Examples 2 in Exponential & Log Functions

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Construct the graphs of the functions as follows.

0. $y = g(x) = \dfrac{2^x + 2^{-x}}{2}$ for x real.

1. $y = h(x) = \log_2 |x|$.

Suggestions or Solutions
To the Problem in the Example 0

Construct the graph of $y = g(x) = \dfrac{2^x + 2^{-x}}{2}$ for x real.

The given function $g(x)$ is an exponential function. What then, is the base?

We can put the function g this way: $y = g(x) = \frac{1}{2}(2^x + 2^{-x}) = 2^{x-1} + 2^{-x-1}$.

So assuming $g_1(x) = 2^{x-1}$, and $g_2(x) = 2^{-x-1}$, we can take g as the sum of the two functions g_1 and g_2.

And we know $g_2(x) = 2^{-x-1} = 2^{-(x+1)} = (2^{-1})^{x+1}$.

So in the function g_1, the base is 2, and in the function g_2, the base is 2^{-1}.

How then, can we get the curve of the function g?

Setting $f_1(x) = 2^x$, and $f_2(x) = 2^{-x}$, we can notice that of the function $g(x)$, each output is the average of the two outputs from the two functions $f_1(x)$ and $f_2(x)$.

And thus, getting first, the two curves of f_1 and f_2, and next, taking the average of the two curves of f_1 and f_2, we can get the curve of g.   How then, do we get the average?

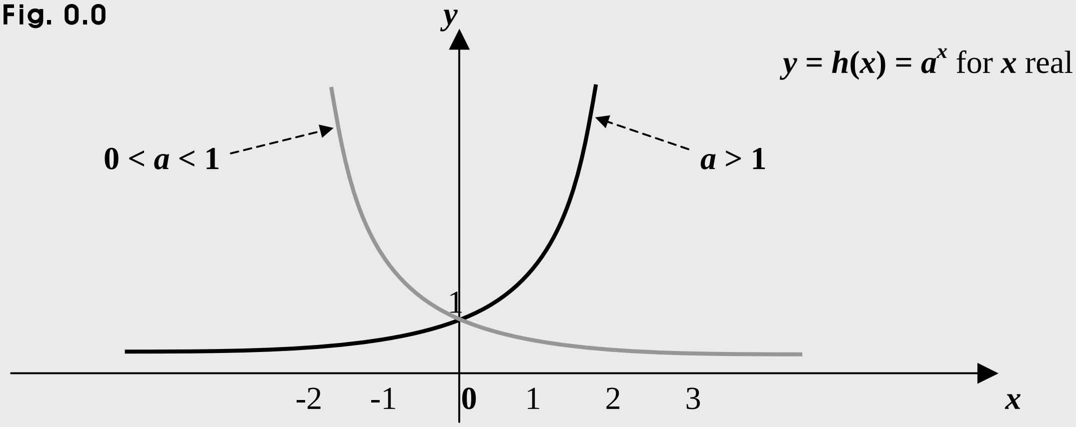
Taking the average of two outputs from f_1 and f_2, for the same input, we get the output of g for the same input. For instance, taking the average of $f_1(1)$ and $f_2(1)$, we get $g(1)$.

So from the two curves of f_1 and f_2, taking the midpoint between two points where the x-coordinates are the same, we can get a point in the curve of g.
And we do the same for all the other x-coordinates, too.

Then, connecting every midpoint of every pair of such two points in the curves of f_1 and f_2, we can get the average, which is the curve of the function g.

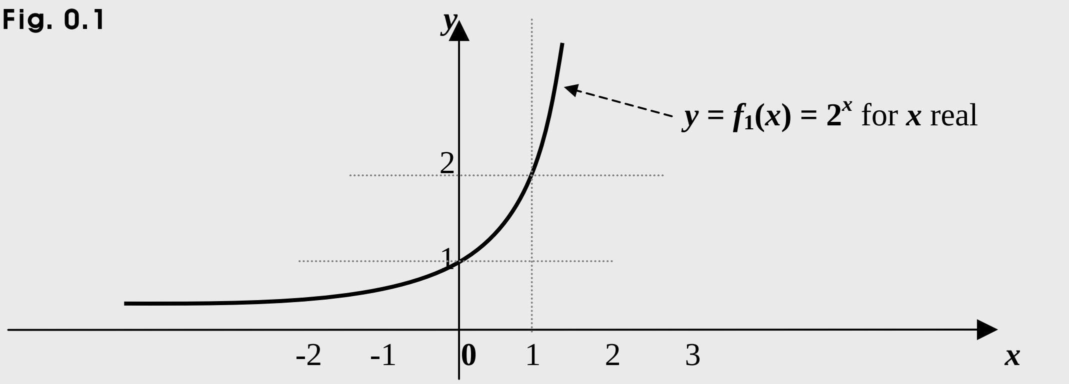
Now, let's to begin with, put in a graph the curve of $\boldsymbol{y = h(x) = a^x}$, first. Then, we get

Fig. 0.0



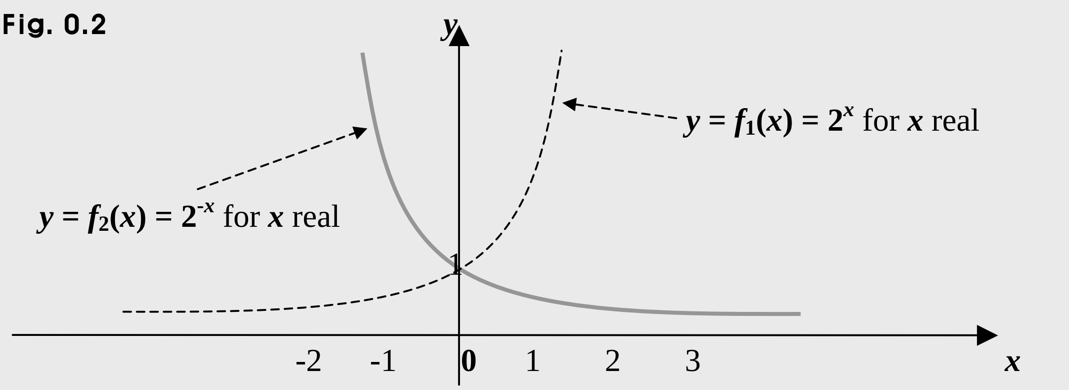
So next, taking 2 as the base $\boldsymbol{a}$, we can see that the curve below is the curve of $\boldsymbol{f_1(x) = 2^x}$.

Fig. 0.1



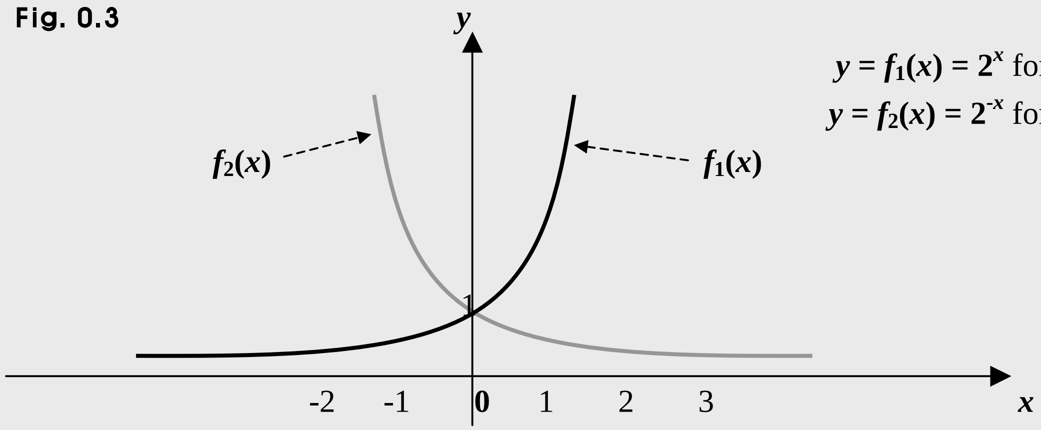
And next, we have $\boldsymbol{f_2(x) = 2^{-x} = (2^{-1})^x}$. So in **Fig. 0.0**, taking 2^{-1} as the base $\boldsymbol{a}$, we can see that the gray curve below is the curve of $\boldsymbol{f_2(x) = 2^{-x}}$.

Fig. 0.2



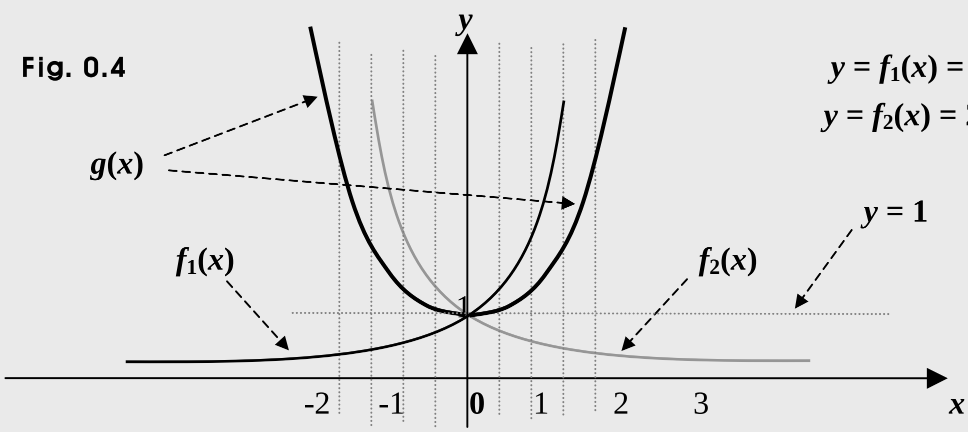
So next, putting in a graph the two curves of $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$, we get

Fig. 0.3



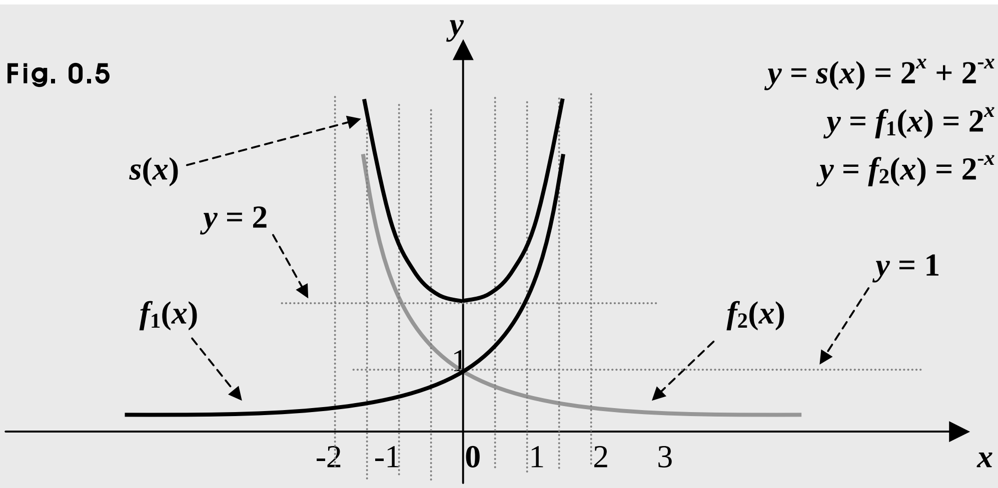
So taking each of all the midpoints, we get the curve of $\boldsymbol{g(x)}$, and the curve is as below.

Fig. 0.4



And if $\boldsymbol{s(x)}$ is the sum of the two functions $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$, the curve of $\boldsymbol{s(x)}$ is as below.

Fig. 0.5



Suggestions or Solutions

To the Problem in the Example 1

Construct the graph of $y = h(x) = \log_2 |x|$.

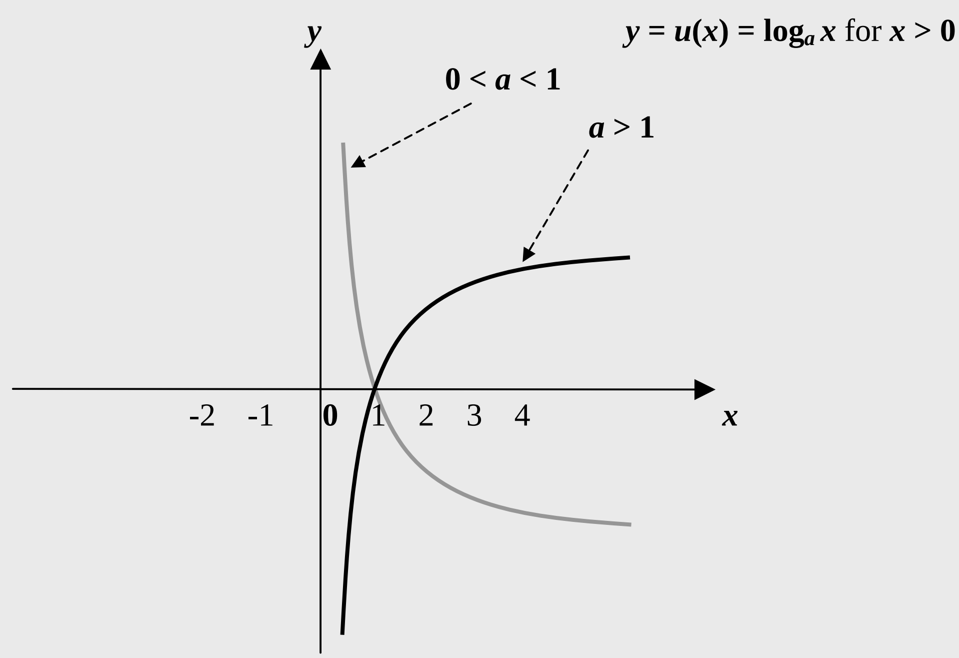
This time, we have a log function, where absolute sign is applied to the input variable. So getting the curve of this function: $y = f(x) = \log_2 x$ first, we can easily get the curve of the function h. To begin with, what is the domain of the function $f(x)$?

In a log, the antilog is positive, and in f, the antilog is x, so we get $x > 0$, which is the domain. If not specified, the domain is the set of all the values the function can be defined. And the log function f can be defined for any x-value positive.
So the domain of f is $x > 0$. And assuming R is a set of all real numbers, we can say that the domain of h is $R - \{0\}$, because the antilog in a log cannot be 0.

So we can put the function h this way, too: $y = h(x) = \log_2 |x|$ for $x \neq 0$.

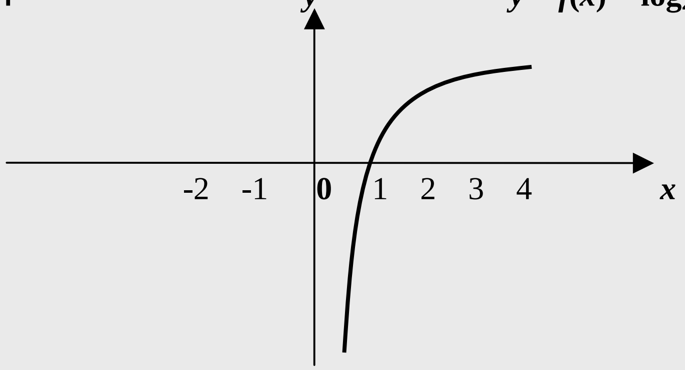
Let's now first, put in a graph, the curve of $y = u(x) = \log_a x$ for $x > 0$. Then, we get

Fig. 1.0



So taking 2 as the base ***a***, we can see that the curve below is the curve of the function ***f***.

Fig. 1.1 $\boldsymbol{y = f(x) = \log_2 x}$ for $\boldsymbol{x > 0}$



So let's now move on to the function $\boldsymbol{y = h(x) = \log_2 |x|}$.

In the **Examples 1**, we have covered the case below.

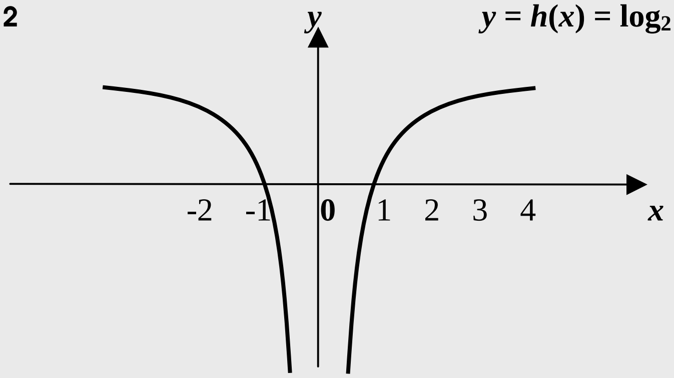
• Suppose we want to get the curve of a function $\boldsymbol{y = g(x) = |x|^2 - |x| - 2}$.

Then first, get the curve of $\boldsymbol{y = p(x) = x^2 - x - 2}$., and then, remove the portion on the left of the origin if such a portion exists, of course.

And next, copy and paste symmetrically about the ***y***-axis, the portion on the right of the origin.

So we can get the curve of $\boldsymbol{y = h(x) = \log_2 |x|}$ the way as follows.

Fig. 1.2 $\boldsymbol{y = h(x) = \log_2 |x|}$ for $\boldsymbol{x \neq 0}$



• What then, about the curve of a function $\boldsymbol{y = k(x) = |\log_2 x|}$?

In the **Examples 1**, we have covered the case below.

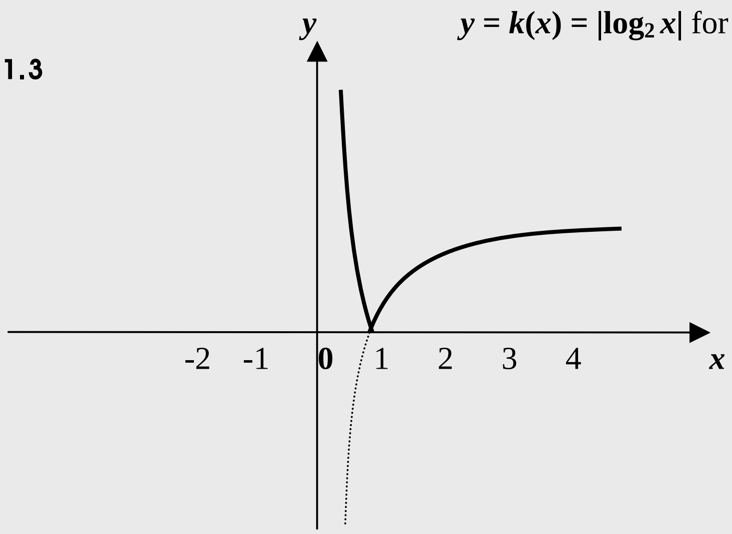
• Suppose we want to get the curve of a function $\boldsymbol{y = |p(x)| = |x^2 - x - 2|}$.

Then first, we get the curve of a function $\boldsymbol{y = p(x) = x^2 - x - 2}$.

And next, if the curve has a portion below the $\boldsymbol{x}$-axis, copy and paste the portion symmetrically about the $\boldsymbol{x}$-axis, and then, remove the portion below the $\boldsymbol{x}$-axis.

So we can get the curve of $\boldsymbol{y = k(x) = |\log_2 x|}$ the way below.

Fig. 1.3



• And let's next, move on to a function $\boldsymbol{y = r(x) = |\log_2 |x||}$.

In the set of **Examples 1**, we have covered the fact below.

• Suppose we want to get the curve of $\boldsymbol{y = |p(|x|)| = ||x|^2 - 3|x| + 2|}$.

Then, first, get the curve of $\boldsymbol{y = p(x) = x^2 - 3x + 2}$, and then, remove the portion on the left of the origin if such a portion exists, of course.

Then again, in the curve of $\boldsymbol{p}$, copy the portion on the right of the origin, and then, put on the left of the origin, the portion copied.

Then, we will get a curve symmetric about the $\boldsymbol{y}$-axis, which is the curve of another function $\boldsymbol{y = p(|x|) = |x|^2 - 3|x| + 2}$.

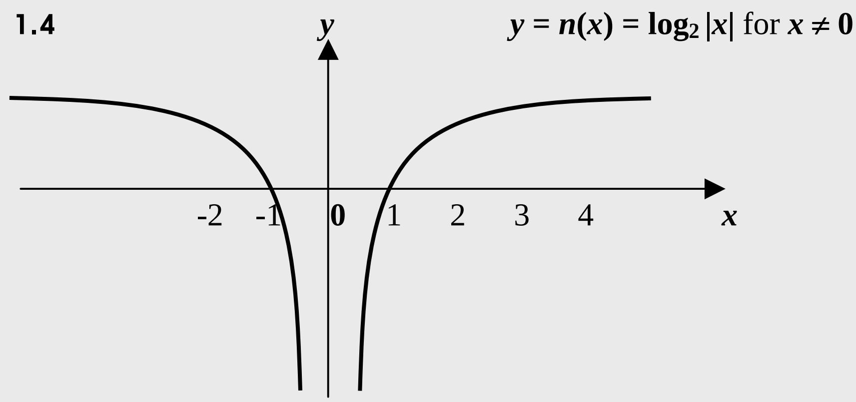
And next, if the curve of $p(|x|)$ has a portion below the x-axis, copy and paste the portion symmetrically about the x-axis, and then, remove the portion below the x-axis.

Then, we will get the curve we want, which is the curve of the function $|p(|x|)|$.

So we can quickly get the curve of $y = r(x) = |\log_2 |x||$, the way below.

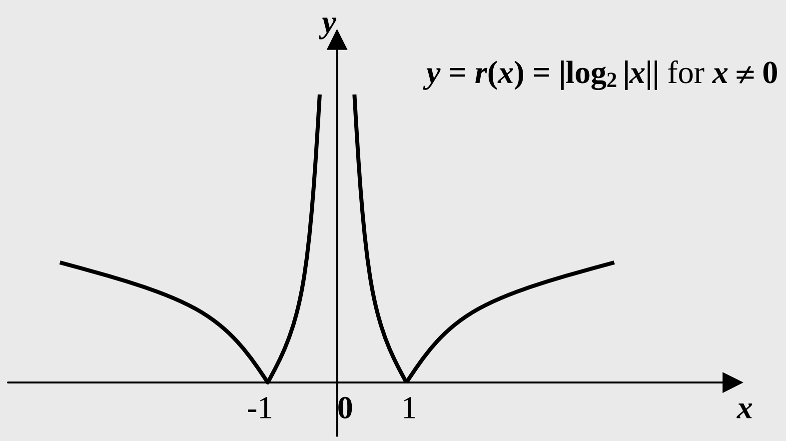
Beginning with the curve of $y = m(x) = \log_2 x$, since no portion on the left of the origin, just copying the curve of m, and pasting it symmetrically about the y-axis, we get the curve of a function $y = n(x) = \log_2 |x|$, which is shown below.

Fig. 1.4



And next, after copying, pasting, and removing the portion below the x-axis, we get the curve of the function $y = r(x) = |\log_2 |x||$, which is shown below.

Fig. 1.5



• How about the curve of $y = g(x) = \log_2 (x - 1)$?

We can readily get it shifting by 1 the curve of a function $y = f(x) = \log_2 x$ in the direction of the x-axis. Why?

Suppose in g, the x-value changes from 2 to 4, for instance.

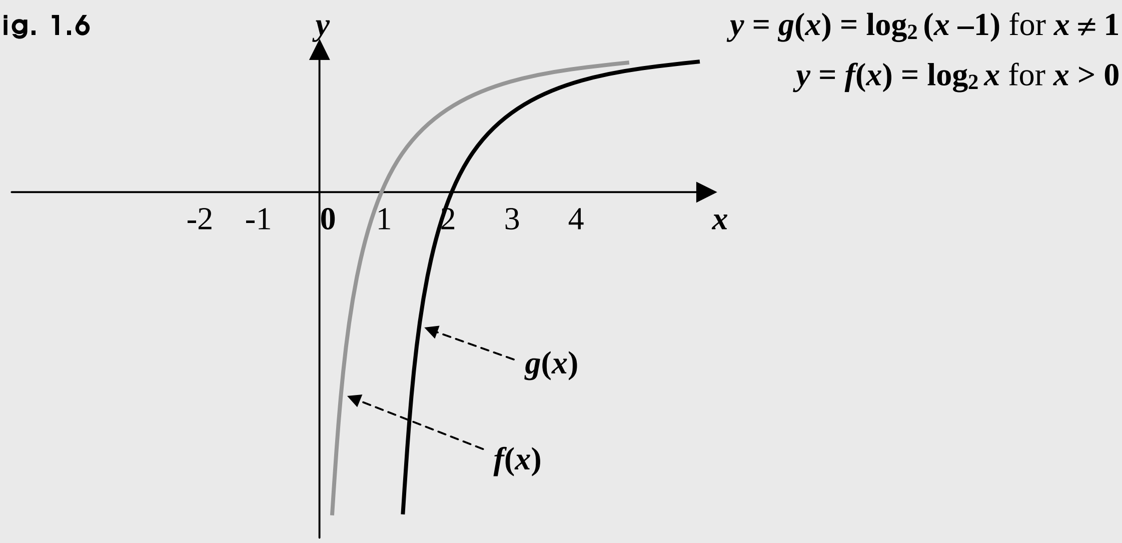
Then, the value of $(x - 1)$ changes from 1 to 3, of course. And we have $y = f(x) = \log_2 x$.

So when x changes from 1 to 3, the value of $f(x)$ is the same as the value of $g(x)$ when x changes from 2 to 4. Thus, for instance, $f(1) = g(2)$, $f(2) = g(3)$, and $f(3) = g(4)$.

So when x changes from 2 to 4, the curve made by $g(x)$ is exactly the same as the curve made by $f(x)$ when x changes from 1 to 3, and behaves in the same manner.

And thus, shifting by 1 the entire curve of f in the direction of the x-axis, we can get the entire curve of g.

Fig. 1.6

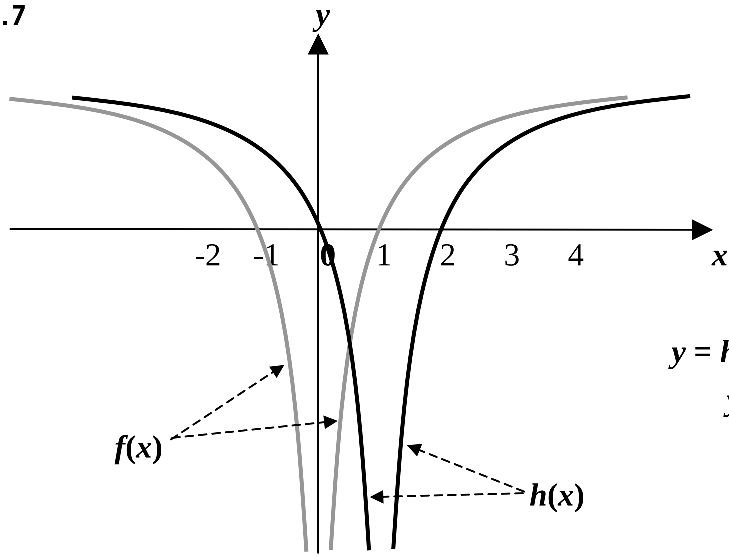


• And the same is true, too, for the curve of a function $y = h(x) = \log_2 |x - 1|$.

That is to say that shifting by 1 the curve of a function $y = f(x) = \log_2 |x|$ in the direction of the x-axis, we can readily get the curve of $y = h(x) = \log_2 |x - 1|$.

So we can quickly get the curve of h the way below.

Fig. 1.7



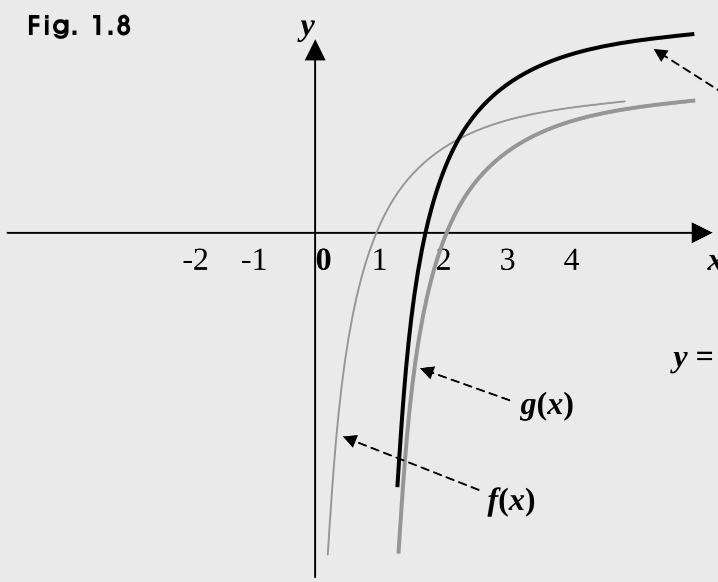
$y = h(x) = \log_2|x-1|$ for $x \neq 1$

$y = f(x) = \log_2|x|$ for $x \neq 0$

• And we can readily get the curve of $y = h(x) = \log_2(x-1) + 1$ shifting by 1 the curve of a function $y = g(x) = \log_2(x-1)$ in the direction of the y-axis. Why?

Every time we add 1 to an output for a particular input in the function g, the sum is the output for the same particular input in the function h. That is, we have $h(x) = f(x) + 1$ because $h(x) = \log_2(x-1) + 1$ and $g(x) = \log_2(x-1)$. And thus, shifting by 1 the curve of g in the direction of the y-axis, we can get the curve of h.

Fig. 1.8



$y = h(x) = \log_2(x-1) + 1$ for $x \neq 0$

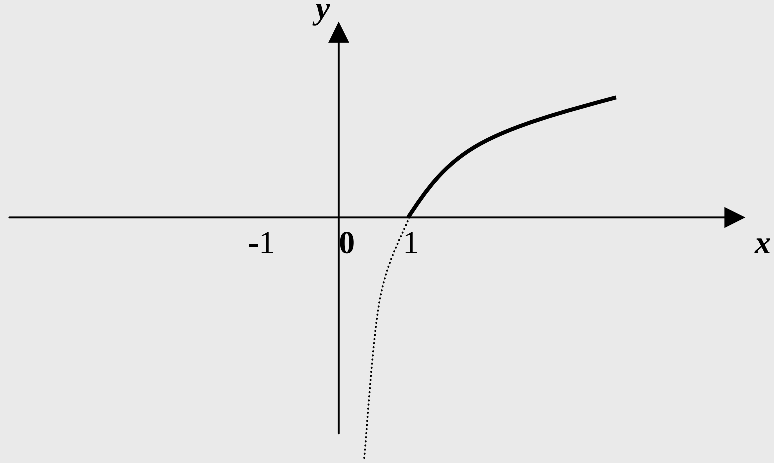
$y = g(x) = \log_2(x-1)$ for $x \neq 1$

$y = f(x) = \log_2 x$ for $x > 0$

• How about the curve of an equation $|y| = \log_2 |x|$?

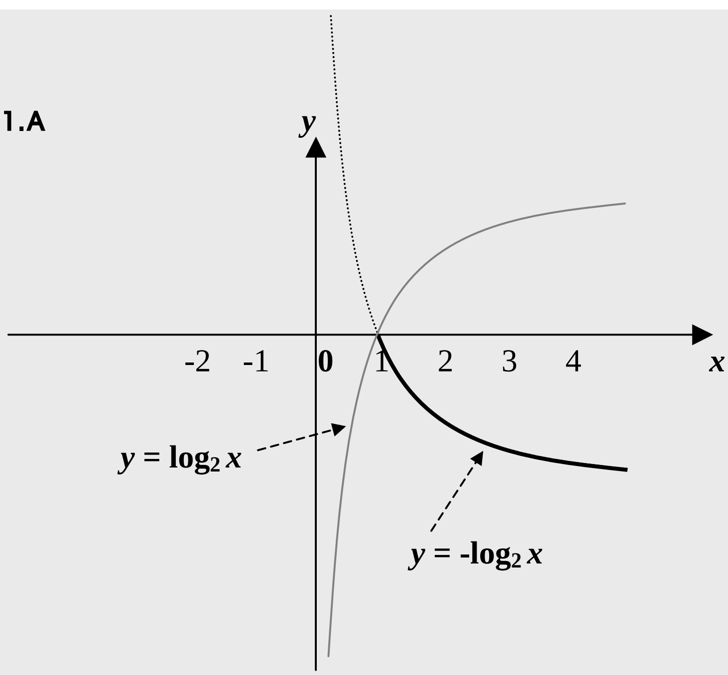
Beginning with an equation $|y| = \log_2 x$, we want to consider two cases, one is a case where $y \geq 0$, and the other is $y < 0$. So first, if $y \geq 0$, we get $|y| = \log_2 x \Rightarrow y = \log_2 x$ for $y \geq 0$ and $x > 0$. and thus, the curve will be

Fig. 1.9



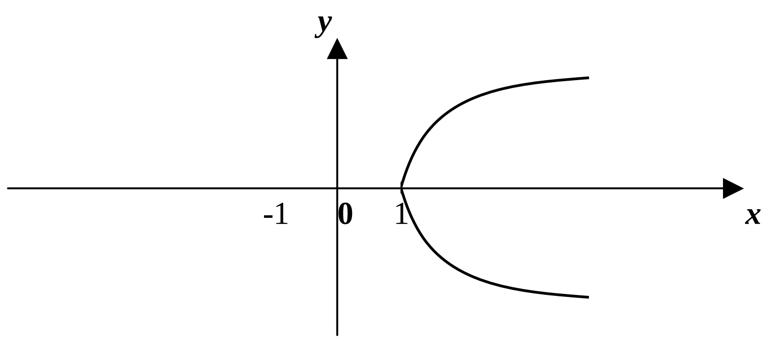
Next, if $y < 0$, we get $|y| = \log_2 x \Rightarrow -y = \log_2 x \Rightarrow y = -\log_2 x$ for $x > 0$. So the curve is

Fig. 1.A



And thus, the curve of the equation $|y| = \log_2 |x|$ is as follows.

Fig. 1.B



- Now, let's next, move on to the curve of the equation $|y| = \log_2 |x|$.

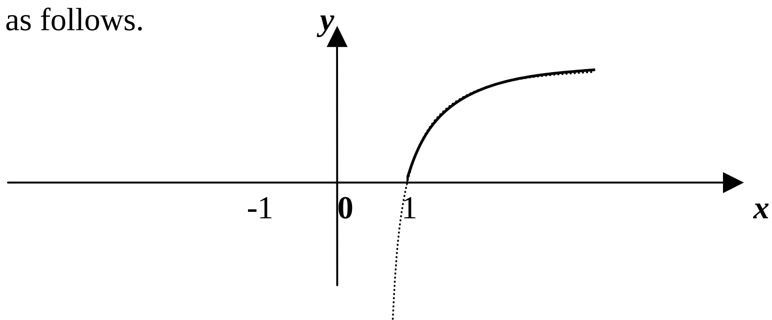
Then, unlike the two equations above, we want to consider four cases.

We know in a log, the antilog is positive.

So one is a case where $y \geq 0$ and $x > 0$, another is a case where $y \geq 0$ and $x < 0$, another is a case where $y < 0$ and $x > 0$, and the other is a case where $y < 0$ and $x < 0$.

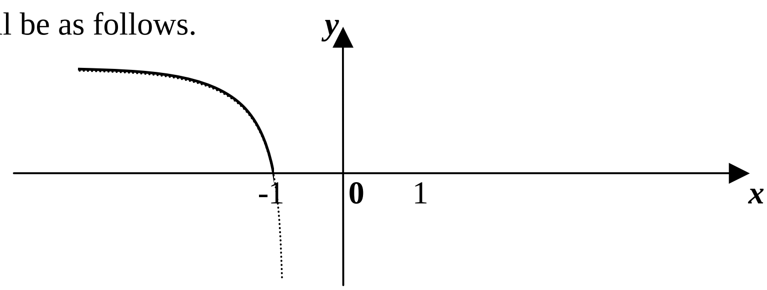
So first, if $y \geq 0$ and $x \geq 0$, we get $|y| = \log_2 |x| \Rightarrow y = \log_2 x$ for $y \geq 0$ and $x > 0$, and thus, the curve will be as follows.

Fig. 1.C



And next, if $y \geq 0$ and $x < 0$, we get $|y| = \log_2 |x| \Rightarrow y = \log_2 (-x)$ for $y \geq 0$ and $x < 0$.
So the curve will be as follows.

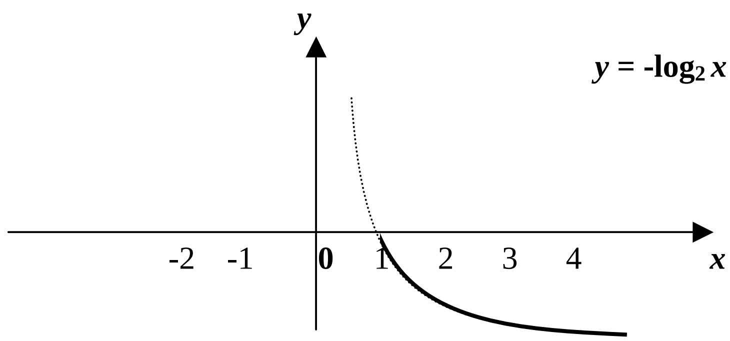
Fig. 1.D



And next, if $y < 0$ and $x > 0$, we get

$|y| = \log_2 |x| \Rightarrow -y = \log_2 x \Rightarrow y = -\log_2 x$ for $y < 0$, and $x > 0$. And thus, the curve will be

Fig. 1.E

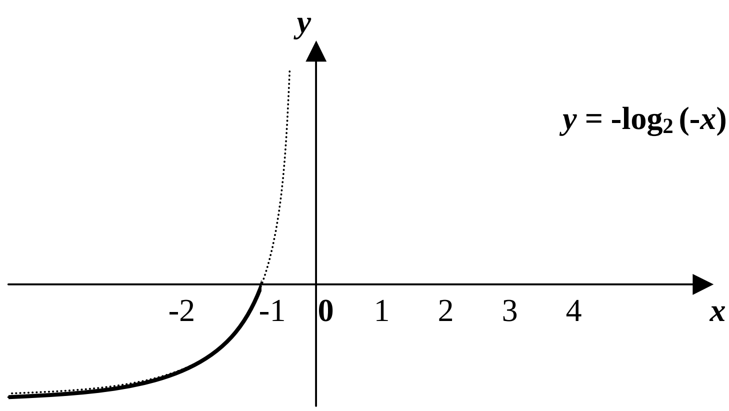


$y = -\log_2 x$ for $y < 0$ and $x > 0$

And next, if $y < 0$ and $x < 0$, we get

$|y| = \log_2 |x| \Rightarrow -y = \log_2 (-x) \Rightarrow y = -\log_2 (-x)$ for $y < 0$ and $x < 0$. So the curve will be

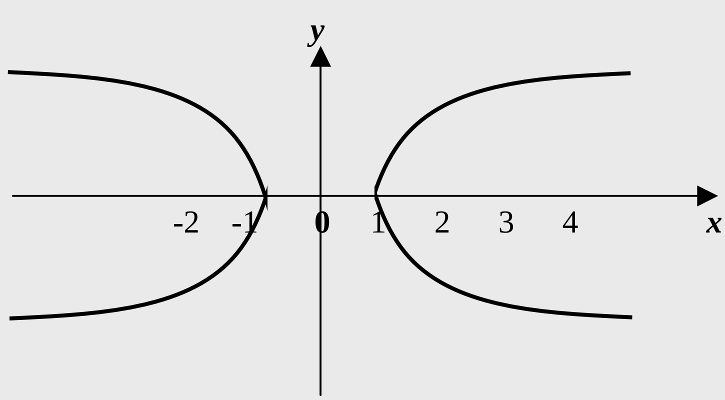
Fig. 1.F



$y = -\log_2 (-x)$ for $y < 0$ and $x < 0$

And thus, the curve of $|y| = \log_2 |x|$ is as follows.

Fig. 1.G

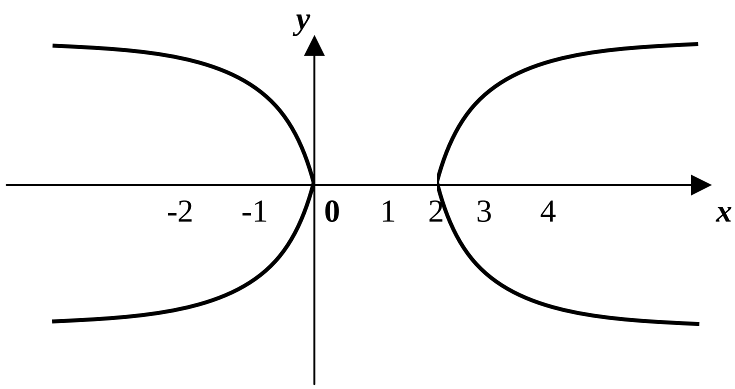


• What about the curve of another equation, $|y| = \log_2 |x + 1|$?

We can readily get it moving (translating) the entire curve above by -1 in the direction of the ***x***-axis. If not quite sure, refer to **Examples 1**.

And by the similar fashion, getting the curve of an equation $|y| = \log_2 |x - 1|$, we can readily get it moving (translating) the entire curve above by 1 in the direction of the ***x***-axis. And the curve is shown below.

Fig. 1.H



• What about the curve of another equation $|y - 1| = \log_2 |x + 1|$?

We can readily get it moving (translating) the entire curve above by -1 in the direction of the ***y***-axis, and the curve is shown below.

(If not sure, refer to **Examples 1** or the section on **Parallel Transformations** in the book, **GRAPH OPERATIONS**.)

Fig. 1.I