

Examples 1 in Lines

Copyright © 2018 by Seong Ryeol Kim. All rights reserved

Click this to start.

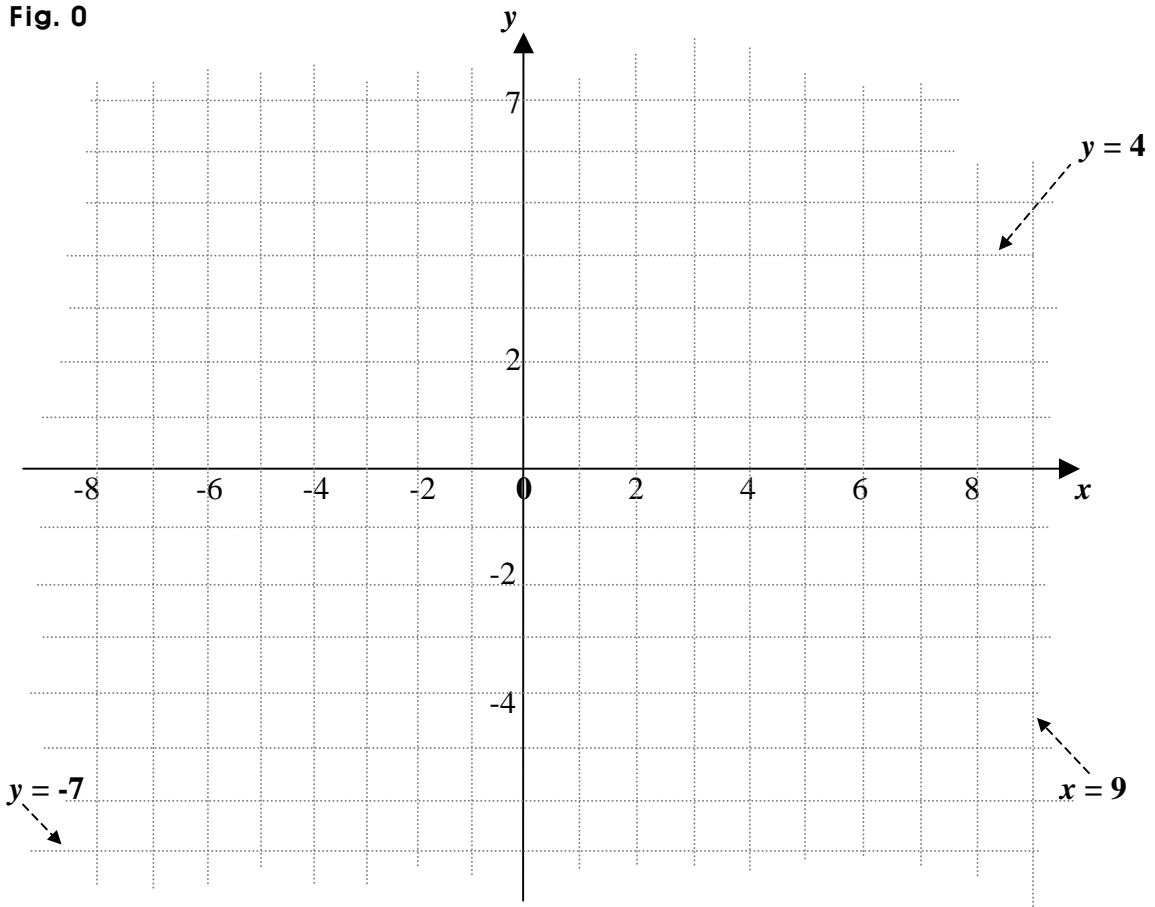
Examples 1 in Lines

Put in a graph, the two points in each case below, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

- 0. (2, 1) and (5, 3) 1. (-2, 1) and (3, 2) 2. (-5, 2) and (-3, 1)
- 3. (-5, -2) and (-1, 3) 4. (-5, -1) and (-3, -7)

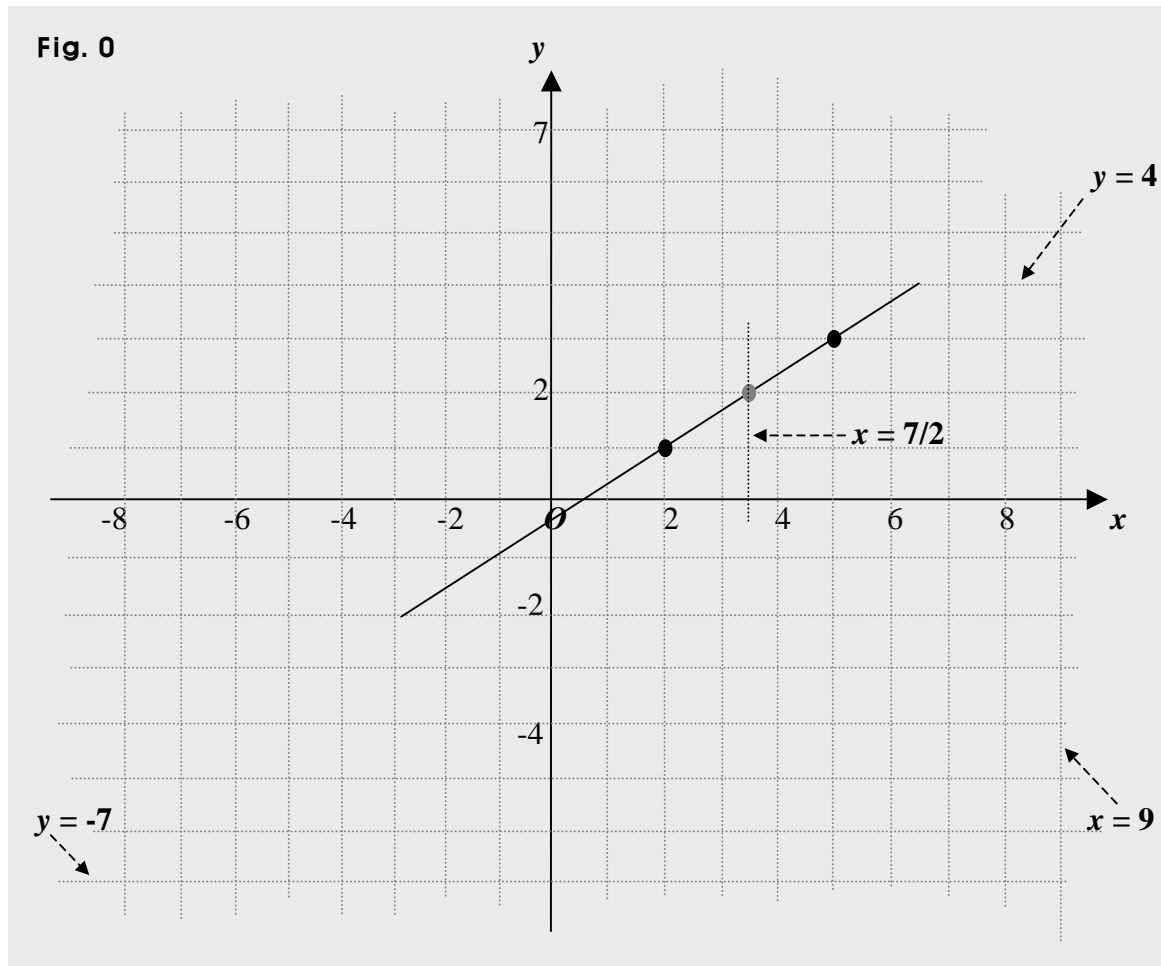
Copy the x - y plane shown below or draw the plane on a copy paper for each case, and do each example above.

Fig. 0



Suggestions or Solutions To the Problem in the Example 0

Put in a graph, two points $(2, 1)$ and $(5, 3)$, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x -coordinate, and t is the average y -coordinate, and finding the midpoint, we get

$$s = (2 + 5)/2 = 7/2, \text{ and } t = (1 + 3)/2 = 2. \text{ So the midpoint is } (7/2, 2).$$

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2) , we get

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So in the case of (2, 1) and (5, 3), we get $d^2 = (2 - 5)^2 + (1 - 3)^2 = 9 + 4 = 13$.

Thus, we get $d = \pm\sqrt{13}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{13}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2) , we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of (2, 1) and (5, 3), we get $a = \frac{1 - 3}{2 - 5} = \frac{-2}{-3} = \frac{2}{3}$.

And next, if a line has a slope of a , and a point (x_1, y_1) , the line is $y - y_1 = a(x - x_1)$.

So using (2, 1), we get $y - 1 = (2/3)(x - 2)$, which is the line, and can be put this way, too: $y - 1 = (2/3)(x - 2) \Rightarrow y = (2/3)x - 4/3 + 1 = (2/3)x - 1/3 \Rightarrow y = (2/3)x - 1/3$.

By the way, setting $x = 0$, we get the y -intercept, so getting the y -intercept, we get

$$x = 0 \Rightarrow y = (2/3)x - 1/3 = 0 - 1/3 = -1/3, \text{ which is the } y\text{-intercept.}$$

Also, setting $y = 0$, we get the x -intercept, so getting the x -intercept, we get

$$0 = (2/3)x - 1/3 \Rightarrow (2/3)x = 1/3 \Rightarrow x = (1/3)(3/2) = \frac{1}{3} \cdot \frac{3}{2} = 1/2, \text{ which is the } x\text{-intercept.}$$

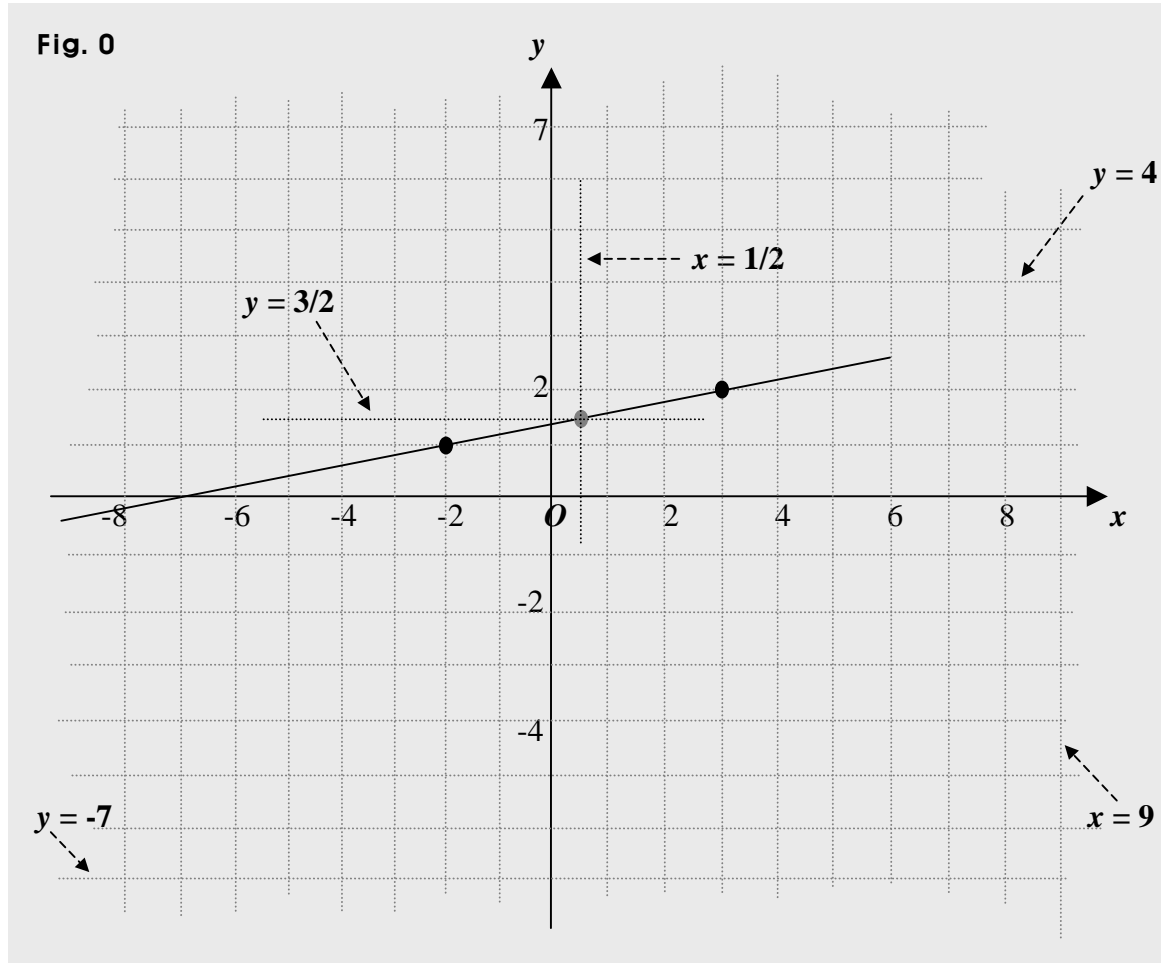
Let's now check to see if the intercept form makes the line using the intercepts.

First, the form is $bx + ay = ab$, where a is the x -intercept, and b is the y -intercept.

So next, we get $(-1/3)x + (1/2)y = (1/2)(-1/3) = -1/6 \Rightarrow 2x - 3y = 1 \Rightarrow y = (2/3)x - 1/3$.

Suggestions or Solutions To the Problem in the Example 1

Put in a graph, two points $(-2, 1)$ and $(3, 2)$, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x -coordinate, and t is the average y -coordinate, and finding the midpoint, we get

$$s = \frac{-2 + 3}{2} = \frac{1}{2}, \text{ and } t = \frac{1 + 2}{2} = \frac{3}{2}. \text{ So the midpoint is } \left(\frac{1}{2}, \frac{3}{2}\right).$$

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2) , we get

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So in the case of $(-2, 1)$ and $(3, 2)$, we get $d^2 = (-2 - 3)^2 + (1 - 2)^2 = 25 + 1 = 26$.

Thus, we get $d = \pm\sqrt{26}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{26}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2) , we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of $(-2, 1)$ and $(3, 2)$, we get $a = \frac{1 - 2}{-2 - 3} = \frac{-1}{-5} = \frac{1}{5}$.

Next, if a line has a slope of a , and a point (x_1, y_1) , the line is $y - y_1 = a(x - x_1)$.

So using $(-2, 1)$, we get $y - 1 = (1/5)(x + 2)$, which is the line, and can be put this way, too: $y - 1 = (1/5)(x + 2) \Rightarrow y = (1/5)x + 2/5 + 1 = (1/5)x + 7/5 \Rightarrow y = (1/5)x + 7/5$.

And next, setting $x = 0$, we get the y -intercept, so getting the y -intercept, we get

$$x = 0 \Rightarrow y = (1/5)x + 7/5 = 0 + 7/5 = 7/5, \text{ which is the } y\text{-intercept.}$$

Also, setting $y = 0$, we get the x -intercept, so getting the x -intercept, we get

$$0 = (1/5)x + 7/5 \Rightarrow (1/5)x = -7/5 \Rightarrow x = (-7/5)(5/1) = \frac{-7}{5} \cdot \frac{5}{1} = -7, \text{ which is the } x\text{-intercept.}$$

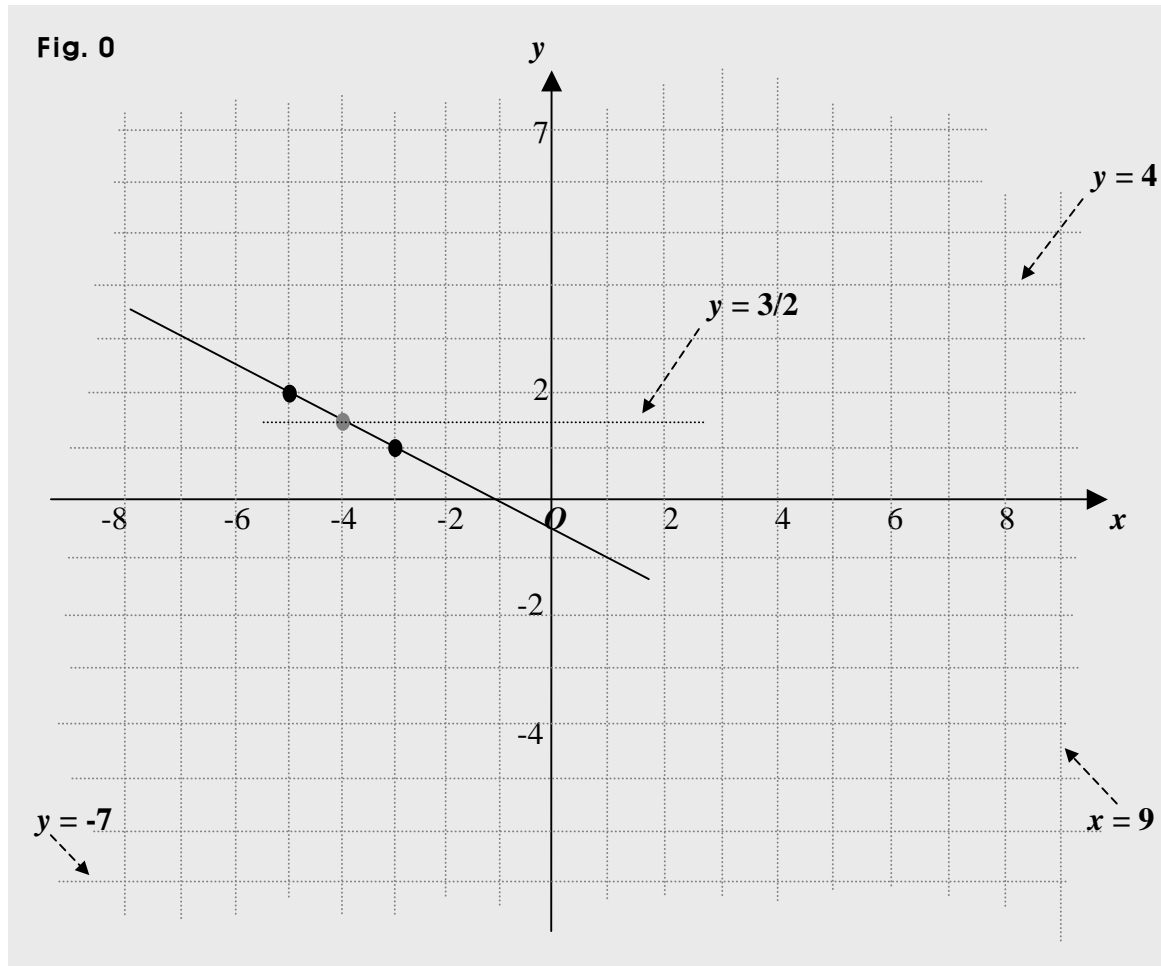
Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $bx + ay = ab$, where a is the x -intercept, and b is the y -intercept.

$$\text{So next, we get } (7/5)x + (-7)y = (-7)(7/5) \Rightarrow 7x - 35y = 49 \Rightarrow y = (1/5)x + 7/5.$$

Suggestions or Solutions To the Problem in the Example 2

Put in a graph, two points $(-5, 2)$ and $(-3, 1)$, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x -coordinate, and t is the average y -coordinate, and finding the midpoint, we get

$$s = \{-5 + (-3)\}/2 = -4, \text{ and } t = (2 + 1)/2 = 3/2. \text{ So the midpoint is } (-4, 3/2).$$

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2) , we get

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So in the case of $(-5, 2)$ and $(-3, 1)$, we get $d^2 = (-5 + 3)^2 + (2 - 1)^2 = 4 + 1 = 5$.

Thus, we get $d = \pm\sqrt{5}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{5}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2) , we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of $(-5, 2)$ and $(-3, 1)$, we get $a = \frac{2-1}{-5-(-3)} = -\frac{1}{2}$.

Next, if a line has a slope of a , and a point (x_1, y_1) , the line is $y - y_1 = a(x - x_1)$.

So using $(-3, 1)$, we get $y - 1 = (-1/2)(x + 3)$, which is the line, and can be put this way, too: $y - 1 = (-1/2)(x + 3) \Rightarrow y = (-1/2)x - 3/2 + 1 = (-1/2)x - 1/2 \Rightarrow y = (-1/2)x - 1/2$.

And next, setting $x = 0$, we get the y -intercept, so getting the y -intercept, we get

$$x = 0 \Rightarrow y = (-1/2)x - 1/2 = 0 - 1/2 = -1/2, \text{ which is the } y\text{-intercept.}$$

Also, setting $y = 0$, we get the x -intercept, so getting the x -intercept, we get

$$0 = (-1/2)x - 1/2 \Rightarrow (1/2)x = -1/2 \Rightarrow x = -1, \text{ which is the } x\text{-intercept.}$$

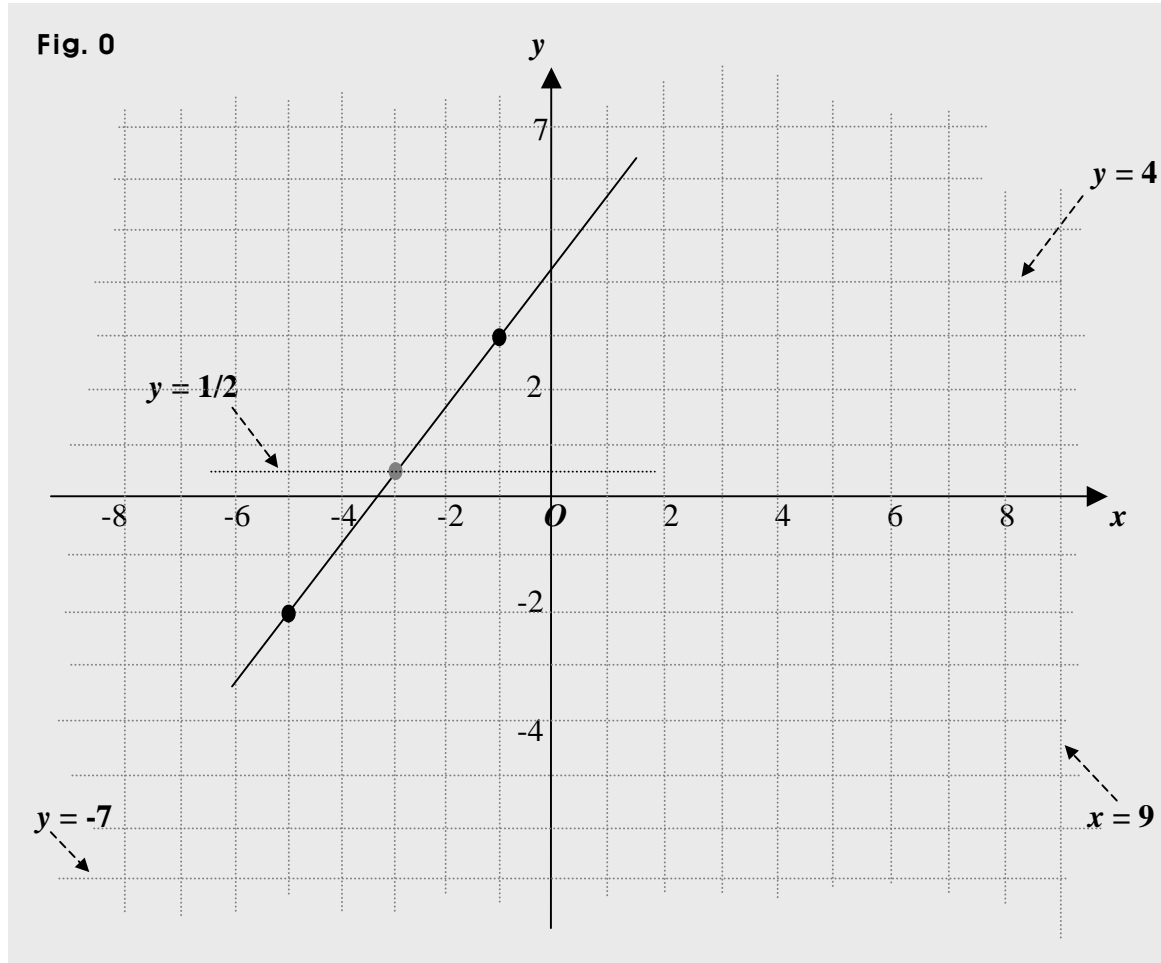
Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $bx + ay = ab$, where a is the x -intercept, and b is the y -intercept.

So next, we get $(-1/2)x + (-1)y = (-1)(-1/2) \Rightarrow x + 2y = -1 \Rightarrow y = (-1/2)x - 1/2$.

Suggestions or Solutions To the Problem in the Example 3

Put in a graph, two points $(-5, -2)$ and $(-1, 3)$, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x -coordinate, and t is the average y -coordinate, and finding the midpoint, we get

$$s = \{-5 + (-1)\}/2 = -3, \text{ and } t = (-2 + 3)/2 = 1/2. \text{ So the midpoint is } (-3, 1/2).$$

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2) , we get

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So in the case of $(-5, -2)$ and $(-1, 3)$, we get $d^2 = (-5 + 1)^2 + (-2 - 3)^2 = 16 + 25 = 41$.

Thus, we get $d = \pm\sqrt{41}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{41}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2) , we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of $(-5, -2)$ and $(-1, 3)$, we get $a = \frac{-2 - 3}{-5 - (-1)} = \frac{-5}{-4} = \frac{5}{4}$.

Next, if a line has a slope of a , and a point (x_1, y_1) , the line is: $y - y_1 = a(x - x_1)$.

So using $(-1, 3)$, we get $y - 3 = (5/4)(x + 1)$, which is the line, and can be put this way, too: $y - 3 = (5/4)(x + 1) \Rightarrow y = (5/4)x + 5/4 + 3 = (5/4)x + 17/4 \Rightarrow y = (5/4)x + 17/4$.

And next, setting $x = 0$, we get the y -intercept, so getting the y -intercept, we get

$$x = 0 \Rightarrow y = (5/4)x + 17/4 = 0 + 17/4 = 17/4, \text{ which is the } y\text{-intercept.}$$

Also, setting $y = 0$, we get the x -intercept, so getting the x -intercept, we get

$$0 = (5/4)x + 17/4 \Rightarrow 5x = -17 \Rightarrow x = -\frac{17}{5}, \text{ which is the } x\text{-intercept.}$$

Let's now check to see if we get the line putting the intercepts into the intercept form.

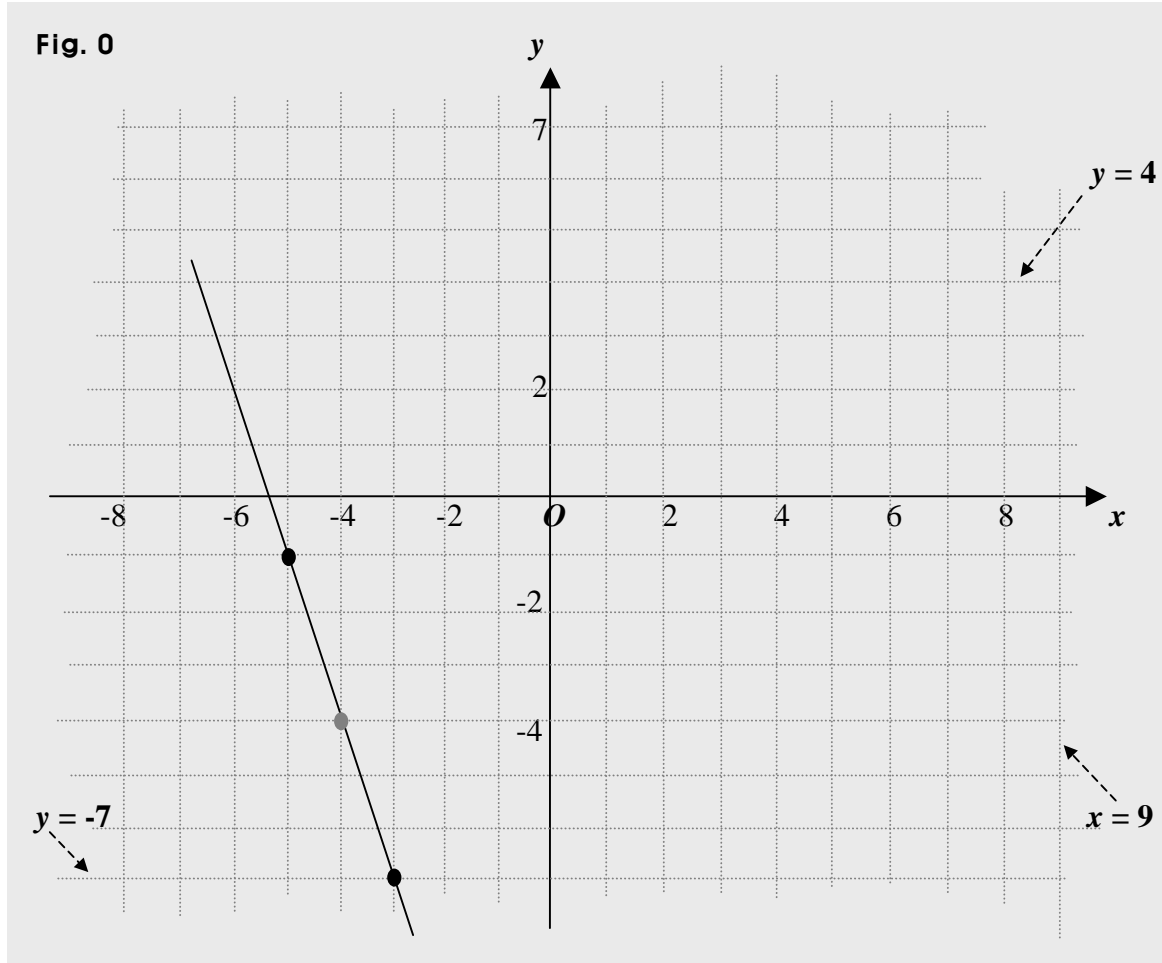
First, the form is $bx + ay = ab$, where a is the x -intercept, and b is the y -intercept.

$$\text{So next, we get } (17/4)x + (-17/5)y = (-17/5)(17/4) \Rightarrow x/4 - y/5 = -17/20$$

$$\Rightarrow 5x - 4y = -17 \Rightarrow y = (5/4)x + 17/4.$$

Suggestions or Solutions
To the Problem in the Example 4

Put in a graph, two points $(-5, -1)$ and $(-3, -7)$, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x -coordinate, and t is the average y -coordinate, and finding the midpoint, we get

$$s = \frac{-5 - 3}{2} = -4, \text{ and } t = \frac{-1 - 7}{2} = -4. \text{ So the midpoint is } (-4, -4).$$

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2) , we get

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

So in the case of $(-5, -1)$ and $(-3, -7)$, we get $d^2 = (-2 + 3)^2 + (-1 + 7)^2 = 1 + 36 = 37$.

Thus, we get $d = \pm\sqrt{37}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{37}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2) , we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of $(-5, -1)$ and $(-3, -7)$, we get $a = \frac{-1 - (-7)}{-5 - (-3)} = \frac{6}{-2} = -3$.

Next, if a line has a slope of a , and a point (x_1, y_1) , the line is $y - y_1 = a(x - x_1)$.

So using $(-5, -1)$, we get $y + 1 = -3(x + 5)$, which is the line, and can be put this way, too:
 $y + 1 = -3(x + 5) \Rightarrow y = -3x - 15 - 1 = -3x - 16 \Rightarrow y = -3x - 16$.

And next, setting $x = 0$, we get the y -intercept, so getting the y -intercept, we get

$$x = 0 \Rightarrow y = -3x - 16 = 0 - 16 = -16, \text{ which is the } y\text{-intercept.}$$

And also, setting: $y = 0$, we get the x -intercept, so getting the x -intercept, we get:

$$0 = -3x - 16 \Rightarrow 3x = -16 \Rightarrow x = -\frac{16}{3}, \text{ which is the } x\text{-intercept.}$$

Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $bx + ay = ab$, where a is the x -intercept, and b is the y -intercept.

So next, we get $-16x - (16/3)y = (-16/3)(-16) \Rightarrow x + y/3 = -16/3 \Rightarrow y = -3x - 16$.