Examples 2 in Lines

Click this to start.

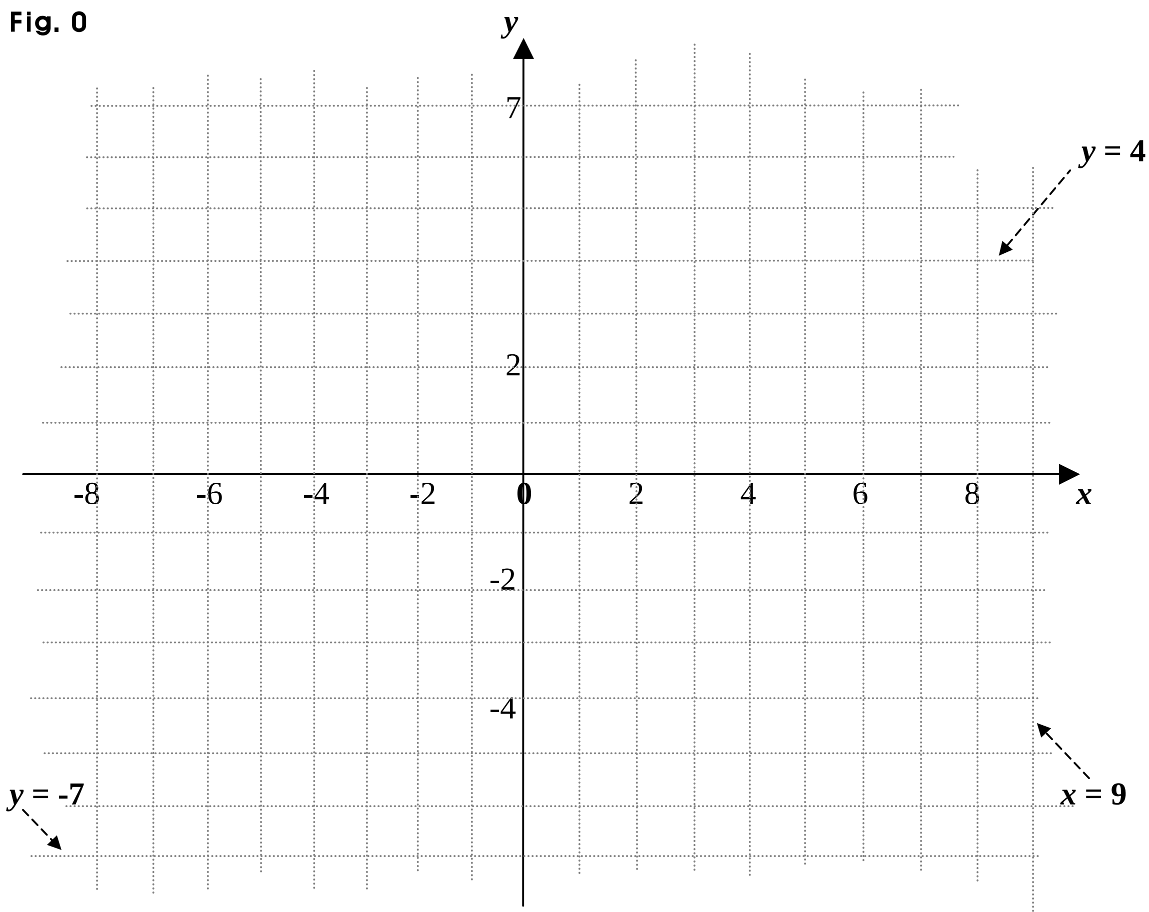
Examples 2 in Lines

Put in a graph, the two points in each case below, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

5. (-2, -1) and (3, -5)
6. (2, -6) and (4, -1)
7. (5, -1) and (2, 3)
8. (1, 5) and (6, 2)
9. (1, -3) and (4, 3)

Copy the ***x-y*** plane shown below or draw the plane on a copy paper for each case, and do each example above.

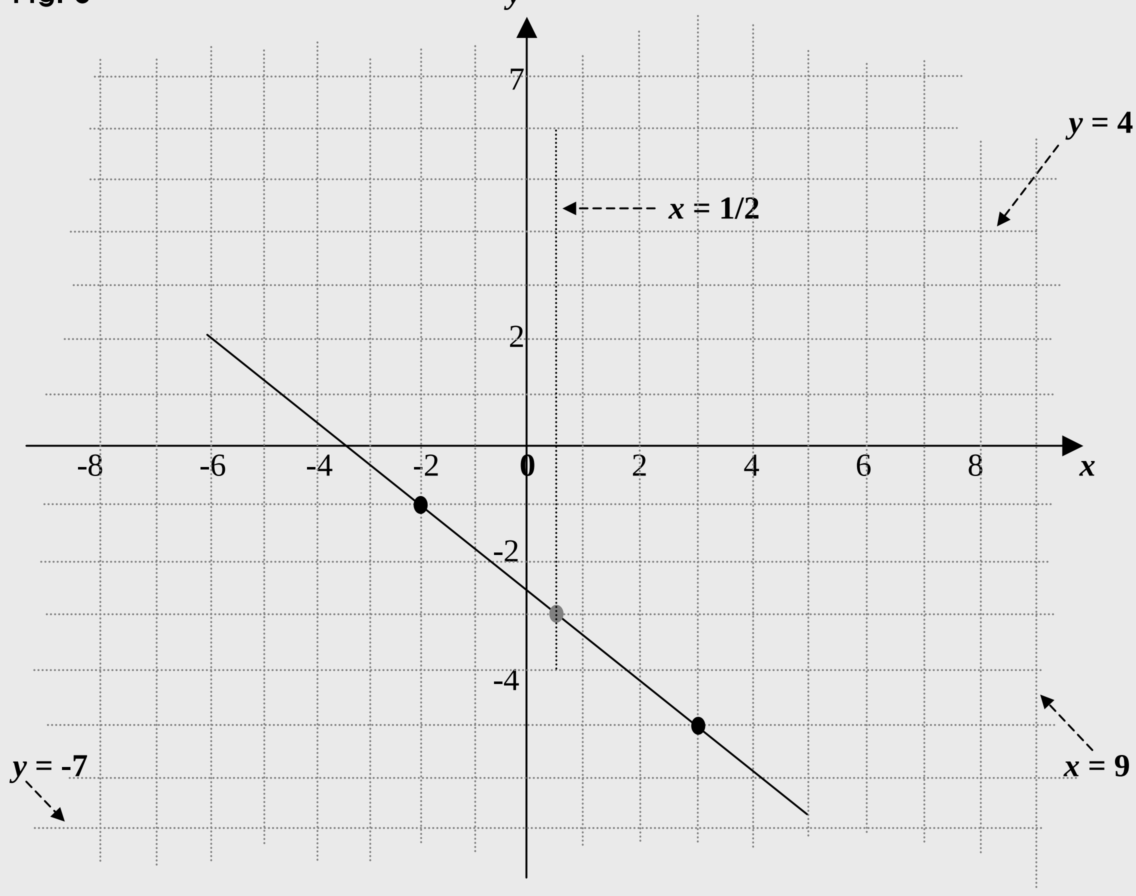
Fig. 0



Suggestions or Solutions
To the Problem in the Example 0

Put in a graph, two points (-2, -1) and (3, -5), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming ***s*** is the average ***x***-coordinate, and ***t*** is the average ***y***-coordinate, and finding the midpoint, we get

$\boldsymbol{s = (-2 + 3)/2 = 1/2}$, and $\boldsymbol{t = (-1 - 5)/2 = -3}$.   So the midpoint is (1/2, -3).

Next, assuming ***d*** is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (-2, -1) and (3, -5), we get $\boldsymbol{d^2 = (-2 - 3)^2 + (-1 + 5)^2 = 25 + 16 = 41}$.

Thus, we get $\boldsymbol{d = \pm\sqrt{41}}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = \sqrt{41}}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}$.

So in the case of (-2, -1) and (3, -5), we get $\boldsymbol{a = \frac{-1+5}{-2-3} = -\frac{4}{5}}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (-2, -1), we get $\boldsymbol{y + 1 = (-4/5)(x + 2)}$, which is the line, and can be put this way, too: $\boldsymbol{y + 1 = (-4/5)(x + 2) \Rightarrow y = (-4/5)x - 8/5 - 1 = (-4/5)x - 13/5 \Rightarrow y = (-4/5)x - 13/5}$.

And next, setting: $\boldsymbol{x = 0}$, we get the $\boldsymbol{y}$-intercept, so getting the $\boldsymbol{y}$-intercept, we get

$\boldsymbol{x = 0 \Rightarrow y = (-4/5)x - 13/5 = 0 - 13/5 = -13/5}$, which is the $\boldsymbol{y}$-intercept.

And also, setting $\boldsymbol{y = 0}$, we get the $\boldsymbol{x}$-intercept, so getting the $\boldsymbol{x}$-intercept, we get

$\boldsymbol{0 = (-4/5)x - 13/5 \Rightarrow 4x = -13 \Rightarrow x = -\frac{13}{4}}$, which is the $\boldsymbol{x}$-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $\boldsymbol{bx + ay = ab}$, where $\boldsymbol{a}$ is the $\boldsymbol{x}$-intercept, and $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept.

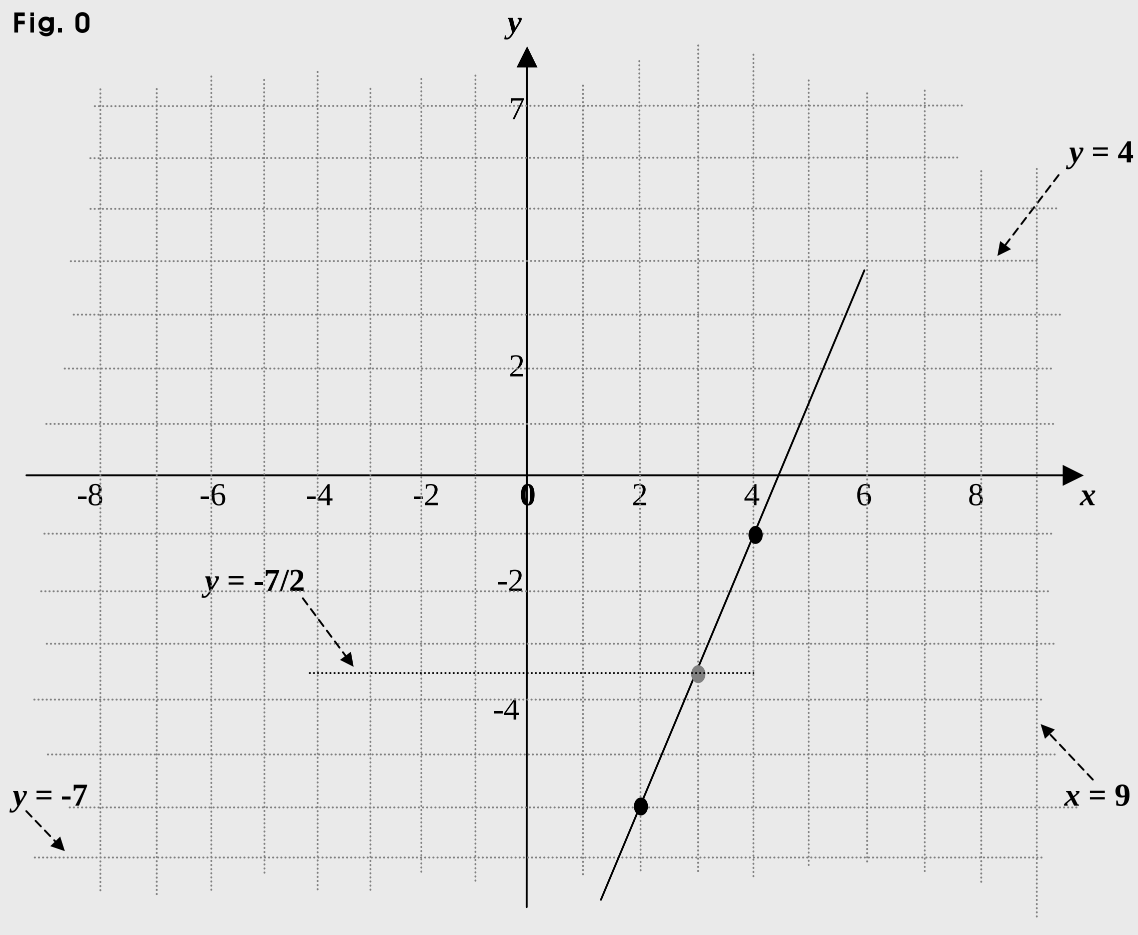
So next, we get $\boldsymbol{(-13/5)x + (-13/4)y = (-13/4)(-13/5) \Rightarrow x/5 + y/4 = -13/20}$

$\boldsymbol{\Rightarrow 4x + 5y = -13 \Rightarrow y = (-4/5)x - 13/5}$.

Suggestions or Solutions
To the Problem in the Example 1

Put in a graph, two points (2, -6) and (4, -1), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming ***s*** is the average ***x***-coordinate, and ***t*** is the average ***y***-coordinate, and finding the midpoint, we get

$\boldsymbol{s = (2 + 4)/2 = 3}$, and $\boldsymbol{t = (-6 - 1)/2 = -7/2}$.   So the midpoint is (3, -7/2).

Next, assuming ***d*** is the distance between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$\boldsymbol{d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$.

So in the case of (2, -6) and (4, -1), we get $\boldsymbol{d^2 = (2 - 4)^2 + (-6 + 1)^2 = 4 + 25 = 29}$.

Thus, we get $\boldsymbol{d = \pm\sqrt{29}}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = \sqrt{29}}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (2, -6) and (4, -1), we get $\boldsymbol{a = \frac{-6+1}{2-4} = \frac{5}{2}}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (4, -1), we get $\boldsymbol{y + 1 = (5/2)(x - 4)}$, which is the line, and can be put this way, too: $\boldsymbol{y + 1 = (5/2)(x - 4) \Rightarrow y = (5/2)x - 10 - 1 = (5/2)x - 11 \Rightarrow y = (5/2)x - 11}$.

And next, setting $\boldsymbol{x = 0}$, we get the $\boldsymbol{y}$-intercept, so getting the $\boldsymbol{y}$-intercept, we get

$\boldsymbol{x = 0 \Rightarrow y = (5/2)x - 11 = 0 - 11 = -11}$, which is the $\boldsymbol{y}$-intercept.

Also, setting $\boldsymbol{y = 0}$, we get the $\boldsymbol{x}$-intercept, so getting the $\boldsymbol{x}$-intercept, we get
$\boldsymbol{0 = (5/2)x - 11 \Rightarrow 5x = 22 \Rightarrow x = \frac{22}{5}}$, which is the $\boldsymbol{x}$-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

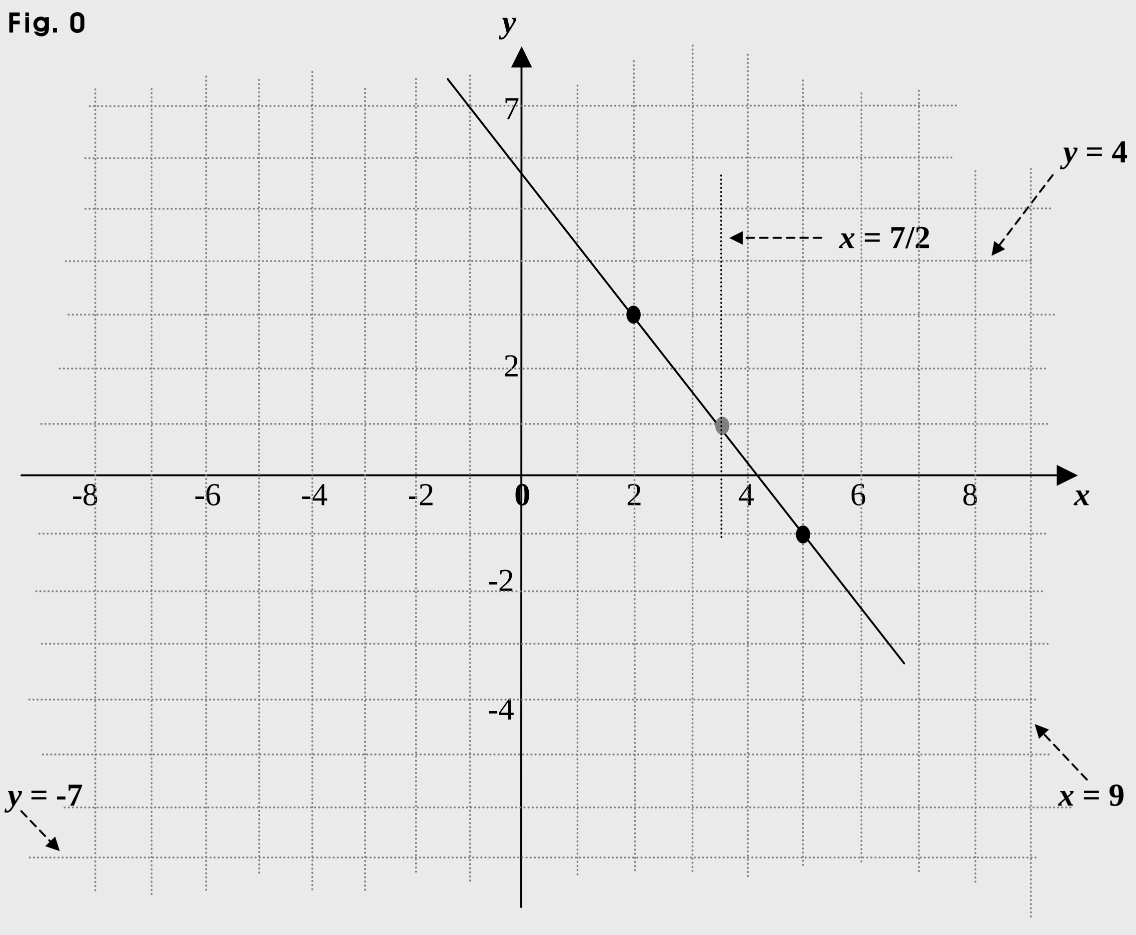
First, the form is $\boldsymbol{bx + ay = ab}$, where $\boldsymbol{a}$ is the $\boldsymbol{x}$-intercept, and $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept.

So next, we get $\boldsymbol{-11x + (22/5)y = (22/5)(-11) \Rightarrow x - (2/5)y = 22/5 \Rightarrow 5x - 2y = 22}$
$\boldsymbol{\Rightarrow y = (5/2)x - 11}$.

Suggestions or Solutions
To the Problem in the Example 2

Put in a graph, two points (5, -1) and (2, 3), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming ***s*** is the average ***x***-coordinate, and ***t*** is the average ***y***-coordinate, and finding the midpoint, we get

s = (5 + 2)/2 = 7/2, and **t = (-1 + 3)/2 = 1**.   So the midpoint is (7/2, 1).

Next, assuming ***d*** is the distance between **(x_1, y_1)** and **(x_2, y_2)**, we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (5, -1) and (2, 3), we get $\boldsymbol{d^2 = (5 - 2)^2 + (-1 - 3)^2 = 9 + 16 = 25}$.

Thus, we get $\boldsymbol{d = \pm 5}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = 5}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (5, -1) and (2, 3), we get $\boldsymbol{a = \frac{-1 - 3}{5 - 2} = -\frac{4}{3}}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (2, 3), we get $\boldsymbol{y - 3 = (-4/3)(x - 2)}$, which is the line, and can be put this way, too: $\boldsymbol{y - 3 = (-4/3)(x - 2) \Rightarrow y = (-4/3)x + 8/3 + 3 = (-4/3)x + 17/3 \Rightarrow y = (-4/3)x + 17/3}$.

And next, setting $\boldsymbol{x = 0}$, we get the $\boldsymbol{y}$-intercept, so getting the $\boldsymbol{y}$-intercept, we get

$\boldsymbol{x = 0 \Rightarrow y = (-4/3)x + 17/3 = 0 + 17/3 = 17/3}$, which is the $\boldsymbol{y}$-intercept.

And also, setting $\boldsymbol{y = 0}$, we get the $\boldsymbol{x}$-intercept, so getting the $\boldsymbol{x}$-intercept, we get

$\boldsymbol{0 = (-4/3)x + 17/3 \Rightarrow 4x = 17 \Rightarrow x = 17/4}$, which is the $\boldsymbol{x}$-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $\boldsymbol{bx + ay = ab}$, where $\boldsymbol{a}$ is the $\boldsymbol{x}$-intercept, and $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept.

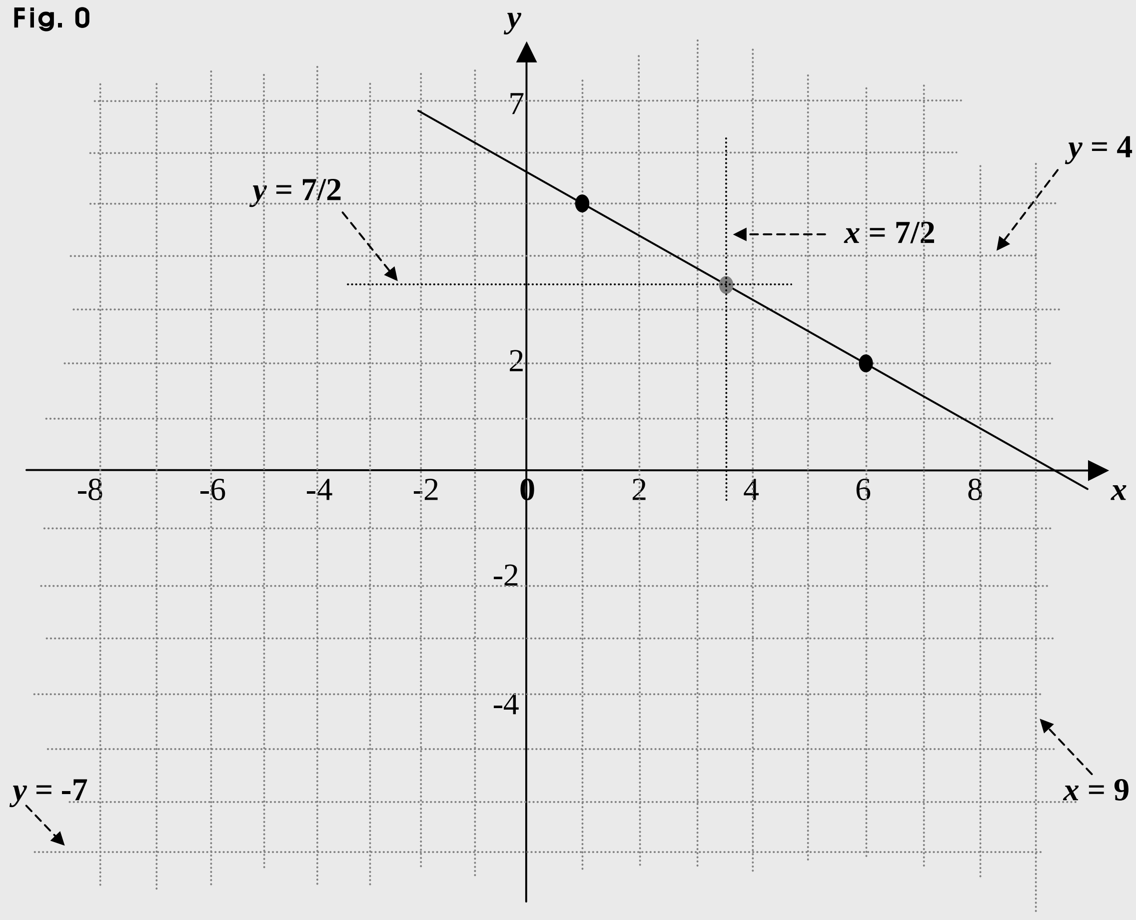
So next, we get $\boldsymbol{(17/3)x + (17/4)y = (17/4)(17/3) \Rightarrow (17/3)(4/17)x + y = 17/3}$

$\boldsymbol{\Rightarrow (4/17)x + (3/17)y = 1 \Rightarrow 4x + 3y = 17 \Rightarrow y = (-4/3)x + 17/3}$.

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph, two points (1, 5) and (6, 2), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming ***s*** is the average ***x***-coordinate, and ***t*** is the average ***y***-coordinate, and finding the midpoint, we get

$s = (1 + 6)/2 = 7/2$, and $t = (5 + 2)/2 = 7/2$. So the midpoint is (7/2, 7/2).

Next, assuming ***d*** is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (1, 5) and (6, 2), we get $\boldsymbol{d^2 = (1 - 6)^2 + (5 - 2)^2 = 25 + 9 = 34}$.

Thus, we get $\boldsymbol{d = \pm\sqrt{34}}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = \sqrt{34}}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (1, 5) and (6, 2), we get $\boldsymbol{a = \frac{5-2}{1-6} = -\frac{3}{5}}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (1, 5), we get $\boldsymbol{y - 5 = (-3/5)(x - 1)}$, which is the line, and can be put this way, too: $\boldsymbol{y - 5 = (-3/5)(x - 1) \Rightarrow y = (-3/5)x + 3/5 + 5 = (-3/5)x + 28/5 \Rightarrow y = (-3/5)x + 28/5}$.

And next, setting $\boldsymbol{x = 0}$, we get the $\boldsymbol{y}$-intercept, so getting the $\boldsymbol{y}$-intercept, we get

$\boldsymbol{x = 0 \Rightarrow y = (-3/5)x + 28/5 = 0 + 28/5 = 28/5}$, which is the $\boldsymbol{y}$-intercept.

Also, setting $\boldsymbol{y = 0}$, we get the $\boldsymbol{x}$-intercept, so getting the $\boldsymbol{x}$-intercept, we get

$\boldsymbol{0 = (-3/5)x + 28/5 \Rightarrow 3x = 28 \Rightarrow x = 28/3}$, which is the $\boldsymbol{x}$-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $\boldsymbol{bx + ay = ab}$, where $\boldsymbol{a}$ is the $\boldsymbol{x}$-intercept, and $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept.

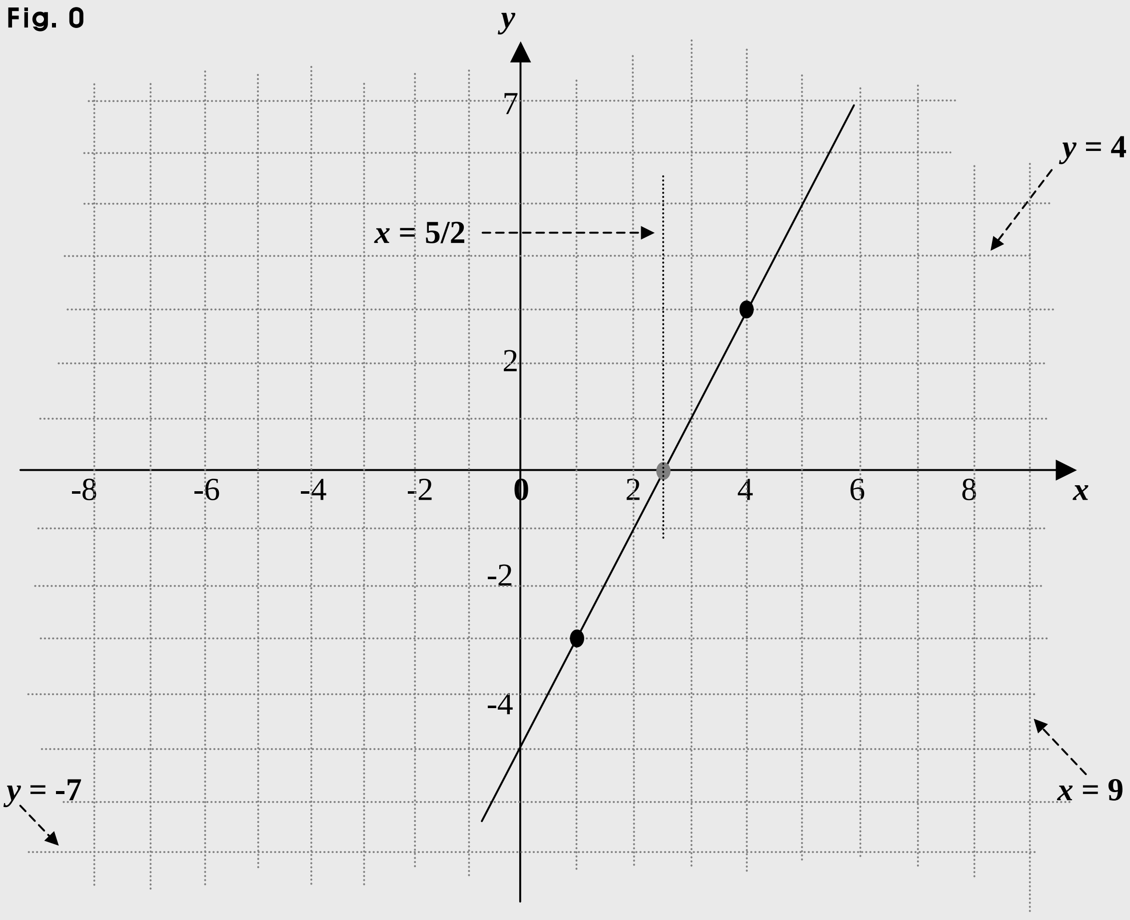
So next, we get $\boldsymbol{(28/5)x + (28/3)y = (28/3)(28/5) \Rightarrow (28/5)(3/28)x + y = 28/5}$

$\boldsymbol{\Rightarrow (3/28)x + (5/28)y = 1 \Rightarrow 3x + 5y = 28 \Rightarrow y = (-3/5)x + 28/5}$.

Suggestions or Solutions
To the Problem in the Example 4

Put in a graph, two points (1, -3) and (4, 3), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming ***s*** is the average ***x***-coordinate, and ***t*** is the average ***y***-coordinate, and finding the midpoint, we get

$\boldsymbol{s = (1 + 4)/2 = 5/2}$, and $\boldsymbol{t = (-3 + 3)/2 = 0}$.   So the midpoint is (5/2, 0).

Next, assuming ***d*** is the distance between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get
$\boldsymbol{d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$.

So in the case of (1, -3) and (4, 3), we get $\boldsymbol{d}^2 = \mathbf{(1 - 4)^2 + (-3 - 3)^2 = 9 + 36 = 45}$.

Thus, we get $\boldsymbol{d} = \pm\sqrt{\mathbf{45}}$. And $\boldsymbol{d} \geq \mathbf{0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d} = \sqrt{\mathbf{45}}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get

$$\boldsymbol{a} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of (1, -3) and (4, 3), we get $\boldsymbol{a} = \dfrac{-3-3}{1-4} = \mathbf{2}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point (x_1, y_1), the line is $y - y_1 = a(x - x_1)$.

So using (1, -3), we get $y + 3 = 2(x - 1)$, which is the line, and can be put this way, too:
$y + 3 = 2(x - 1) \Rightarrow y = 2x - 2 - 3 = 2x - 5 \Rightarrow y = 2x - 5$.

And next, setting $x = 0$, we get the $\boldsymbol{y}$-intercept, so getting the $\boldsymbol{y}$-intercept, we get

$x = 0 \Rightarrow y = 2x - 5 = 0 - 5 = -5$, which is the $\boldsymbol{y}$-intercept.

Also, setting $y = 0$, we get the $\boldsymbol{x}$-intercept, so getting the $\boldsymbol{x}$-intercept, we get

$0 = 2x - 5 \Rightarrow 2x = 5 \Rightarrow x = 5/2$, which is the $\boldsymbol{x}$-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

First, the form is $bx + ay = ab$, where $\boldsymbol{a}$ is the $\boldsymbol{x}$-intercept, and $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept.

So next, we get $-5x + (5/2)y = (5/2)(-5) \Rightarrow x - (1/2)y = 5/2 \Rightarrow y = 2x - 5$.