Examples 4 in Lines

Click this to start.

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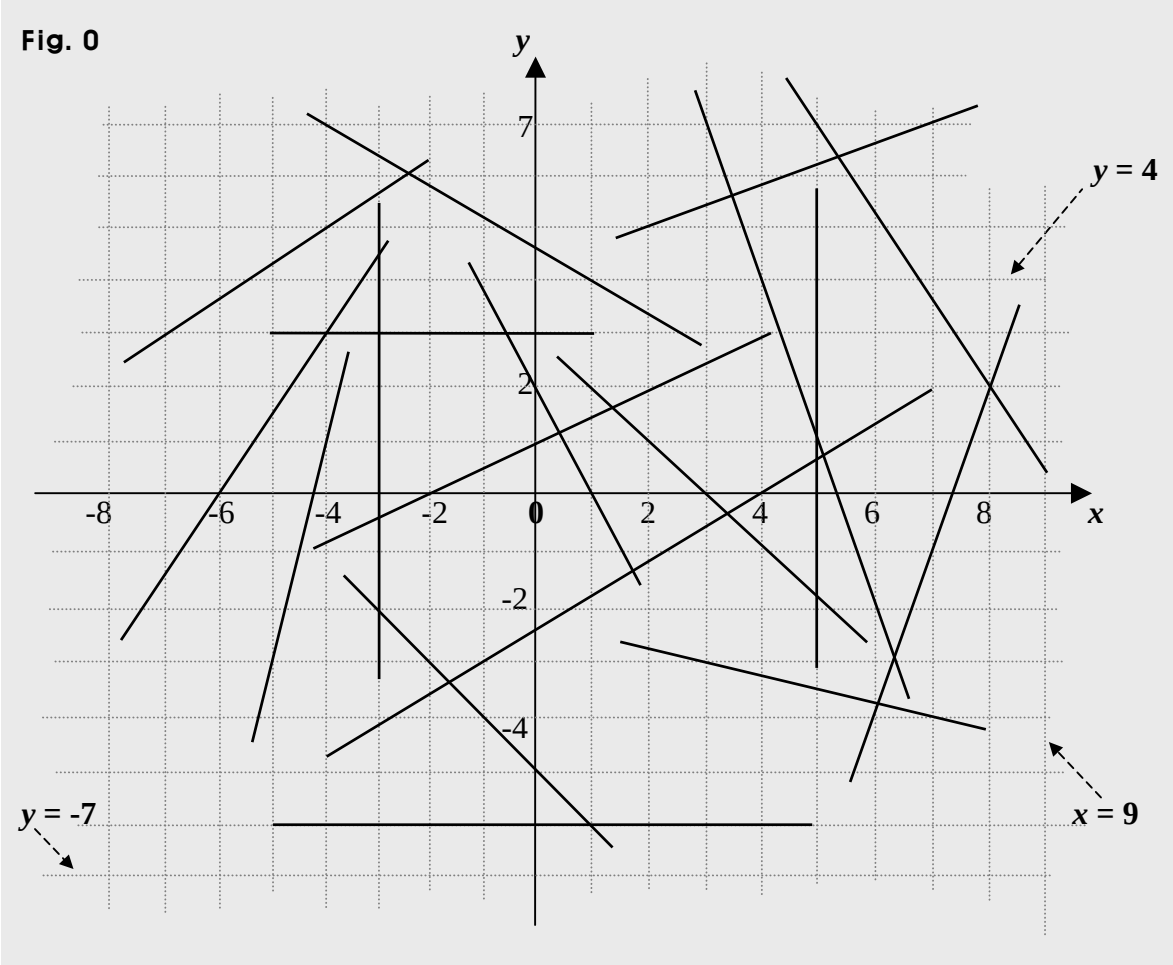
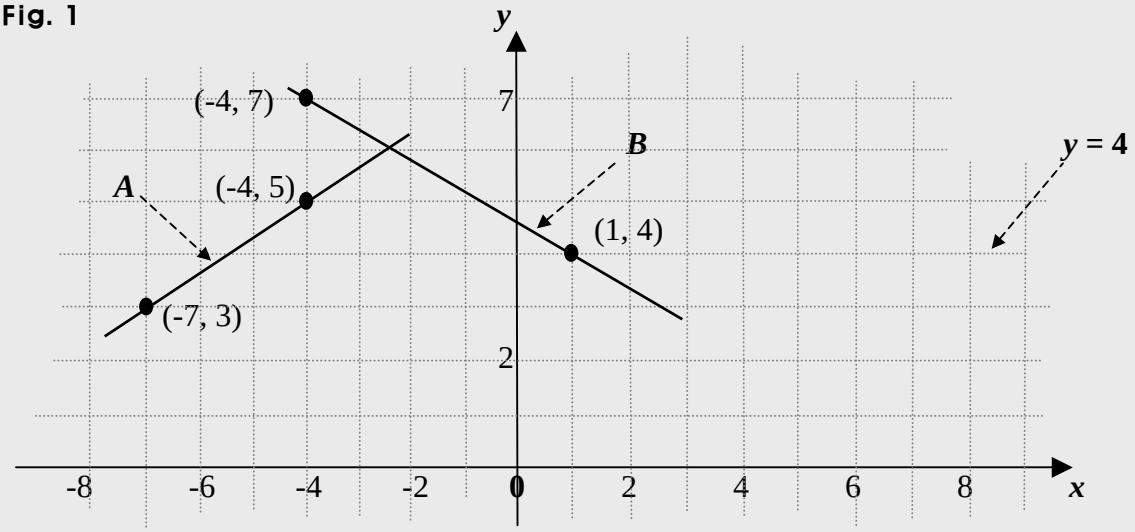
Label each line in the graph below, and then find the equation of each of the lines.



Suggestions or Solutions
To the Problems in the Examples

Beginning with the two lines below, we can see the line ***A*** includes two points (-4, 5), and (-7, 3), and the line ***B*** passes through two points (-4, 7) and (1, 4).

Fig. 1



Next, given the slope and a point in a line, we can get the equation of the line using the point-slope form: $(y - t) = a(x - s)$, where a is the slope, and (s, t) is the point given.

And we can get the slope using the two points found in each line. And assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get $a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.

So next, in the case of ***A***, using (-4, 5) and (-7, 3), we get $a = \frac{5-3}{-4-(-7)} = \frac{2}{-4+7} = \frac{2}{3}$.

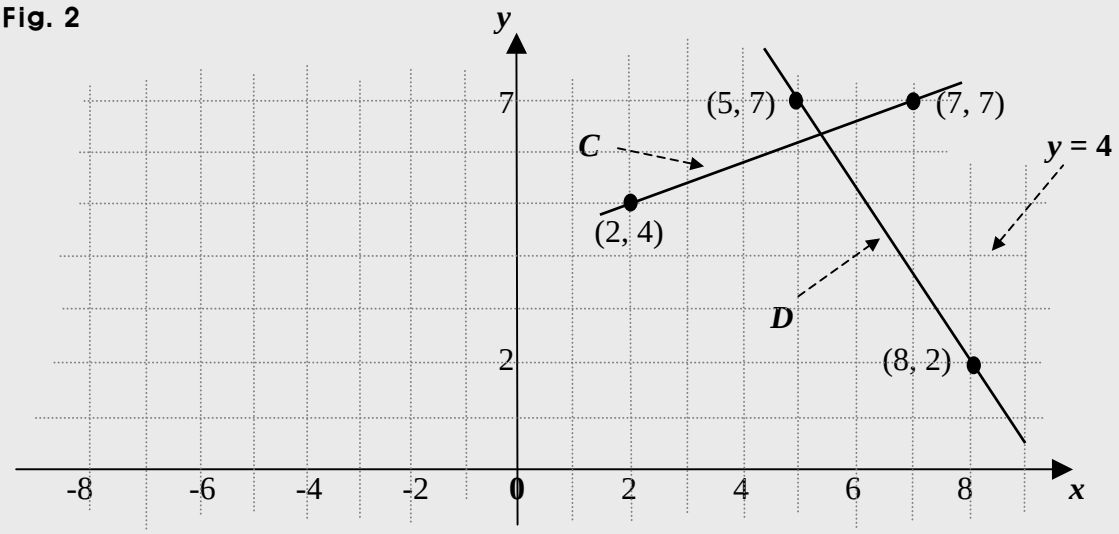
Next, using (-4, 5), we get $y - 5 = (2/3)(x + 4)$, which is the line ***A***, can be put this way, too: $y - 5 = (2/3)(x + 4) \Rightarrow y = (2/3)x + 8/3 + 5 = (2/3)x + 23/3 \Rightarrow y = (2/3)x + 23/3$.

Next, in the case of ***B***, using (-4, 7) and (1, 4), we get $a = \frac{7-4}{-4-1} = \frac{3}{-5} = -\frac{3}{5}$.

Next, using (1, 4), we get $y - 4 = (-3/5)(x - 1)$, which is the line ***B***, can be put this way, too: $y - 4 = (-3/5)(x - 1) \Rightarrow y = (-3/5)x + 3/5 + 4 = (-3/5)x + 23/5 \Rightarrow y = (-3/5)x + 23/5$.

Next, moving on to the two lines below, we can see the line ***C*** includes two points (2, 4), and (7, 7), and the line ***D*** passes through two points (5, 7) and (8, 2).

Fig. 2



Next, if a is the slope of a line, and (s, t) is a point in the line, we can get the equation of the line using the point-slope form, $(y - t) = a(x - s)$.

And we can get the slope using the two points found in each line. And assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get $a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.

So next, in the case of ***C***, using (2, 4) and (7, 7), we get $a = \frac{4-7}{2-7} = \frac{-3}{-5} = \frac{3}{5}$.

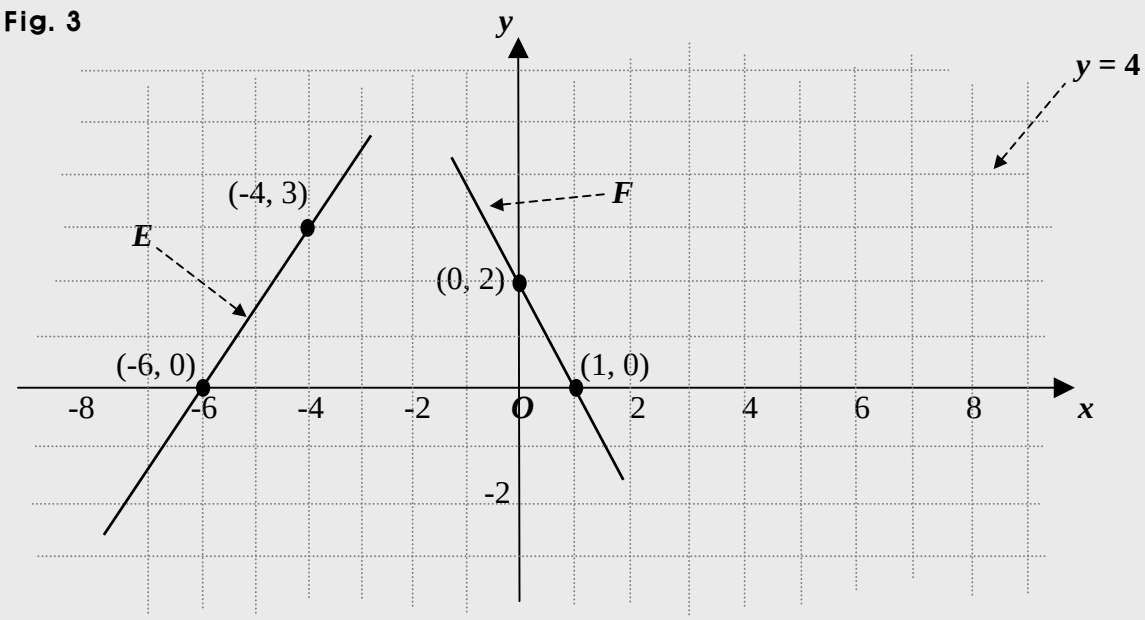
Next, using (2, 4), we get $y - 4 = (3/5)(x - 2)$, which is the line ***C***, can be put this way, too: $y - 4 = (3/5)(x - 2) \Rightarrow y = (3/5)x - 6/5 + 4 = (3/5)x + 14/5 \Rightarrow y = (3/5)x + 14/5$.

Next, in the case of ***D***, using (5, 7) and (8, 2), we get $a = \frac{7-2}{5-8} = \frac{5}{-3} = -\frac{5}{3}$.

Next, using (5, 7), we get $y - 7 = (-5/3)(x - 5)$, which is the line ***D***, can be put this way, too: $y - 7 = (-5/3)(x - 5) \Rightarrow y = (-5/3)x + 25/3 + 7 = (-5/3)x + 46/3 \Rightarrow y = (-5/3)x + 46/3$.

Next, moving on to the two lines below, we can see the line ***E*** includes two points (-6, 0), and (-4, 3), and the line ***F*** passes through two points (0, 2) and (1, 0).

Fig. 3



First, looking at the line ***E***, we can notice that ***E*** meets the ***x***-axis at (-6, 0), so its ***x***-intercept is 6.
And given the slope and an intercept of a line, we can get the equation of the line using the slope-intercept form, too, $y = ax + b$, where a is the slope, and b is the y-intercept. Though the form has the y-intercept instead of the x-intercept, we can get b using the x-intercept, too, if we have the slope, of course.

And we can get the slope using the two points found in the line ***E***. And assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get $a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.

So next, in the case of ***E***, using (-6, 0) and (-4, 3), we get $a = \frac{0 - 3}{-6 - (-4)} = \frac{-3}{-6 + 4} = \frac{3}{2}$.

Thus, we get $y = ax + b \Rightarrow y = (3/2)x + b$.
And we know the ***x***-intercept is the ***x***-coordinate at the point where the line meets the ***x***-axis, so the ***y***-coordinate at the point is 0.

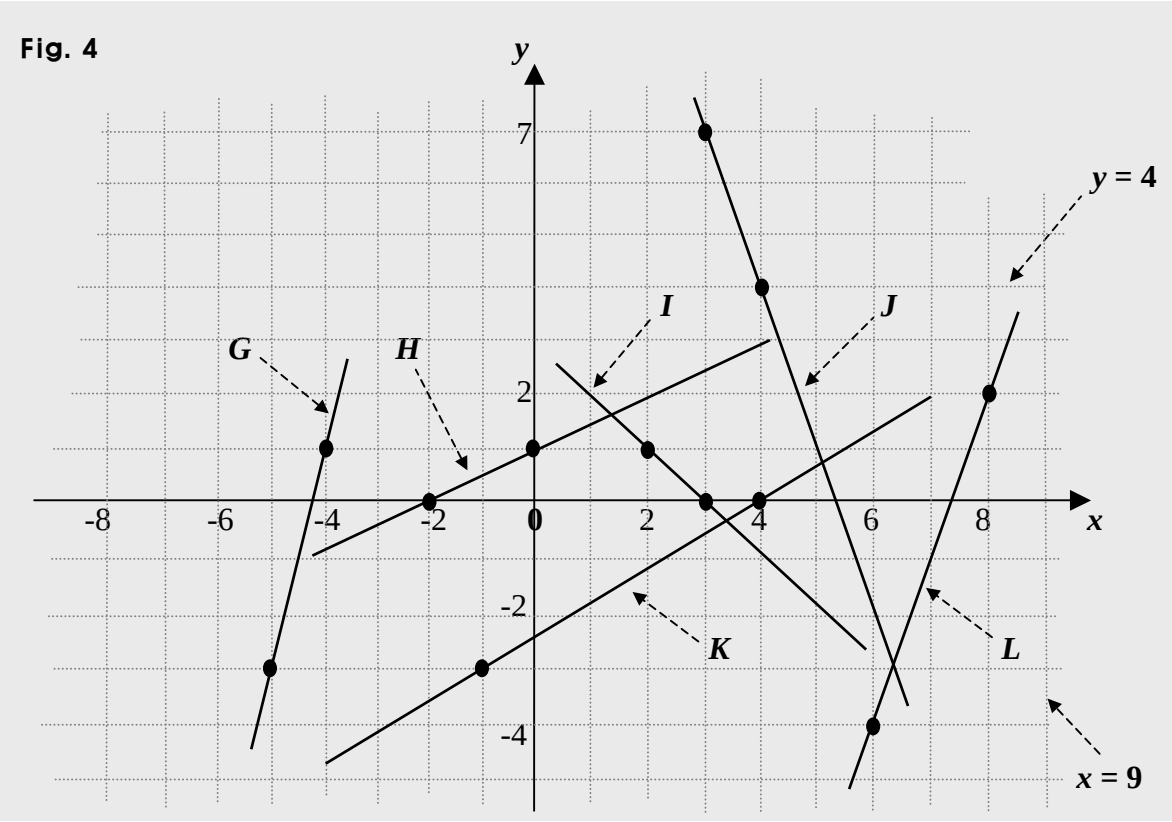
So next, using the ***x***-intercept, that is, the point (-6, 0), we get
$y = (3/2)x + b \Rightarrow 0 = (3/2)(-6) + b \Rightarrow b = (3/2)\cdot 6 = 9 \Rightarrow y = (3/2)x + 9$, which is the line ***E***, and of course, the ***y***-intercept is 9.

Also, given both the ***x*** and ***y*** intercepts, we can use the intercept form as follows: $\frac{x}{s}+\frac{y}{t}=1$, where ***s*** is the ***x***-intercept, and ***t*** is the ***y***-intercept. And looking at the line ***F***, we can its both intercepts, the ***x***-intercept is 1, and the ***y***-intercept is 2. So using the intercept form above, we get $\frac{x}{1}+\frac{y}{2}=1$, which is the line ***F***, and of course, we can quickly put it this way, too: $y = -2x + 2$. What if we forget the form though?

We can just use the point-slope form, $(y - t) = a(x - s)$, where ***a*** is the slope, and (s, t) is a point in the line.
Also, finding the equation of the line ***E***, we can just use the point-slope form, too.

Next, moving on to the lines below, we can see the line ***G*** includes two points (-5, -3), and (-4, 1), and the line ***H*** passes through two points (-2, 0) and (0, 1).

Fig. 4



And the line ***I*** has two points (2, 1), and (3, 0), the line ***J*** has (3, 7) and (4, 4), the line ***K*** has (-1, -3) and (4, 0), and the line ***L*** has (6, -4) and (8, 2).

And finding all the equations, we can just use the point-slope form, $(y - t) = a(x - s)$. Or we can use either of the other forms: the slope-intercept form and the intercept form depending on points the line passes through.

To begin with, assuming a is the slope of the segment between (x_1, y_1) and (x_2, y_2), we get $a = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.

So first, in the case of G, using (-5, -3), and (-4, 1), we get $a = \frac{-3 - 1}{-5 - (-4)} = \frac{-4}{-5 + 4} = 4$.

And next, using (-4, 1), we get $y - 1 = 4(x + 4)$, which is the line G, can be put this way, too: $y - 1 = 4(x + 4) \Rightarrow y = 4x + 16 + 1 = 4x + 17 \Rightarrow y = 4x + 17$.

Next, in the case of H, using the x-intercept at (-2, 0) and the y-intercept at (0, 1), we get $\frac{x}{-2} + \frac{y}{1} = 1$, which is the line H, and we can quickly put it this way, too: $y = x/2 + 1$.

Next, in the case of I, using (2, 1), and (3, 0), we get $a = \frac{1 - 0}{2 - 3} = -1$.

And next, using the x-intercept at (3, 0), together with the slope-intercept form, we get $y = ax + b \Rightarrow y = -x + b \Rightarrow 0 = -3 + b \Rightarrow b = 3 \Rightarrow y = -x + 3$, which is the line I, where the y-intercept is 3, of course.

Next, in the case of J, using (3, 7) and (4, 4), we get $a = \frac{7 - 4}{3 - 4} = \frac{3}{-1} = -3$.

And next, using (4, 4), we get $y - 4 = -3(x - 4)$, which is the line J, can be put this way, too: $y - 4 = -3(x - 4) \Rightarrow y = -3x + 12 + 4 = -3x + 16 \Rightarrow y = -3x + 16$.

Next, in the case of K, using (-1, -3) and (4, 0), we get $a = \frac{-3 - 0}{-1 - 4} = \frac{3}{5}$.

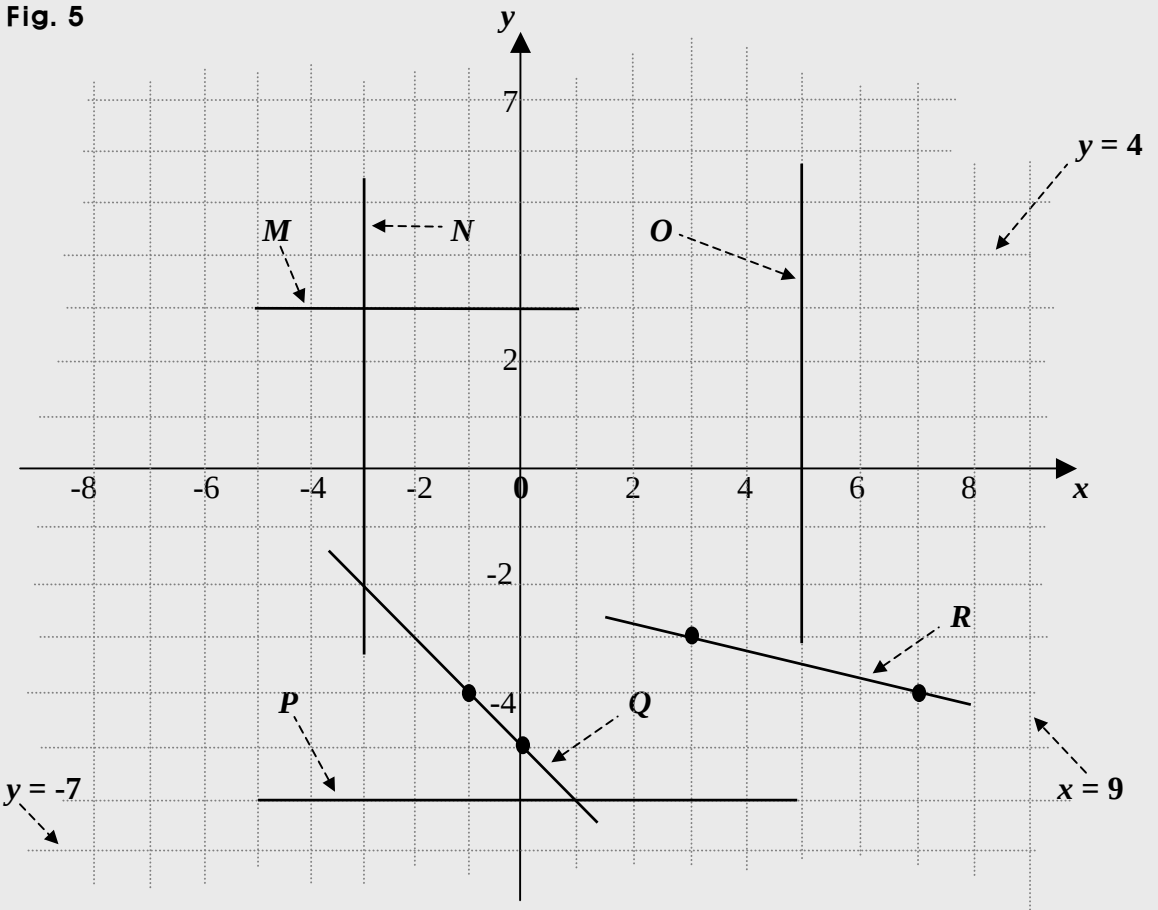
And next, using the x-intercept at (4, 0), together with the slope-intercept form, we get $y = ax + b \Rightarrow y = (3/5)x + b \Rightarrow 0 = (3/5)4 + b \Rightarrow b = -12/5 \Rightarrow y = (3/5)x - 12/5$, which is the line K, where the y-intercept is –12/5, of course.

Next, in the case of L, using (6, -4) and (8, 2), we get $a = \frac{-4 - 2}{6 - 8} = \frac{-6}{-2} = 3$.

And next, using (6, -4), we get $y + 4 = 3(x - 6)$, which is the line L, can be put this way, too: $y + 4 = 3(x - 6) \Rightarrow y = 3x - 18 - 4 = 3x - 22 \Rightarrow y = 3x - 22$.

Next, moving on to the lines below, we can see the line M is parallel to the x-axis, and includes a point (0, 3), so M is a line, $y = 3$, and the line N is parallel to the y-axis, and includes a point (-3, 0), so N is a line, $x = -3$, and by the same token, O is a line, $x = 5$. Also, likewise, P is a line, $y = -6$.

Fig. 5



And the line Q includes two points (-1, -4), and (0, -5), and the line R passes through two points (3, -3) and (7, -4).

First, in the case of Q, using (-1, -4) and (0, -5), we get $a = \frac{-4-(-5)}{-1-0} = \frac{1}{-1} = -1$.

And next, using the y-intercept at (0, -5), together with the slope-intercept form, we can simply get $y = ax + b \Rightarrow y = -x + (-5) \Rightarrow y = -x - 5$, which is the line Q.

Next, in the case of ***R***, using (3, -3) and (7, -4), we get $\boldsymbol{a = \frac{-3-(-4)}{3-7} = \frac{-3+4}{-4} = -\frac{1}{4}}$.

And next, using (3, -3), we get $\boldsymbol{y + 3 = -(x - 3)/4}$, which is the line ***R***, can be put this way, too: $\boldsymbol{y + 3 = -(x - 3)/4 \Rightarrow y = -x/4 + 3/4 - 3 = -x/4 - 9/4 \Rightarrow y = -x/4 - 9/4}$.