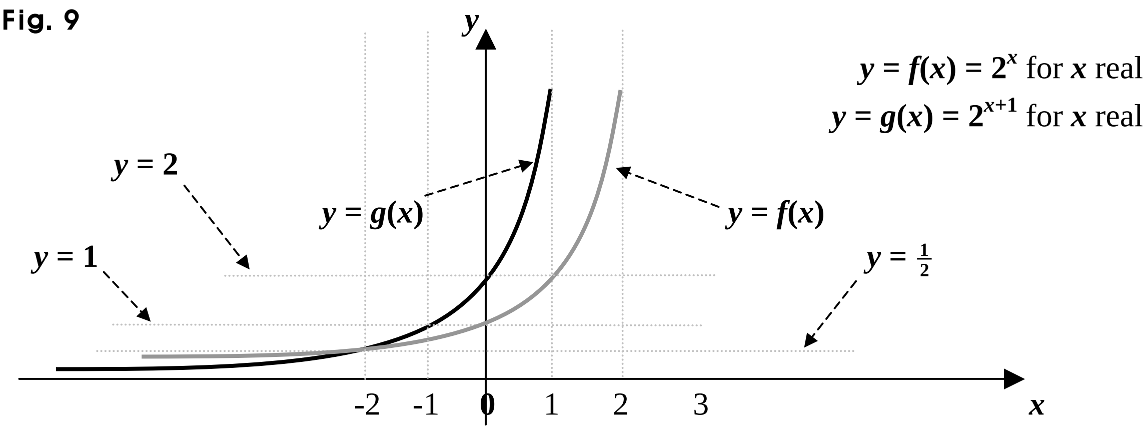
So when x changes from 0 to 3, the value of $f(x)$ is the same as the value of $g(x)$ when x changes from -1 to 2.

Thus, for instance, $f(0) = g(-1)$, $f(1) = g(0)$, and $f(3) = g(2)$.

So when x changes from -1 to 2, the curve made by $g(x)$ is exactly the same as the curve made by $f(x)$ when x changes from 0 to 3, and behaves the same manner.

And thus, shifting by -1 the entire curve of f in the direction of the x-axis, we can get the entire curve of g.

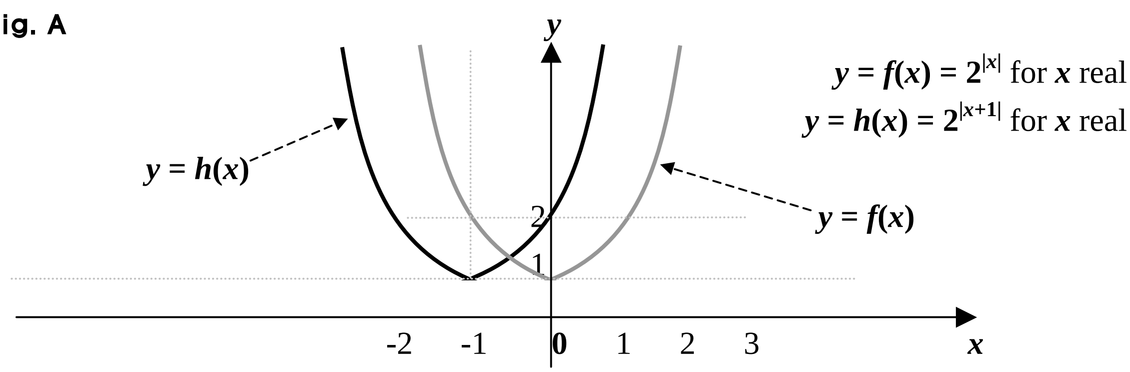
Fig. 9



And the same is true, too, for the curve of another function $y = h(x) = 2^{|x+1|}$.

That is, shifting by -1 the curve of a function $y = f(x) = 2^{|x|}$ in the direction of the x-axis, we can readily get the curve of $y = h(x) = 2^{|x+1|}$.

Fig. A



Next, how about the curve of another function $y = g(x) = 2^x + 1$?

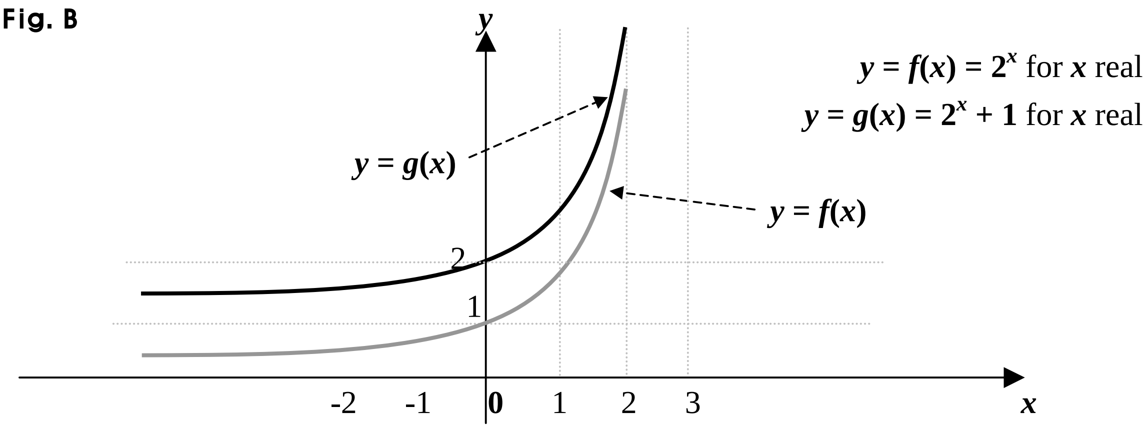
We can readily get it shifting by 1 the curve of a function $y = f(x) = 2^x$ in the direction of the y-axis. Why?

Every time we add 1 to an output for a particular input of the function f, the sum is the output for the same particular input of the function g.

That is, we have $\boldsymbol{g(x) = f(x) + 1}$, because $\boldsymbol{g(x) = 2^x + 1}$ and $\boldsymbol{f(x) = 2^x}$.

And thus, shifting by 1 the curve of $\boldsymbol{f}$ in the direction of the $\boldsymbol{y}$-axis, we can get the curve of $\boldsymbol{g}$.

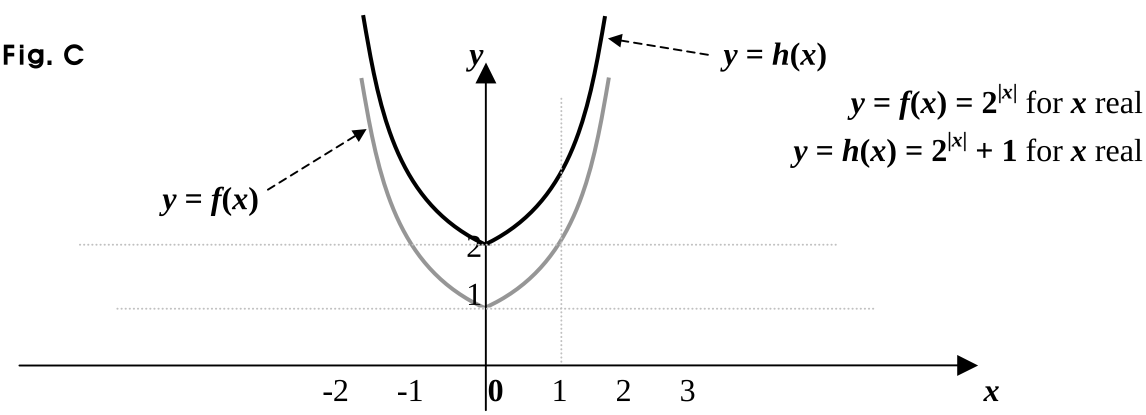
Fig. B



And the same is true, too, for the curve of another function $\boldsymbol{y = h(x) = 2^{|x|} + 1}$.

That is, shifting by 1 the curve of a function $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{y}$-axis, we can readily get the curve of $\boldsymbol{y = h(x) = 2^{|x|} + 1}$.

Fig. C



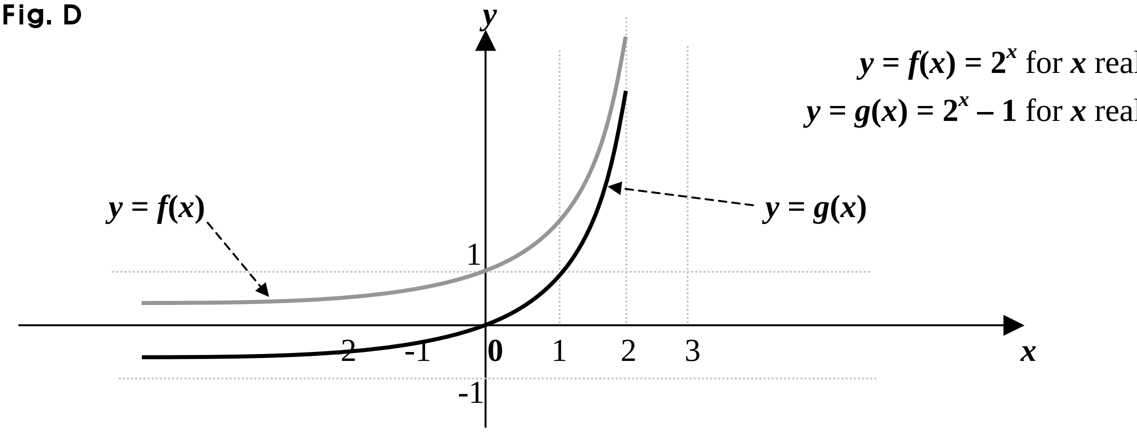
How then, about the curve of this function: $\boldsymbol{y = g(x) = 2^x - 1}$?

We can readily get it shifting by -1 the curve of a function $\boldsymbol{y = f(x) = 2^x}$ in the direction of the $\boldsymbol{y}$-axis.   Why?

Every time we subtract 1 from an output for a particular input of the function $\boldsymbol{f}$, the difference is the output for the same particular input of the function $\boldsymbol{g}$.

That is, we have $\boldsymbol{g(x) = f(x) - 1}$, because $\boldsymbol{g(x) = 2^x - 1}$ and $\boldsymbol{f(x) = 2^x}$. And thus, shifting by -1 the curve of $\boldsymbol{f}$ in the direction of the $\boldsymbol{y}$-axis, we can get the curve of $\boldsymbol{g}$.

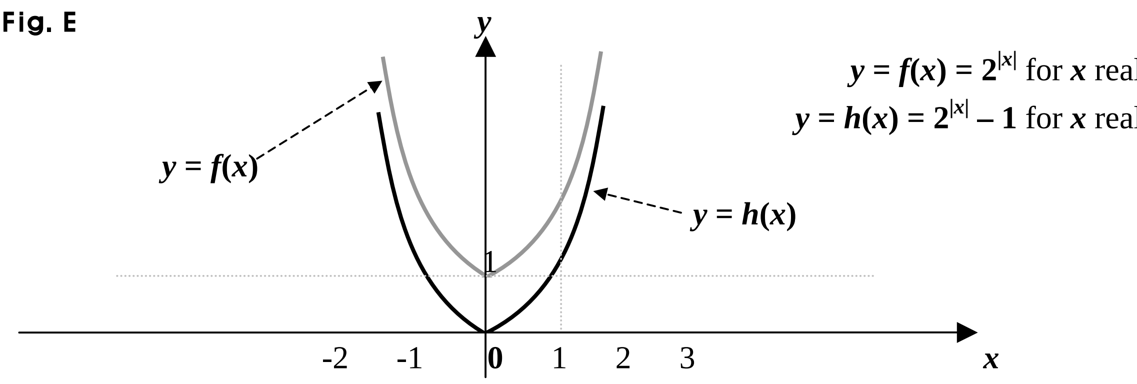
Fig. D



And the same is true, too, for the curve of another function $\boldsymbol{y = h(x) = 2^{|x|} - 1}$.

That is, shifting by -1 the curve of a function $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{y}$-axis, we can readily get the curve of $\boldsymbol{y = h(x) = 2^{|x|} - 1}$.

Fig. E



Next, how about the curve of another function $\boldsymbol{y = u(x) = 2^{x-1} + 1}$?

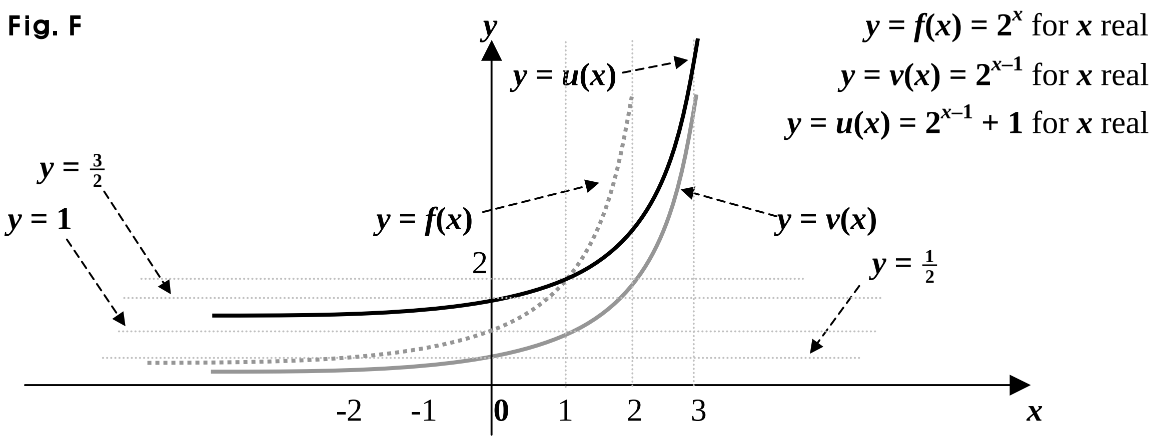
We can readily get it shifting by 1 the curve of a function $\boldsymbol{y = v(x) = 2^{x-1}}$ in the direction of the $\boldsymbol{y}$-axis.   Why?

Every time we add 1 to an output for a particular input of the function $\boldsymbol{v}$, the sum is the output for the same particular input of the function $\boldsymbol{u}$.

That is, we have $\boldsymbol{u(x) = v(x) + 1}$, because $\boldsymbol{u(x) = 2^{x-1} + 1}$ and $\boldsymbol{v(x) = 2^{x-1}}$.
And thus, shifting by 1 the curve of $\boldsymbol{v}$ in the direction of the $\boldsymbol{y}$-axis, we can get the curve of $\boldsymbol{u}$.
And also, we know we can get the curve of $\boldsymbol{v(x) = 2^{x-1}}$ shifting by 1 the curve of a function $\boldsymbol{y = f(x) = 2^x}$ in the direction of the $\boldsymbol{x}$-axis

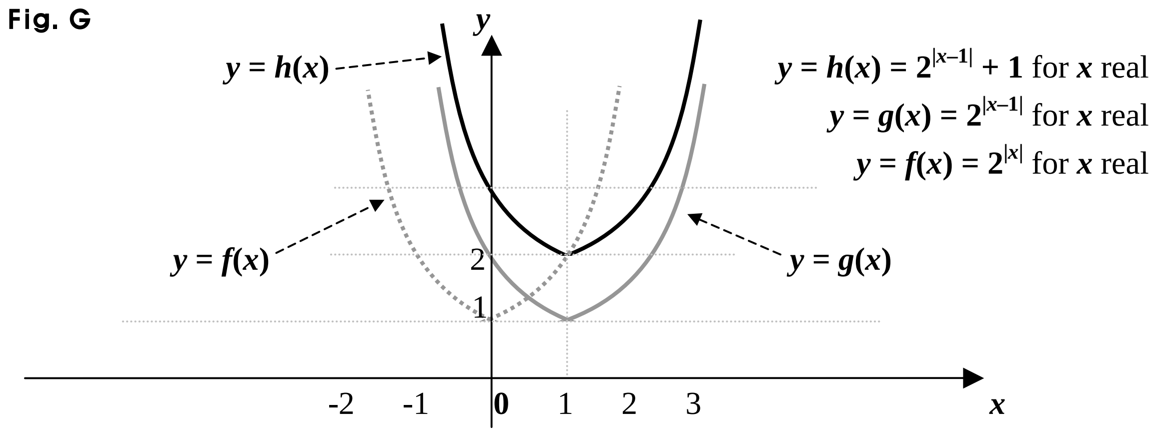
So we can readily get the curve of $\boldsymbol{u(x) = 2^{x-1} + 1}$ shifting by 1 the curve of $\boldsymbol{y = f(x) = 2^x}$ in the direction of the $\boldsymbol{x}$-axis, and then, shifting by 1 in the direction of the $\boldsymbol{y}$-axis.

Fig. F



And the same is true, too, for the curve of another function $\boldsymbol{y = h(x) = 2^{|x-1|} + 1}$. That is, shifting by 1 the curve of $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{x}$-axis, and then, shifting by 1 in the direction of the $\boldsymbol{y}$-axis, we can readily get the curve of $\boldsymbol{y = h(x) = 2^{|x-1|} + 1}$.

Fig. G



How then, about the curve of this function: $y = h(x) = 2^{x-1} - 1$?

We can readily get it shifting by -1 the curve of a function $y = g(x) = 2^{x-1}$ in the direction of the y-axis. Why?

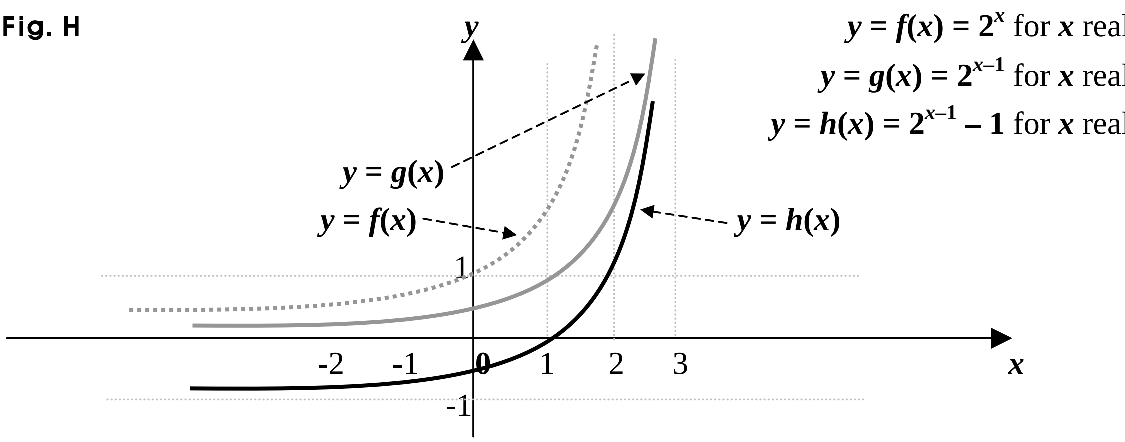
Every time we subtract 1 from the output for a particular input of the function g, the difference is the output for the same particular input of the function h.

That is, we have $h(x) = g(x) - 1$, because $h(x) = 2^{x-1} - 1$ and $g(x) = 2^{x-1}$. And thus, shifting by -1 the curve of g in the direction of the y-axis, we can get the curve of h.

Also, we know we can get the curve of $g(x) = 2^{x-1}$ shifting by 1 the curve of $y = f(x) = 2^x$ in the direction of the x-axis

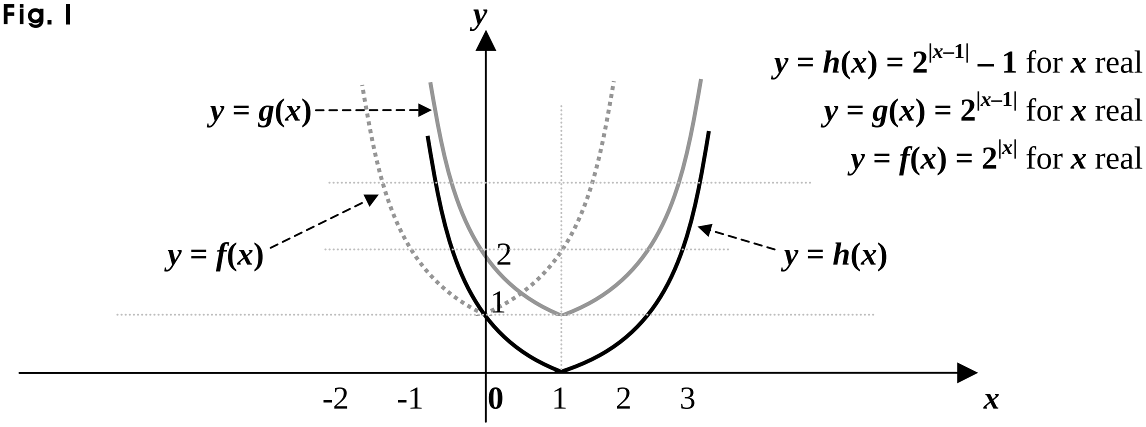
So we can readily get the curve of $h(x) = 2^{x-1} - 1$ shifting by 1 the curve of $y = f(x) = 2^x$ in the direction of the x-axis, and then, shifting by -1 in the direction of the y-axis.

Fig. H



And the same is true, too, for the curve of $y = h(x) = 2^{|x-1|} - 1$. That is, shifting by 1 the curve of $y = f(x) = 2^{|x|}$ in the direction of the x-axis, and then, shifting by -1 in the direction of the y-axis, we can readily get the curve of $y = h(x) = 2^{|x-1|} - 1$.

Fig. I



Next, how about the curve of another function $\boldsymbol{y = h(x) = 2^{|x+1|} - 2}$?

We can readily get it shifting by -2 the curve of a function $\boldsymbol{y = g(x) = 2^{|x+1|}}$ in the direction of the $\boldsymbol{y}$-axis. Why?

Every time we subtract 2 from an output for a particular input of the function $\boldsymbol{g}$, the difference is the output for the same particular input of the function $\boldsymbol{h}$.

That is, we have $\boldsymbol{h(x) = g(x) - 2}$, because $\boldsymbol{h(x) = 2^{|x+1|} - 2}$ and $\boldsymbol{g(x) = 2^{|x+1|}}$.

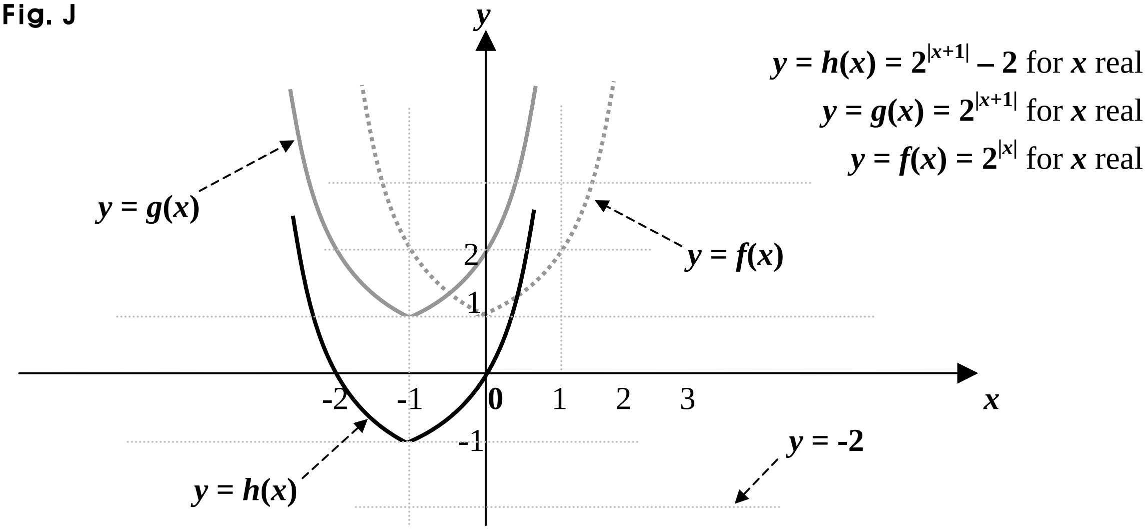
And thus, shifting by -2 the curve of $\boldsymbol{g}$ in the direction of the $\boldsymbol{y}$-axis, we can get the curve of $\boldsymbol{h}$.

And also, we can get the curve of $\boldsymbol{g(x) = 2^{|x+1|}}$ shifting by -1 the curve of $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{x}$-axis.

So shifting by -1 the curve of $\boldsymbol{y = f(x) = 2^{|x|}}$ in the direction of the $\boldsymbol{x}$-axis, and then, shifting by -2 in the direction of the $\boldsymbol{y}$-axis, we can readily get the curve of

$\boldsymbol{y = h(x) = 2^{|x+1|} - 2}$.

Fig. J



Next, how about the curve of another function $y = h(x) = |2^{x+1} - 2|$?

We can readily get it shifting by -2 the curve of a function $y = 2^{x+1}$ in the direction of the y-axis, and then, move symmetrically about the x-axis, the portion of the curve below the x-axis. Why?

We know $|2^{x+1} - 2| \geq 0$ for any value of x.

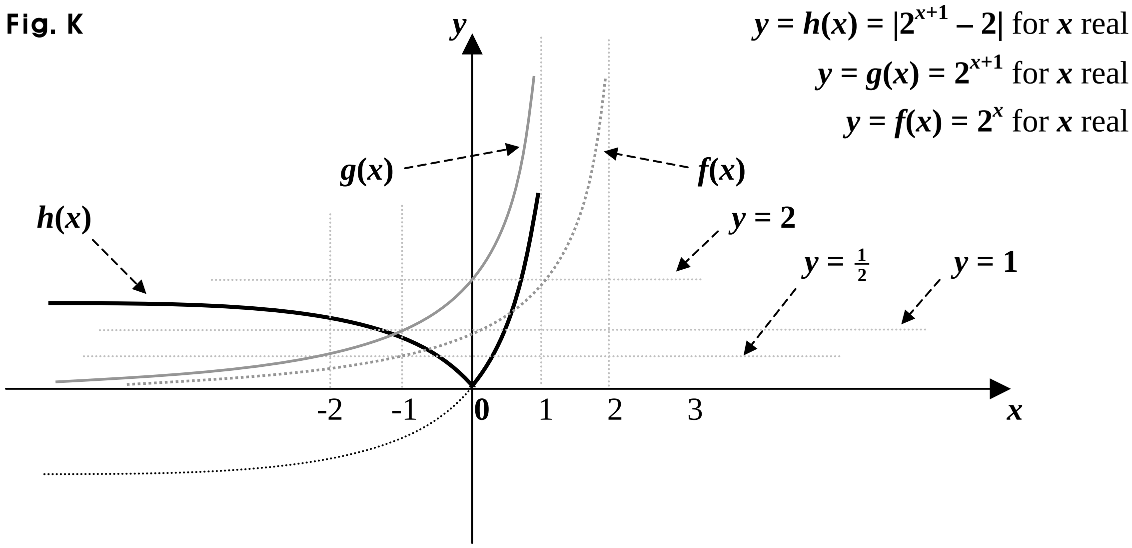
So even though the values of $(2^{x+1} - 2)$ are negative for some values of x, the y-values are positive, that is, the y-coordinates are positive, because we have $y = |2^{x+1} - 2|$.

And thus, no point in the curve of the function $y = h(x) = |2^{x+1} - 2|$ can be below the x-axis.

And in fact, those points that would be below the x-axis will be above the x-axis if there were no absolute sign.

So the curve has to be as shown below.

Fig. K



The curve solid in black is the curve of the function $\boldsymbol{y = h(x) = |2^{x+1} - 2|}$ for $\boldsymbol{x}$ real.

How then, about the curve of another function $\boldsymbol{y = h(x) = |2^{|x+1|} - 2|}$?

We can readily get it shifting by -2 the curve of a function $\boldsymbol{g(x) = 2^{|x+1|}}$ in the direction of the $\boldsymbol{y}$-axis, and then, move symmetrically about the $\boldsymbol{x}$-axis, the portion of the curve below the $\boldsymbol{x}$-axis. Why?

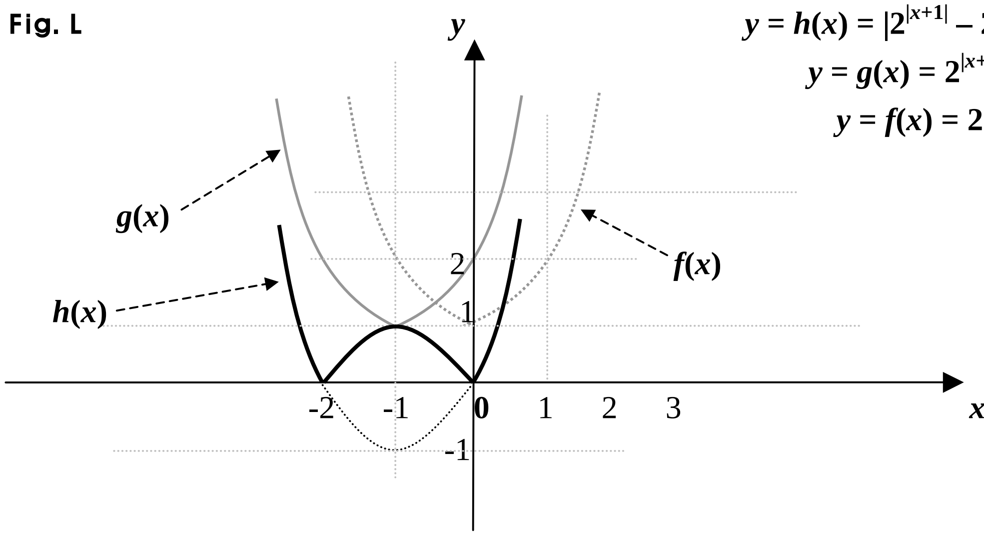
We know $\boldsymbol{|2^{|x+1|} - 2| \geq 0}$.

So even though the values of $(\boldsymbol{2^{|x+1|} - 2})$ are negative for some values of $\boldsymbol{x}$, the $\boldsymbol{y}$-values are positive, that is the $\boldsymbol{y}$-coordinates are positive, because $\boldsymbol{y = |2^{|x+1|} - 2|}$.

So no point in the curve of the function $\boldsymbol{h(x) = |2^{|x+1|} - 2|}$ can be below the $\boldsymbol{x}$-axis.

And therefore, the curve has to be as shown below.

Fig. L



$y = h(x) = |2^{|x+1|} - 2|$ for x real
$y = g(x) = 2^{|x+1|}$ for x real
$y = f(x) = 2^{|x|}$ for x real

The curve solid in black is the curve of the function $h(x) = |2^{|x+1|} - 2|$.

So we can get the curve of a function with absolute sign the way below.

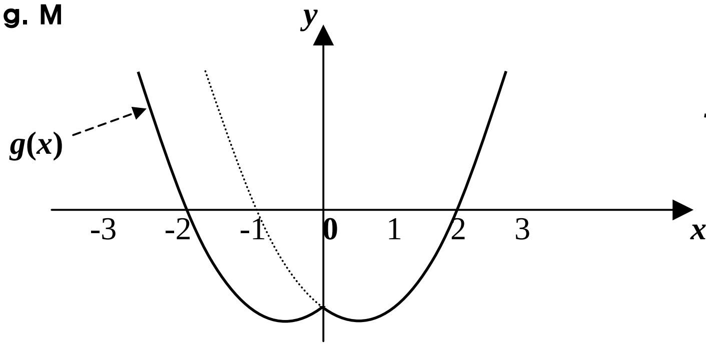
• Suppose we want to get the curve of a function $y = g(x) = |x|^2 - |x| - 2$.

Then first, get the curve of $y = p(x) = x^2 - x - 2$., and then, remove the portion on the left of the origin if such a portion exists, of course.

And next, copy and paste symmetrically about the y-axis, the portion on the right of the origin.

So getting the curve of $y = g(x) = |x|^2 - |x| - 2$, which is equal to $(|x| + 1)(|x| - 2)$, we can quickly get it the way below.

Fig. M



$y = g(x) = |x|^2 - |x| - 2$ for x real

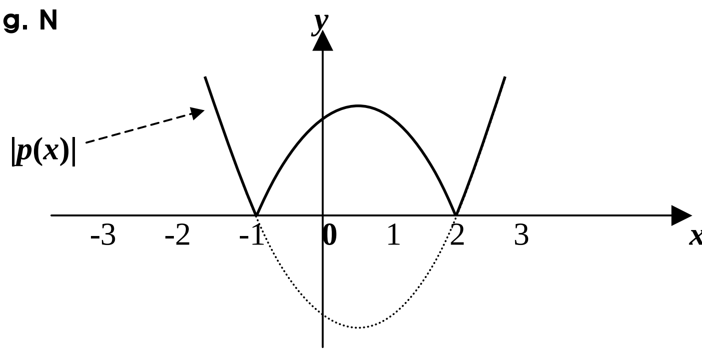
• Suppose next, we want to get the curve of a function $\boldsymbol{y = |p(x)| = |x^2 - x - 2|}$.

Then first, we get the curve of a function $\boldsymbol{y = p(x) = x^2 - x - 2}$.

And next, if the curve has a portion below the $\boldsymbol{x}$-axis, copy and paste the portion symmetrically about the $\boldsymbol{x}$-axis, and then, remove the portion below the $\boldsymbol{x}$-axis.

So we can get the curve of $\boldsymbol{y = |p(x)| = |x^2 - x - 2|}$ the way below.

Fig. N



$\boldsymbol{y = |p(x)| = |x^2 - x - 2|}$ for $\boldsymbol{x}$ real

If however, getting the curve of a function $\boldsymbol{y = h(x) = |x^2 - 2x + 2|}$, we just get the curve of a function $\boldsymbol{y = q(x) = x^2 - 2x + 2}$.

It's because no portion in the curve of $\boldsymbol{y = q(x)}$ is below the $\boldsymbol{x}$-axis. In fact, $\boldsymbol{q(x) = h(x)}$. That is, $\boldsymbol{x^2 - 2x + 2 = |x^2 - 2x + 2|}$, which doesn't need thus, absolute sign.

• Suppose next, we want to get the curve of $\boldsymbol{y = |p(|x|)| = ||x|^2 - 3|x| + 2|}$.

Then, first, get the curve of $\boldsymbol{y = p(x) = x^2 - 3x + 2}$, and then, remove the portion on the left of the origin if such a portion exists, of course.

Then again, in the curve of $\boldsymbol{p}$, copy the portion on the right of the origin, and then, put on the left of the origin, the portion copied.

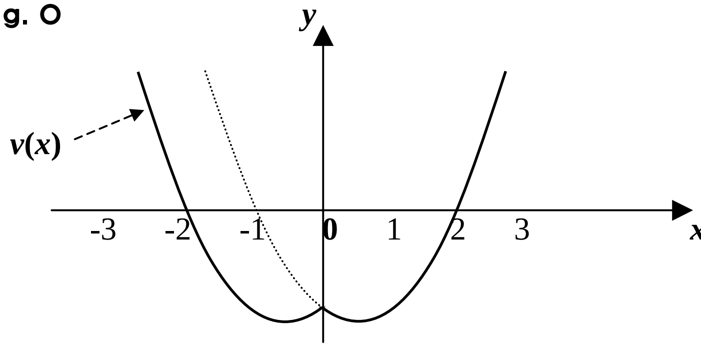
Then, we will get a curve symmetric about the $\boldsymbol{y}$-axis, which is the curve of another function $\boldsymbol{y = p(|x|) = |x|^2 - 3|x| + 2}$.
And next, if the curve of $\boldsymbol{p(|x|)}$ has a portion below the $\boldsymbol{x}$-axis, copy and paste the portion symmetrically about the $\boldsymbol{x}$-axis, and then, remove the portion below the $\boldsymbol{x}$-axis.

Then, we will get the curve we want, which is the curve of the function $|\boldsymbol{p}(|\boldsymbol{x}|)|$.

So for instance, putting in a graph the curve of $\boldsymbol{y} = \boldsymbol{q}(\boldsymbol{x}) = ||\boldsymbol{x}|^2 - |\boldsymbol{x}| - 2|$, which equals $|(|\boldsymbol{x}| + 1)(|\boldsymbol{x}| - 2)|$, we can quickly get the curve of the function $\boldsymbol{q}$ the way below.

Beginning with the curve of $\boldsymbol{y} = \boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{x}^2 - \boldsymbol{x} - 2$, removing the portion on the left of the origin, and then, copying the portion on the right of the origin, and pasting the portion symmetrically about the $\boldsymbol{y}$-axis, we get the curve of a function $\boldsymbol{y} = \boldsymbol{v}(\boldsymbol{x}) = |\boldsymbol{x}|^2 - |\boldsymbol{x}| - 2$, which is shown below.

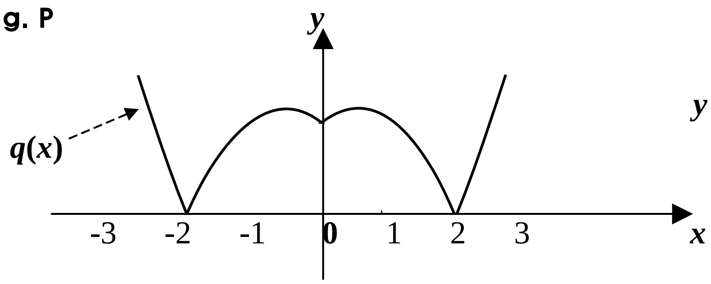
Fig. O



$\boldsymbol{y} = \boldsymbol{v}(\boldsymbol{x}) = |\boldsymbol{x}|^2 - |\boldsymbol{x}| - 2$ for $\boldsymbol{x}$ real

And next, after copying, pasting, and removing the portion below the $\boldsymbol{x}$-axis, we get the curve of the function $\boldsymbol{y} = \boldsymbol{q}(\boldsymbol{x}) = ||\boldsymbol{x}|^2 - |\boldsymbol{x}| - 2|$, which is shown below.

Fig. P



$\boldsymbol{y} = \boldsymbol{q}(\boldsymbol{x}) = ||\boldsymbol{x}|^2 - |\boldsymbol{x}| - 2|$ for $\boldsymbol{x}$ real

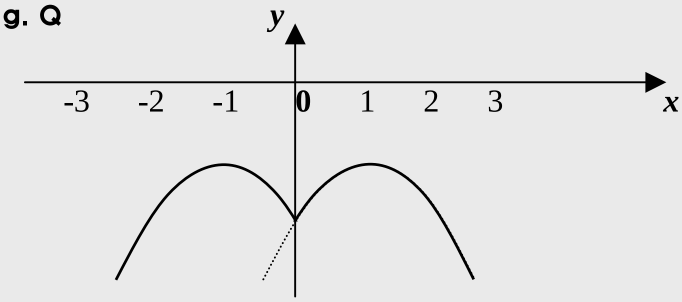
If however, putting in a graph the curve of a function $\boldsymbol{y} = \boldsymbol{s}(\boldsymbol{x}) = ||\boldsymbol{x}|^2 - 2|\boldsymbol{x}| + 2|$, we just get the graph of a function $\boldsymbol{y} = \boldsymbol{t}(\boldsymbol{x}) = |\boldsymbol{x}|^2 - 2|\boldsymbol{x}| + 2$. Why?

The curve of $\boldsymbol{y} = \boldsymbol{t}(\boldsymbol{x})$ does not have any portion below the $\boldsymbol{x}$-axis. In fact, $\boldsymbol{t}(\boldsymbol{x}) = \boldsymbol{s}(\boldsymbol{x})$. That is to say that we have $|\boldsymbol{x}|^2 - 2|\boldsymbol{x}| + 2 = ||\boldsymbol{x}|^2 - 2|\boldsymbol{x}| + 2|$.

• How about then, the curve of a function $y = k(x) = |-|x|^2 + 2|x| - 2|$?

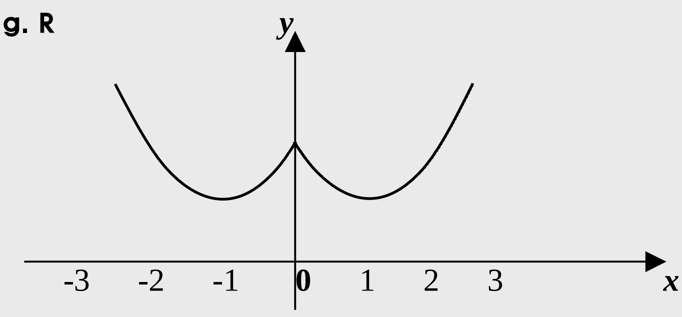
Beginning with a curve of a function $y = f(x) = -x^2 + 2x - 2$, removing the portion on the left of the origin, and then, copying the portion on the right of the origin, and pasting the portion symmetrically about the y-axis, we get the curve of $y = g(x) = -|x|^2 + 2|x| - 2$, and its curve will look like the one shown below.

Fig. Q



So next, since the entire curve above is below the x-axis, we just move the entire curve symmetrically about the x-axis. Then, we will get the curve of $y = k(x) = |-|x|^2 + 2|x| - 2|$, which will look like the one shown below.

Fig. R



We can notice that the curve above is the same as the curve of $y = f(x) = ||x|^2 - 2|x| + 2|$, and is the same as the curve of $y = h(x) = |x|^2 - 2|x| + 2$, too.

That is to say that we have $|-|x|^2 + 2|x| - 2| = ||x|^2 - 2|x| + 2| = |x|^2 - 2|x| + 2$.

So we have $|-|x|^2 + 2|x| - 2| = |x|^2 - 2|x| + 2$.

• What then, about the curve of an equation $|y| = |x|^2 - 2|x| + 2$?

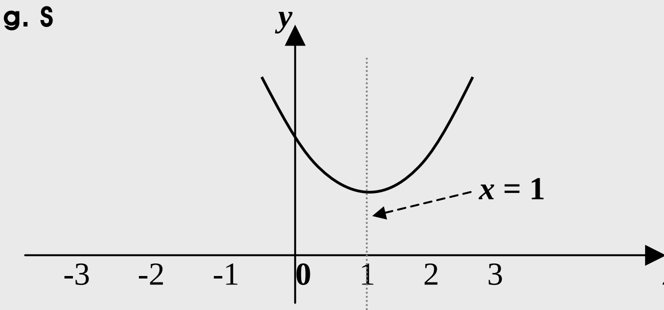
Putting in a graph an equation $y = x^2 - 2x + 2$, we get a parabola.

So let's begin with an equation $|y| = x^2 - 2x + 2$, the curve of which will be made of two parabolas symmetric about the x-axis.

Then, we want to consider two cases, one is a case where $y \geq 0$, and the other is $y < 0$.

So first, if $y \geq 0$, we get $|y| = x^2 - 2x + 2 \Rightarrow y = x^2 - 2x + 2 = (x-1)^2 + 1$, and thus, the curve will be

Fig. S



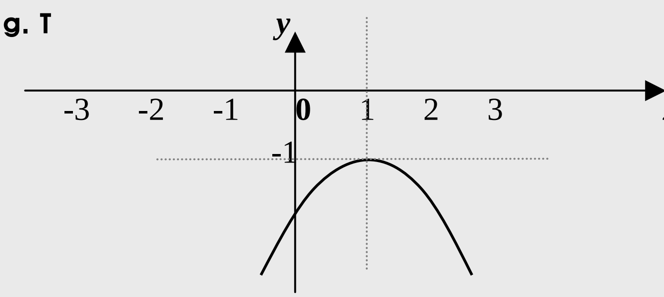
The line $x = 1$ is called the axis of symmetry, so the parabola above is symmetric about the line $x = 1$.

And next, if $y < 0$, we get $|y| = x^2 - 2x + 2 \Rightarrow -y = x^2 - 2x + 2$

$\Rightarrow y = -x^2 + 2x - 2 = -(x-1)^2 - 1$ for $y < 0$.

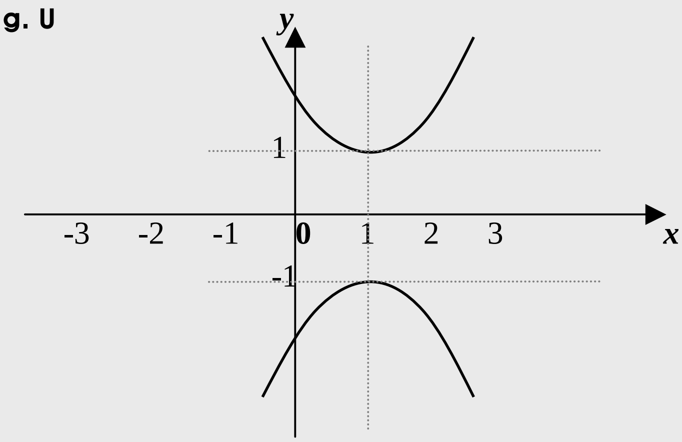
And thus, the curve will be as follows.

Fig. T



And thus, the curve of the equation $|y| = x^2 - 2x + 2$ is as follows.

Fig. U



So the curve is made of two parabolas symmetric about the x-axis.

Let's next, move on to the curve of an equation $y = |x|^2 - 2|x| + 2$.

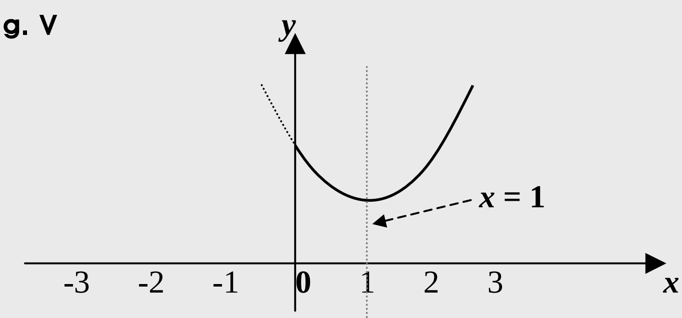
Then again, we want to consider two cases.

This time, one is a case where $x \geq 0$, and the other is $x < 0$.

So first, if $x \geq 0$, we get $y = |x|^2 - 2|x| + 2 \Rightarrow y = x^2 - 2x + 2 = (x - 1)^2 + 1$ for $x \geq 0$.

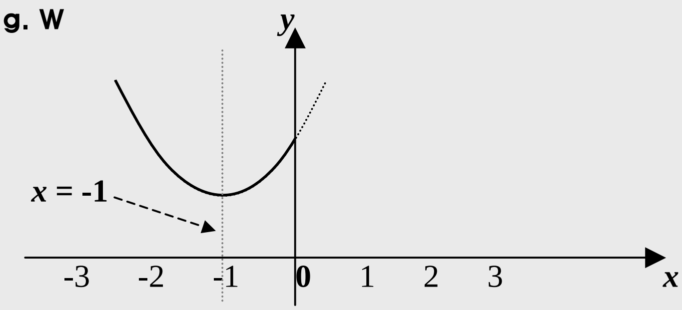
And thus, the curve will be as follows.

Fig. V



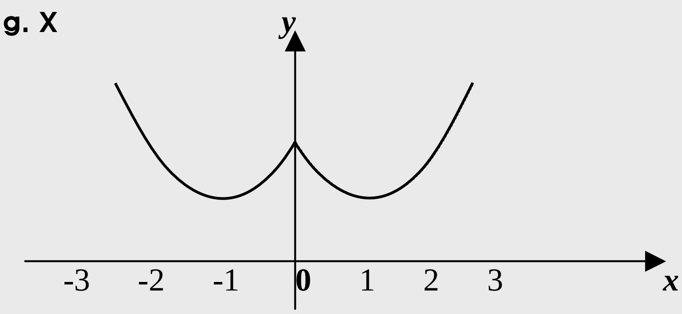
And next, if $x < 0$, we get $y = |x|^2 - 2|x| + 2 \Rightarrow y = x^2 + 2x + 2 = (x + 1)^2 + 1$ for $x < 0$, and thus, the curve will be as below.

Fig. W



And thus, the curve of the equation $\boldsymbol{y = |x|^2 - 2|x| + 2}$ is as follows.

Fig. X



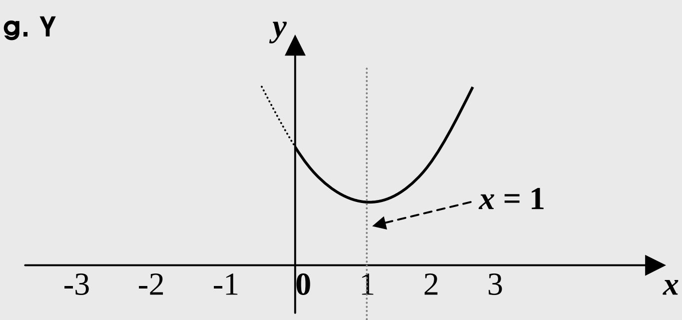
• Now, let's next, move on to the curve of the equation $\boldsymbol{|y| = |x|^2 - 2|x| + 2}$.

Then, unlike the two equations above, we want to consider four cases.

One is a case where $\boldsymbol{y \geq 0}$ and $\boldsymbol{x \geq 0}$, another is a case where $\boldsymbol{y \geq 0}$ and $\boldsymbol{x < 0}$, another is a case where $\boldsymbol{y < 0}$ and $\boldsymbol{x \geq 0}$, and the other is a case where $\boldsymbol{y < 0}$ and $\boldsymbol{x < 0}$.

So first, if $\boldsymbol{y \geq 0}$ and $\boldsymbol{x \geq 0}$, we get $\boldsymbol{|y| = |x|^2 - 2|x| + 2 \Rightarrow y = x^2 - 2x + 2 = (x - 1)^2 + 1}$, and thus, the curve will be as follows.

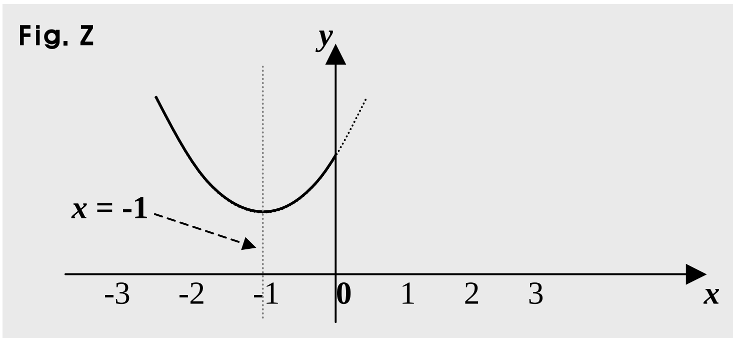
Fig. Y



And next, if $y \geq 0$ and $x < 0$, we get

$|y| = |x|^2 - 2|x| + 2 \Rightarrow y = x^2 + 2x + 2 = (x + 1)^2 + 1$. So the curve will be as below.

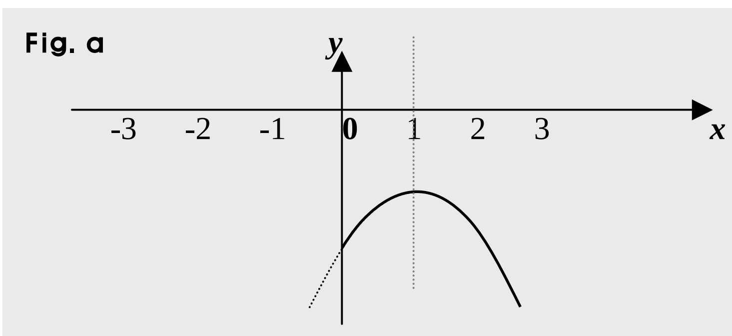
Fig. Z



And next, if $y < 0$ and $x \geq 0$, we get $|y| = |x|^2 - 2|x| + 2 \Rightarrow -y = x^2 - 2x + 2$

$\Rightarrow y = -x^2 + 2x - 2 = -(x - 1)^2 - 1$, and thus, the curve will be as below.

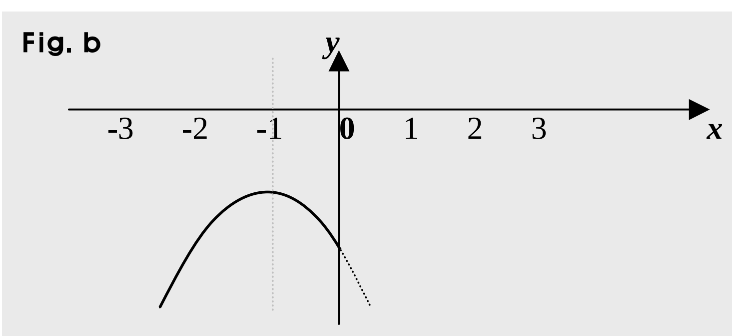
Fig. a



And next, if $y < 0$ and $x < 0$, we get $|y| = |x|^2 - 2|x| + 2 \Rightarrow -y = x^2 + 2x + 2$

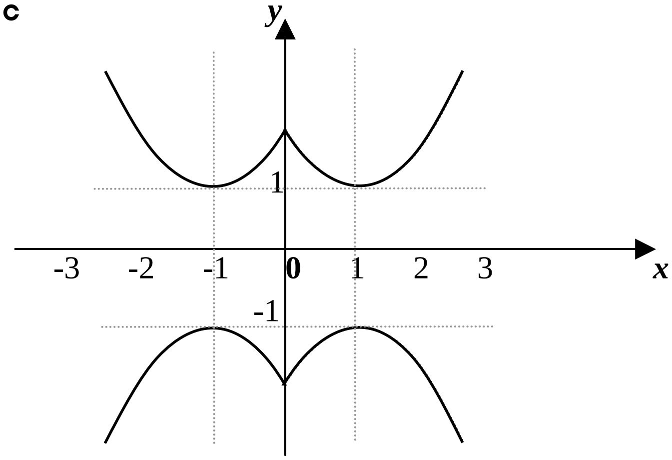
$\Rightarrow y = -x^2 - 2x - 2 = -(x + 1)^2 - 1$, and thus, the curve will be as below.

Fig. b



And thus, the curve of the equation $|y| = |x|^2 - 2|x| + 2$ is as follows.

Fig. c



So the curve above is made of four curves.

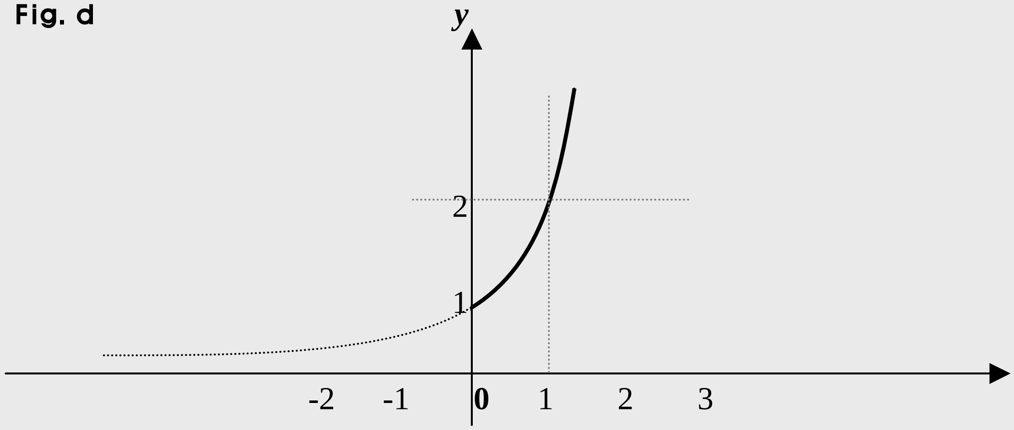
And we can notice that if the curve in the first quadrant is ***C***, the curve in the second quadrant is symmetric to ***C*** about the ***y***-axis, the curve in the fourth quadrant is symmetric to ***C*** about the ***x***-axis, and the curve in the third quadrant is symmetric to ***C*** about the origin.

And the same is true, too, for all the other equations of two variables.

So finding the curve of an equation $|y| = 2^{|x|}$, we want to consider the four cases as explained above.

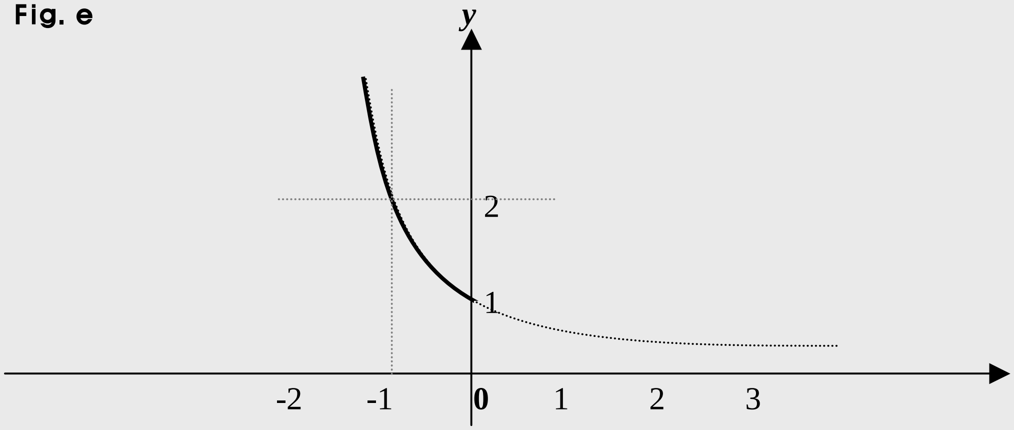
Thus, first, if $y \geq 0$ and $x \geq 0$, we get $|y| = 2^{|x|} \Rightarrow y = 2^x$, and thus, the curve will be

Fig. d



And next, if $\boldsymbol{y} \geq \mathbf{0}$ and $\boldsymbol{x} < \mathbf{0}$, we get $|\boldsymbol{y}| = \mathbf{2}^{|x|} \Rightarrow \boldsymbol{y} = \mathbf{2}^{-x}$, and thus, the curve will be

Fig. e

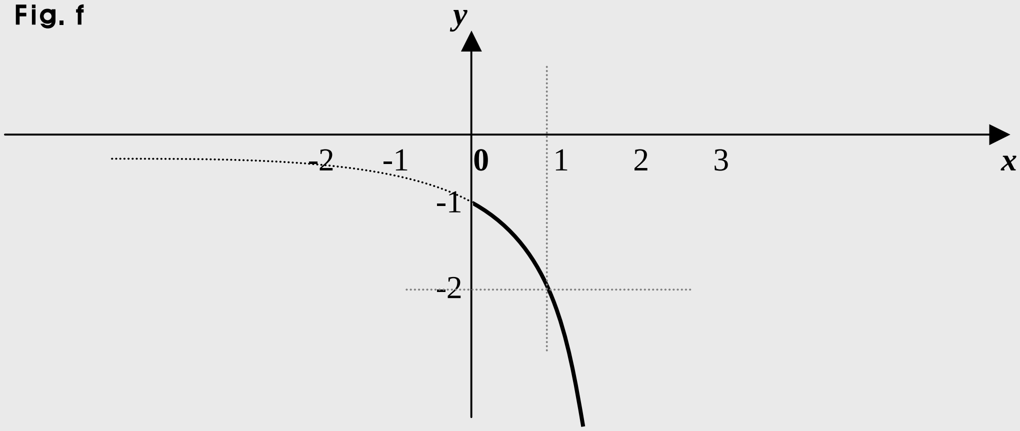


And next, if $\boldsymbol{y} < \mathbf{0}$ and $\boldsymbol{x} \geq \mathbf{0}$, we get

$|\boldsymbol{y}| = \mathbf{2}^{|x|} \Rightarrow -\boldsymbol{y} = \mathbf{2}^{x} \Rightarrow \boldsymbol{y} = -\mathbf{2}^{x}$ for $\boldsymbol{y} < \mathbf{0}$ and $\boldsymbol{x} \geq \mathbf{0}$.

And thus, the curve will be

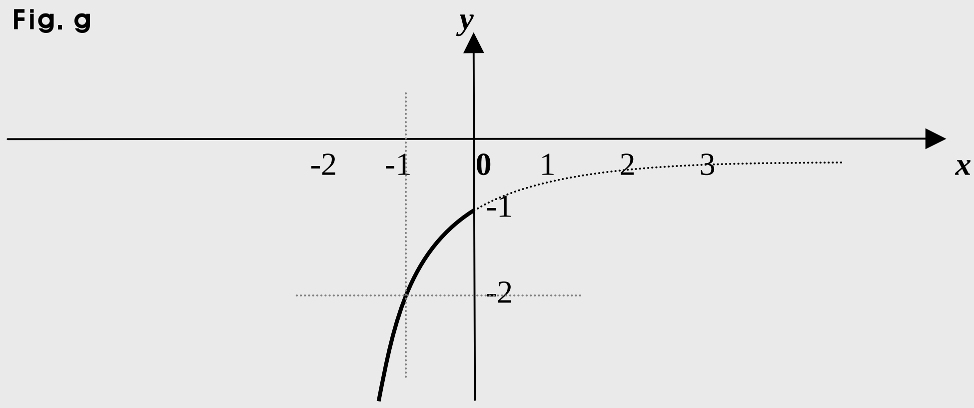
Fig. f



And next, if $\boldsymbol{y} < \mathbf{0}$ and $\boldsymbol{x} < \mathbf{0}$, we get

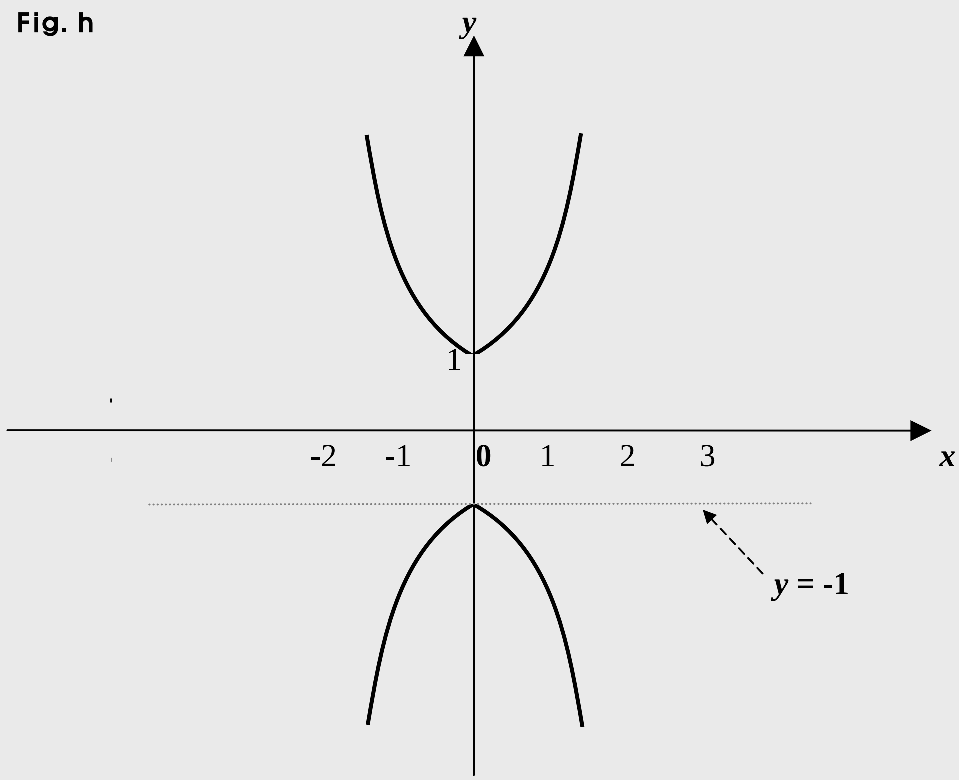
$|\boldsymbol{y}| = \mathbf{2}^{|x|} \Rightarrow -\boldsymbol{y} = \mathbf{2}^{-x} \Rightarrow \boldsymbol{y} = -\mathbf{2}^{-x}$.   And thus, the curve will be as follows.

Fig. g



And thus, the curve of the equation $|y| = 2^{|x|}$ is as follows.

Fig. h



So the curve above is made of four curves.

And if the curve in the first quadrant is ***C***, the curve in the second quadrant is symmetric to ***C*** about the ***y***-axis, the curve in the fourth quadrant is symmetric to ***C*** about the ***x***-axis, and the curve in the third quadrant is symmetric to ***C*** about the origin.