Examples 3 in Lines

Click this to start.

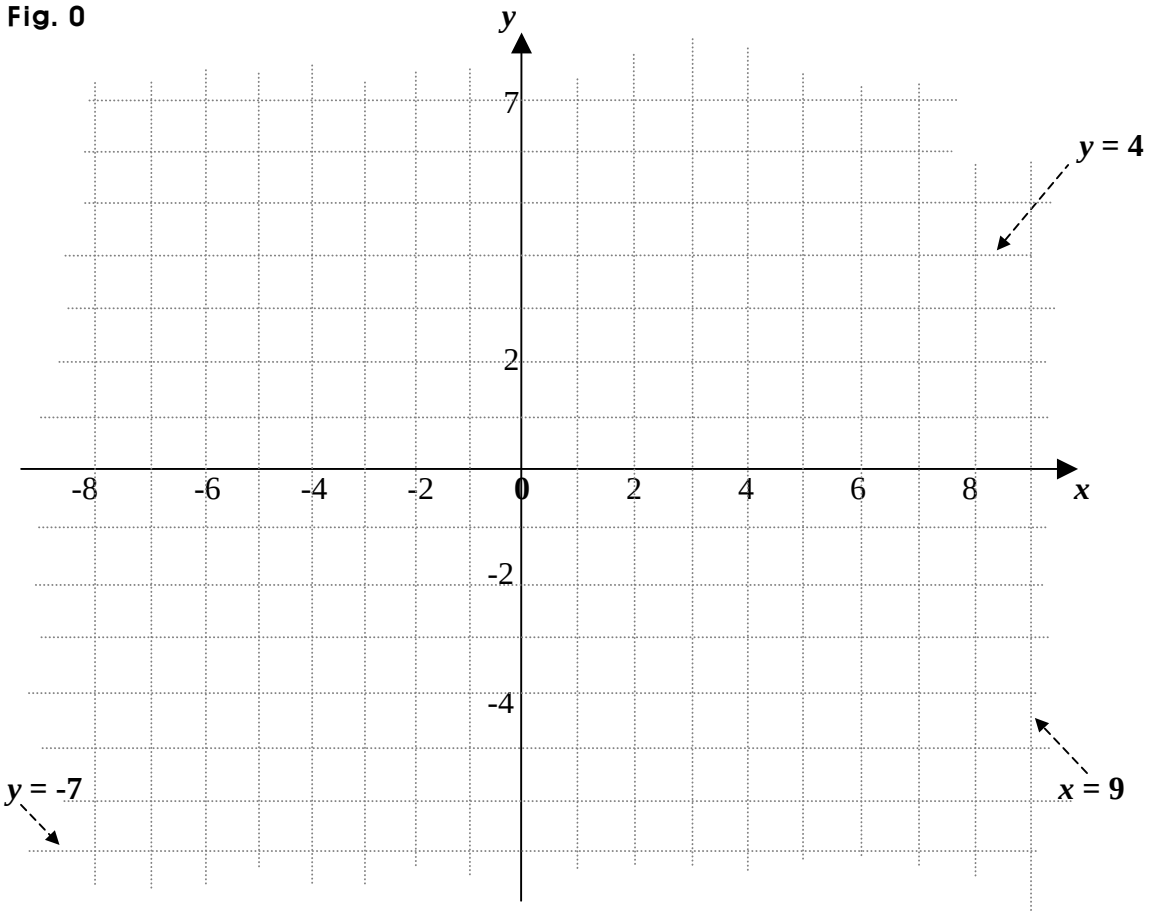
Examples 3 in Lines

Put in a graph, the two points in each case below, and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

0. (3, 4) and (-2, -3)
1. (-3, 4) and (5, -2)
2. (-3, 2) and (5, 2)
3. (2, -3) and (2, 4)
4. (-2, -3) and (2, 3)

Copy the ***x*-*y*** plane shown below or draw the plane on a copy paper for each case, and do each example above.

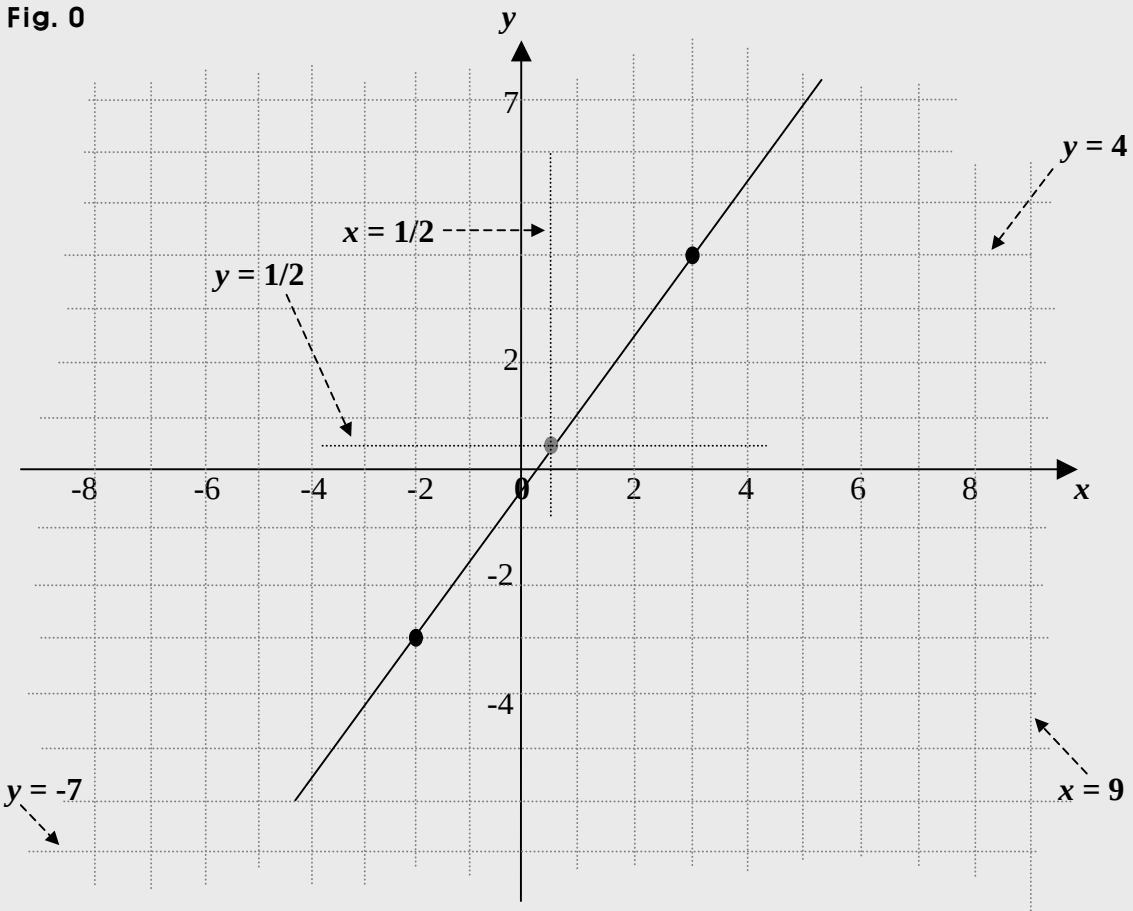
Fig. 0



Suggestions or Solutions
To the Problem in the Example 0

Put in a graph, two points (3, 4) and (-2, -3), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x-coordinate, and t is the average y-coordinate, and finding the midpoint, we get

$s = (3 - 2)/2 = 1/2$, and $t = (4 - 3)/2 = 1/2$. So the midpoint is (1/2, 1/2).

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (3, 4) and (-2, -3), we get: $d^2 = (3 + 2)^2 + (4 + 3)^2 = 25 + 49 = 74$.

Thus, we get $d = \pm\sqrt{74}$. And $d \geq 0$, since d is a distance. So we get $d = \sqrt{74}$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of (3, 4) and (-2, -3), we get $a = \frac{4+3}{3+2} = \frac{7}{5}$.

Next, if a line has a slope of a, and a point (x_1, y_1), the line is $y - y_1 = a(x - x_1)$.

So using (-2, -3), we get $y + 3 = (7/5)(x + 2)$, which is the line, and can be put this way, too: $y + 3 = (7/5)(x + 2) \Rightarrow y = (7/5)x + 14/5 - 3 = (7/5)x - 1/5 \Rightarrow y = (7/5)x - 1/5$.

And next, setting $x = 0$, we get the y-intercept, so getting the y-intercept, we get

$x = 0 \Rightarrow y = (7/5)x - 1/5 = 0 - 1/5 = -1/5$, which is the y-intercept.

And also, setting $y = 0$, we get the x-intercept, so getting the x-intercept, we get

$0 = (7/5)x - 1/5 \Rightarrow 7x = 1 \Rightarrow x = 1/7$, which is the x-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

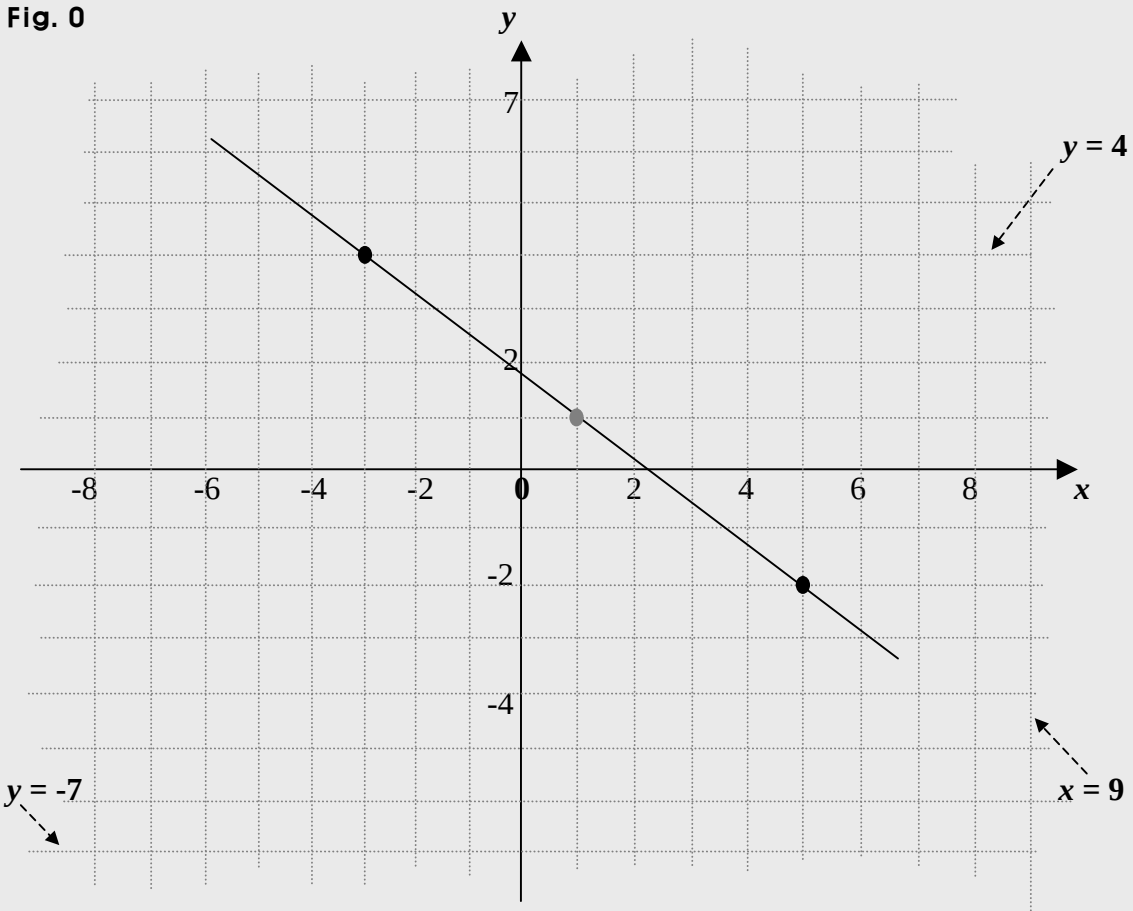
First, the form is $bx + ay = ab$, where a is the x-intercept, and b is the y-intercept.

So next, we get $(-1/5)x + (1/7)y = (1/7)(-1/5) \Rightarrow 7x - 5y = 1 \Rightarrow y = (7/5)x - 1/5$.

Suggestions or Solutions
To the Problem in the Example 1

Put in a graph, two points (-3, 4) and (5, -2), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x-coordinate, and t is the average y-coordinate, and finding the midpoint, we get

$s = (-3 + 5)/2 = 1$, and $t = (4 - 2)/2 = 1$. So the midpoint is (1, 1).

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (-3, 4) and (5, -2), we get $d^2 = (-3 - 5)^2 + (4 + 2)^2 = 64 + 36 = 100$.

Thus, we get $d = \pm 10$. And $d \geq 0$, since d is a distance. So we get $d = 10$.

Next, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get

$$a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

So in the case of (-3, 4) and (5, -2), we get $a = \frac{4+2}{-3-5} = \frac{6}{-8} = -\frac{3}{4}$.

Next, if a line has a slope of a, and a point (x_1, y_1), the line is $y - y_1 = a(x - x_1)$.

So using (-3, 4), we get $y - 4 = (-3/4)(x + 3)$, which is the line, and can be put this way, too: $y - 4 = (-3/4)(x + 3) \Rightarrow y = (-3/4)x - 9/4 + 4 = (-3/4)x + 7/4 \Rightarrow y = (-3/4)x + 7/4$.

And next, setting $x = 0$, we get the y-intercept, so getting the y-intercept, we get

$x = 0 \Rightarrow y = (-3/4)x + 7/4 = 0 + 7/4 = 7/4$, which is the y-intercept.

And also, setting $y = 0$, we get the x-intercept, so getting the x-intercept, we get

$0 = (-3/4)x + 7/4 \Rightarrow 3x = 7 \Rightarrow x = 7/3$, which is the x-intercept.

Let's now check to see if we get the line putting the intercepts into the intercept form.

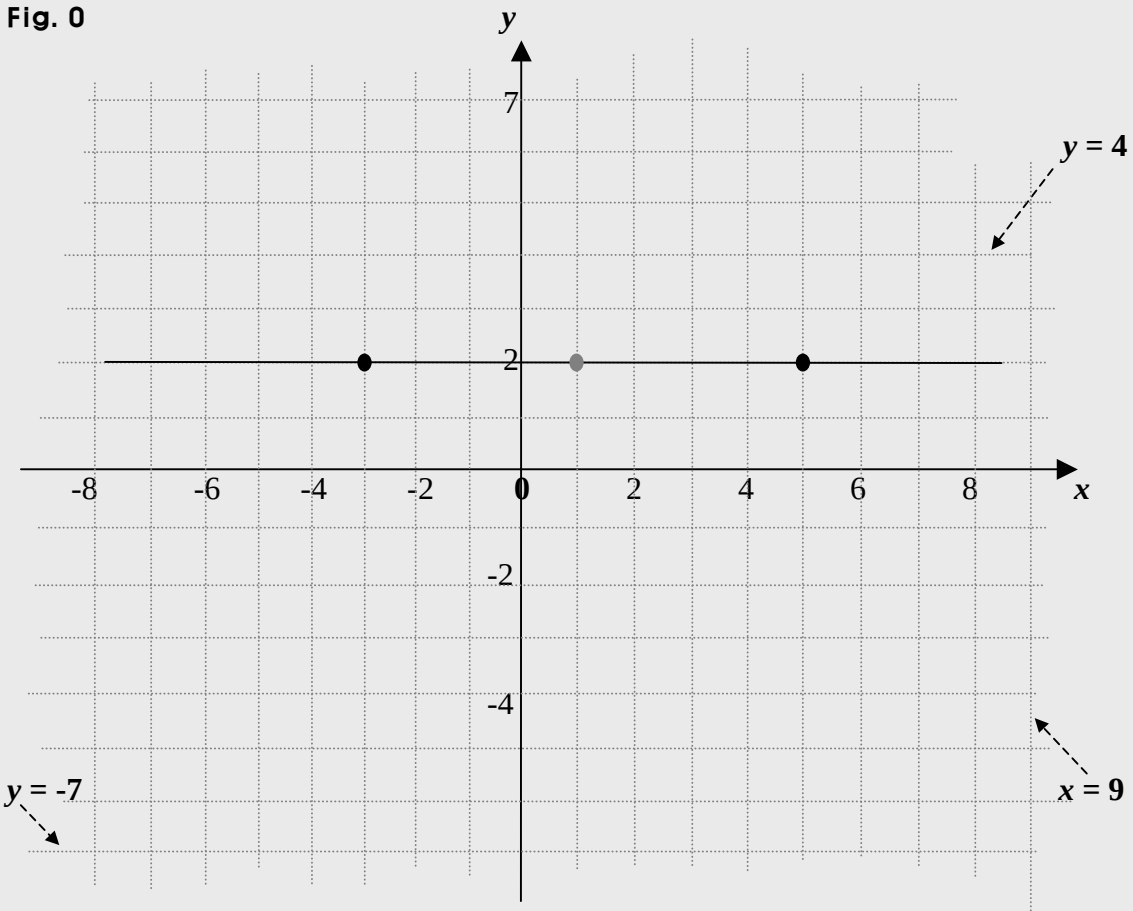
First, the form is $bx + ay = ab$, where a is the x-intercept, and b is the y-intercept.

So next, we get $(7/4)x + (7/3)y = (7/3)(7/4) \Rightarrow (3/7)x + (4/7)y = 1 \Rightarrow 3x + 4y = 7$
$\Rightarrow y = (-3/4)x + 7/4$.

Suggestions or Solutions
To the Problem in the Example 2

Put in a graph, two points (-3, 2) and (5, 2), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x-coordinate, and t is the average y-coordinate, and finding the midpoint, we get

$s = (-3 + 5)/2 = 1$, and $t = (2 + 2)/2 = 2$. So the midpoint is (1, 2).

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (-3, 2) and (5, 2), we get $\boldsymbol{d^2 = (-3 - 5)^2 + (2 - 2)^2 = 64}$.

Thus, we get $\boldsymbol{d = \pm 8}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = 8}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (-3, 2) and (5, 2), we get $\boldsymbol{a = \frac{2 - 2}{-3 - 5} = 0}$.

What line then is it?

It's a horizontal line, and passes through a point where the $\boldsymbol{y}$-coordinate is 2.

So the line is $\boldsymbol{y = 2}$.

Let's check though, to see if the point-slope form works in this case, too.

If a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (-3, 2), we get $\boldsymbol{y - 2 = 0(x + 3) = 0 \Rightarrow y = 2}$, which is the line. So it works.

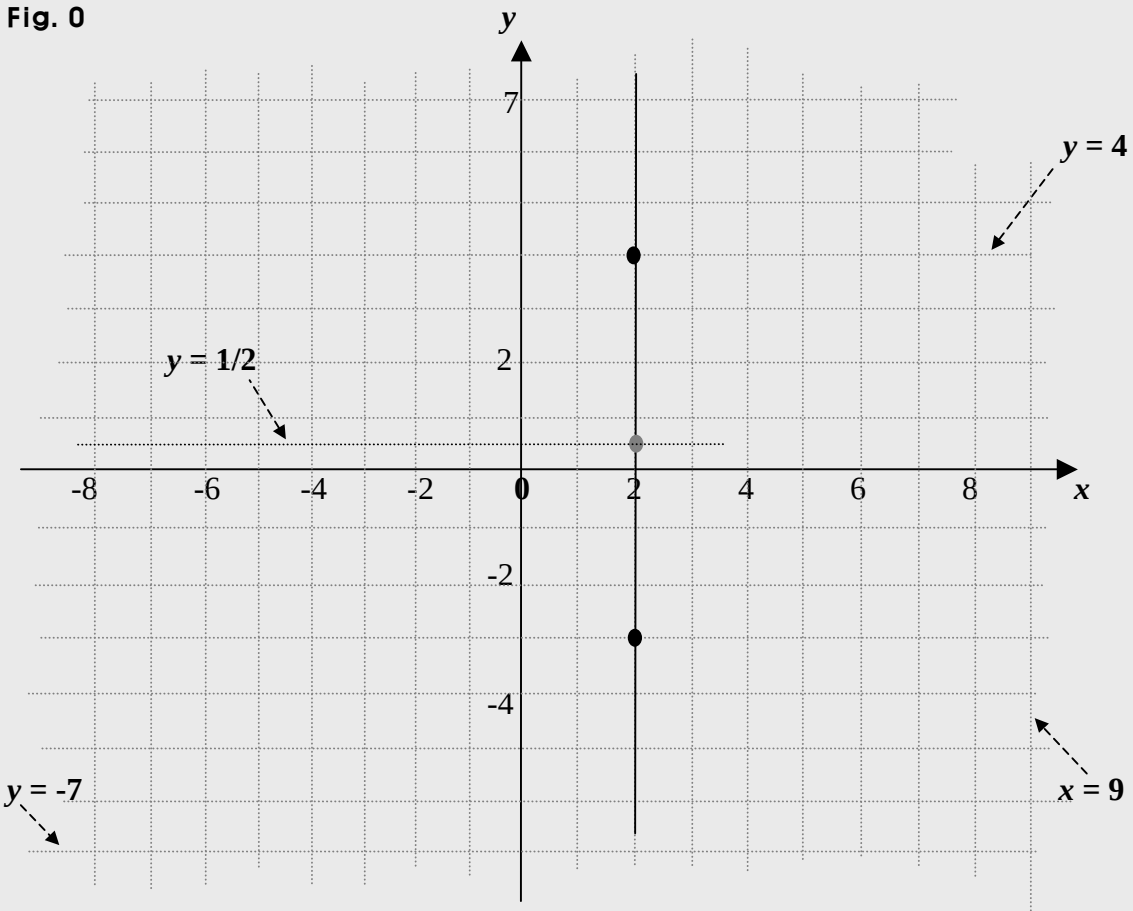
And of course, when $\boldsymbol{x = 0}$, we get $\boldsymbol{y = 2}$, which is thus, the $\boldsymbol{y}$-intercept.

And since $\boldsymbol{y = 2}$ only, $\boldsymbol{y}$ cannot be 0, so we get no $\boldsymbol{x}$-intercept.

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph, two points (2, -3) and (2, 4), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x-coordinate, and t is the average y-coordinate, and finding the midpoint, we get

$s = (2 + 2)/2 = 2$, and $t = (-3 + 4)/2 = 1/2$. So the midpoint is (2, 1/2).

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (2, -3) and (2, 4), we get $\boldsymbol{d^2 = (2-2)^2 + (-3-4)^2 = 49}$.

Thus, we get $\boldsymbol{d = \pm 7}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = 7}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (2, -3) and (2, 4), we get $\boldsymbol{a = \frac{-3-4}{2-2}}$, which is not allowed, because the denominator is 0. So it has no slope. What line then is it?

It's a vertical line, and passes through a point where the $\boldsymbol{x}$-coordinate is 2.

So the line is $\boldsymbol{x = 2}$.

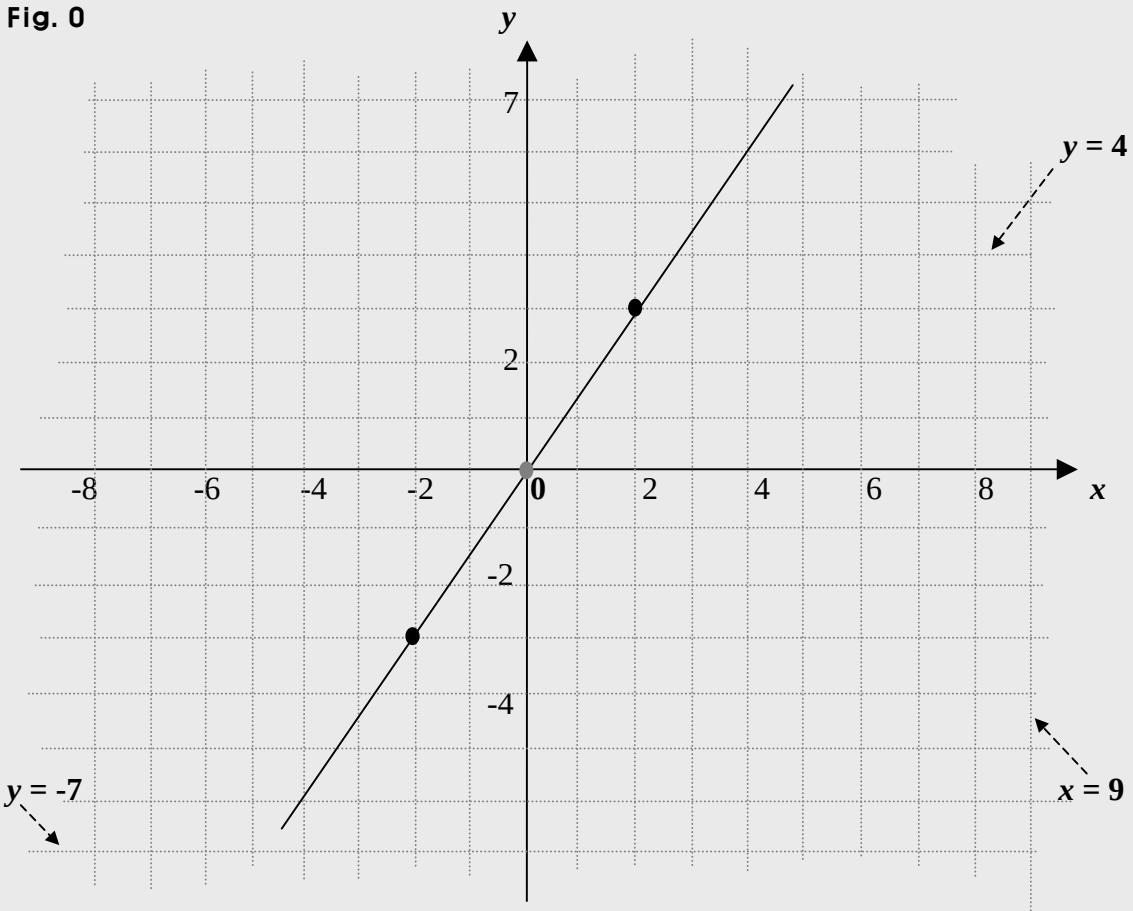
And of course, when $\boldsymbol{y = 0}$, we get $\boldsymbol{x = 2}$, which is thus, the $\boldsymbol{x}$-intercept.

And since $\boldsymbol{x = 2}$ only, $\boldsymbol{x}$ cannot be 0, so we get no $\boldsymbol{y}$-intercept.

Suggestions or Solutions
To the Problem in the Example 4

Put in a graph, two points (-2, -3) and (2, 3), and then, find the midpoint, distance, and slope between the two points, and the line passing through the two points.

Fig. 0



The midpoint is the average of the two points. And taking the average, find each average coordinate. So assuming s is the average x-coordinate, and t is the average y-coordinate, and finding the midpoint, we get

$s = (-2 + 2)/2 = 0$, and $t = (-3 + 3)/2 = 0$. So the midpoint is (0, 0).

Next, assuming d is the distance between (x_1, y_1) and (x_2, y_2), we get
$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

So in the case of (-2, -3) and (2, 3), we get $\boldsymbol{d^2 = (-2 - 2)^2 + (-3 - 3)^2 = 16 + 36 = 52}$.

Thus, we get $\boldsymbol{d = \pm 2\sqrt{13}}$. And $\boldsymbol{d \geq 0}$, since $\boldsymbol{d}$ is a distance. So we get $\boldsymbol{d = 2\sqrt{13}}$.

Next, assuming $\boldsymbol{a}$ is the slope of the segment between $\boldsymbol{(x_1, y_1)}$ and $\boldsymbol{(x_2, y_2)}$, we get

$$\boldsymbol{a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}}.$$

So in the case of (-2, -3) and (2, 3), we get $\boldsymbol{a = \frac{-3 - 3}{-2 - 2} = \frac{3}{2}}$.

Next, if a line has a slope of $\boldsymbol{a}$, and a point $\boldsymbol{(x_1, y_1)}$, the line is $\boldsymbol{y - y_1 = a(x - x_1)}$.

So using (2, 3), we get $\boldsymbol{y - 3 = (3/2)(x - 2)}$, which is the line, and can be put this way, too: $\boldsymbol{y - 3 = (3/2)(x - 2) \Rightarrow y = (3/2)x - 6/2 + 3 = (3/2)x \Rightarrow y = (3/2)x}$, which is the line, which passes through the origin. So both the $\boldsymbol{x}$ and $\boldsymbol{y}$ intercepts are 0, of course.