Examples B in Lines

Click this to start.

Examples B in Lines

Graphing matters, and it does do a lot when we solve many problems.
So it's a good idea to make graphs when you solve problems.
Graphs can help see problems better.

They can help picture the problem clearer, so you can see better what you can do about it, and in turn, whereabouts the solution is around.

Quite often, you can get across the solution while you are making the graph.
The more accurate, the better, of course. It doesn't be very accurate though at the beginning. It is often enough to do draw it schematically to get the idea.

So let's now, put some lines in graphs.

Construct the graph of each of the equations as follows.

1.0. $\boldsymbol{y = |3 - 2x| - 1}$.

1.1. $\boldsymbol{y = |2x - 3| + 3x + 1}$.

1.2. $\boldsymbol{y = x - \frac{|3x-2|}{2}}$.

1.3. $\boldsymbol{2x + \left|\frac{y}{2}\right| = 2}$.

1.4. $\boldsymbol{|2x| + \frac{y}{2} = 2}$.

1.5. $\boldsymbol{|2x| + \left|\frac{y}{2}\right| = 2}$.

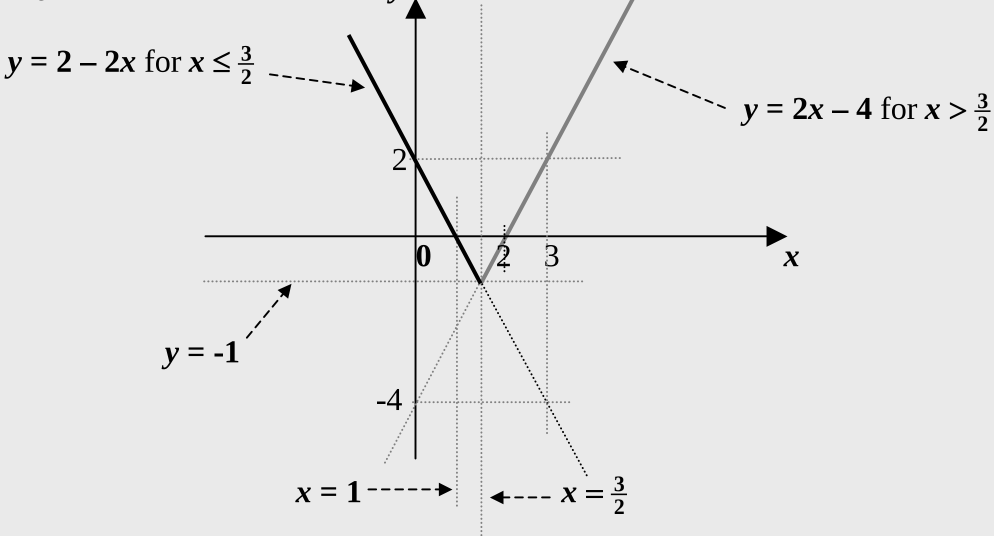
Suggestions or Solutions
To the Problem in the Example 0

The equation is $\boldsymbol{y = |3 - 2x| - 1}$.   And we can get it's graph the way as follows.

$$\mathbf{3 - 2}\boldsymbol{x} \ge \mathbf{0} \Rightarrow \mathbf{2}\boldsymbol{x} \le \mathbf{3} \Rightarrow \boldsymbol{x} \le \tfrac{3}{2} \Rightarrow \boldsymbol{y} = \mathbf{3 - 2}\boldsymbol{x} \mathbf{- 1 = 2 - 2}\boldsymbol{x} \text{ for } \boldsymbol{x} \le \tfrac{3}{2}.$$

$$\mathbf{3 - 2}\boldsymbol{x} < \mathbf{0} \Rightarrow \mathbf{2}\boldsymbol{x} > \mathbf{3} \Rightarrow \boldsymbol{x} > \tfrac{3}{2} \Rightarrow \boldsymbol{y} = \mathbf{-(3 - 2}\boldsymbol{x}\mathbf{) - 1 = -3 + 2}\boldsymbol{x} \mathbf{- 1 = 2}\boldsymbol{x} \mathbf{- 4} \text{ for } \boldsymbol{x} > \tfrac{3}{2}.$$

Fig. 0



If not quite sure of the idea behind the solution above, follow the steps below.

Values can be positive, 0, or negative.
An absolute value though, is a magnitude, which therefore, can only be 0 or positive.
Indicating the magnitude of a value, we use a pair of short vertical bars.

For instance, $|\boldsymbol{v}|$ indicates the absolute value (the magnitude) of $\boldsymbol{v}$, so a set of two vertical bars **| |** is called the absolute sign.

For instance, we get $|\boldsymbol{t}| = \mathbf{1}$ when $\boldsymbol{t} = \mathbf{-1}$ as well as $\boldsymbol{t} = \mathbf{1}$, since $|\boldsymbol{x}|$ indicates the magnitude of the value $\boldsymbol{t}$ has.   So for any value of $\boldsymbol{x}$, we get $|\boldsymbol{x}| \ge \mathbf{0}$.

Suppose for another instance, $A = x - 3$, and $B = x - y$.

Then, if $x = 3$ and $y = 5$, we get $|A| = |x - 3| = 0$, and $|B| = |x - y| = 2$.

Thus, absolute values are greater than or equal to 0.

So for instance, when working with $|K|$, we need to consider two cases.

One is the case where $K \geq 0$. And the other is the case where $K < 0$.

By the way, we have $|-x| = |x| \geq 0$, too. So what?

If $x \geq 0$, we get $|-x| = |x| = x$, and if $x < 0$, we get $|-x| = |x| = -x$.

The fact above might seem quite obvious, but it is the fact we use when we remove or take care of the absolute sign.

Now, in this example, the equation given is $y = |3 - 2x| - 1$, and in the equation, the absolute sign is applied.

So we want to first, remove the absolute sign from $|3 - 2x|$.

We have two cases for $(3 - 2x)$.

In one, we have $3 - 2x \geq 0$, and in the other, we have $3 - 2x < 0$.

So first, taking care of this: $3 - 2x \geq 0$, and removing the absolute sign, we get

$|3 - 2x| = 3 - 2x$, simply because $3 - 2x \geq 0$, and $|3 - 2x| \geq 0$.

And next, taking care of this: $3 - 2x < 0$, and removing the absolute sign, we get

$|3 - 2x| = -(3 - 2x)$, simply because $3 - 2x < 0$, and $|3 - 2x| \geq 0$.

Next, getting back to this again: $3 - 2x \geq 0$, we get $3 - 2x \geq 0 \Rightarrow 2x \leq 3 \Rightarrow x \leq \frac{3}{2}$.

So we get $3 - 2x \geq 0 \Rightarrow 2x \leq 3 \Rightarrow x \leq \frac{3}{2} \Rightarrow |3 - 2x| = 3 - 2x$.

And next, going back to this case: $3 - 2x < 0$, we get $3 - 2x < 0 \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2}$.

So we get $3 - 2x < 0 \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2} \Rightarrow |3 - 2x| = -(3 - 2x)$.

Still not quite clear why we get **$|3 - 2x| = -(3 - 2x)$** when **$3 - 2x < 0$**?

That's because **$|3 - 2x|$** is the magnitude of (**$3 - 2x$**), and if (**$3 - 2x$**) is negative, we need to make it positive, since a magnitude can only be 0 or positive.

So now, removing the absolute sign in **$y = |3 - 2x| - 1$**, we get two equations as follows.

$3 - 2x \geq 0 \Rightarrow 2x \leq 3 \Rightarrow x \leq \frac{3}{2} \Rightarrow y = 3 - 2x - 1 = 2 - 2x$ for $x \leq \frac{3}{2}$.

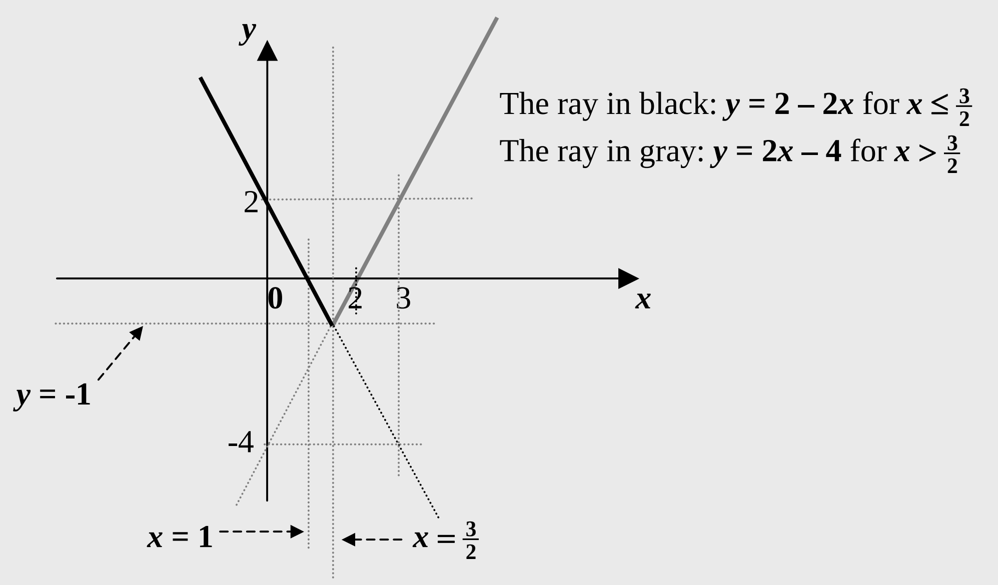
$3 - 2x < 0 \Rightarrow 2x > 3 \Rightarrow x > \frac{3}{2} \Rightarrow y = -(3 - 2x) - 1 = -3 + 2x - 1 = 2x - 4$ for $x > \frac{3}{2}$.

Therefore, we can say that the equation **$y = |3 - 2x| - 1$** is made of two equations.

One is **$y = 2 - 2x$** for **$x \leq \frac{3}{2}$**, and the other is **$y = 2x - 4$** for **$x > \frac{3}{2}$**.

So now, putting **$y = |3 - 2x| - 1$** in a graph, we can get

Fig. 0



The rays in gray and black are symmetric to each other about a line **$x = \frac{3}{2}$**.

The ray in black includes a point **$(\frac{3}{2}, -1)$**, but the one in gray doesn't include it, which is important when you do algebra.

Suggestions or Solutions
To the Problem in the Example 1

The equation given is $\boldsymbol{y = |2x - 3| + 3x + 1}$.

The absolute sign is applied to $(\boldsymbol{2x - 3})$, so we get two cases for $(\boldsymbol{2x - 3})$.
In one, we get $\boldsymbol{2x - 3 \geq 0}$, and in the other, we get $\boldsymbol{2x - 3 < 0}$.

So removing the absolute sign in the equation above, we get

$\boldsymbol{2x - 3 \geq 0 \Rightarrow 2x \geq 3 \Rightarrow x \geq \frac{3}{2}}$, and thus, $\boldsymbol{y = 2x - 3 + 3x + 1 = 5x - 2}$ for $\boldsymbol{x \geq \frac{3}{2}}$.

$\boldsymbol{2x - 3 < 0 \Rightarrow 2x < 3 \Rightarrow x < \frac{3}{2}}$, and thus, $\boldsymbol{y = -(2x - 3) + 3x + 1 = x + 4}$ for $\boldsymbol{x < \frac{3}{2}}$.

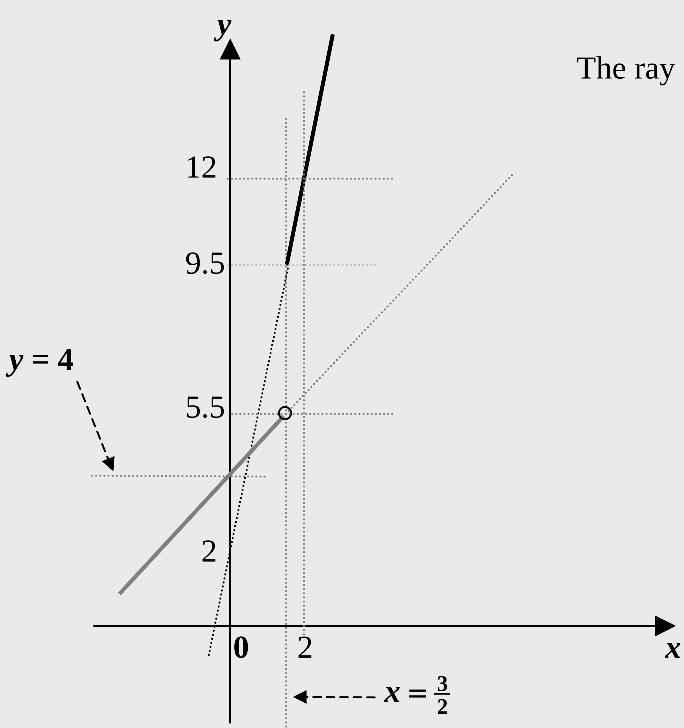
Therefore, we get $\boldsymbol{y = 5x + 2}$ for $\boldsymbol{x \geq \frac{3}{2}}$, and $\boldsymbol{y = x + 4}$ for $\boldsymbol{x < \frac{3}{2}}$.

And putting in a graph $\boldsymbol{y = |2x - 3| + 3x + 1}$, we can put it the way below.

Fig. 0

The ray in black: $\boldsymbol{y = 5x + 2}$ for $\boldsymbol{x \geq \frac{3}{2}}$

The ray in gray: $\boldsymbol{y = x + 4}$ for $\boldsymbol{x < \frac{3}{2}}$



The ray in black includes a point at **(1.5, 9.5)**, but the one in gray doesn't include a point at **(1.5, 5.5)**.

Suggestions or Solutions
To the Problem in the Example 2

The equation given is $y = x - \frac{|3x-2|}{2}$, which has the absolute sign applied to (**3*x* – 2**).

So the sign of the value of (**3*x* – 2**) changes depending on the value of ***x***. Thus, we want to put (**3*x* – 2**) in two cases. One is $3x - 2 \geq 0$, and in the other is $3x - 2 < 0$.

So removing the absolute sign in the equation, we get

$3x - 2 \geq 0 \Rightarrow 3x \geq 2 \Rightarrow x \geq \frac{3}{2}$, and thus, $y = x - \frac{3x-2}{2} = \frac{2x-3x+2}{2} = \frac{2-x}{2} = 1 - \frac{x}{2}$ for $x \geq \frac{3}{2}$.

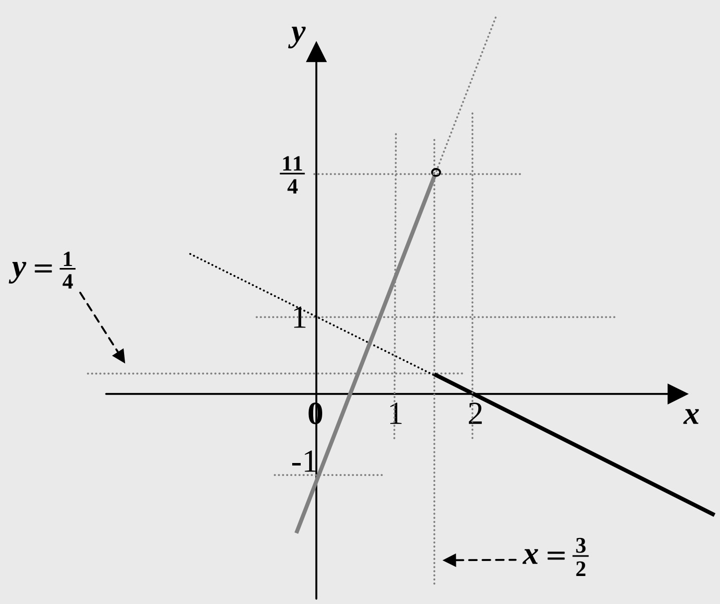
$3x - 2 < 0 \Rightarrow x < \frac{3}{2}$, and thus, $y = x - \frac{-(3x-2)}{2} = \frac{2x+3x-2}{2} = \frac{5x-2}{2} = \frac{5}{2}x - 1$ for $x < \frac{3}{2}$.

Therefore, we get $y = -\frac{1}{2}x + 1$ for $x \geq \frac{3}{2}$, and $y = \frac{5}{2}x - 1$ for $x < \frac{3}{2}$.

So we can graph the equation $y = x - \frac{|3x-2|}{2}$ the way below.

Fig. 0

The ray in black: $y = -\frac{1}{2}x + 1$ for $x \geq \frac{3}{2}$
The ray in gray: $y = \frac{5}{2}x - 1$ for $x < \frac{3}{2}$



The ray in gray does not have a point at $(\frac{3}{2}, \frac{11}{4})$.
However, the ray in black has a point at $(\frac{3}{2}, \frac{1}{4})$.

Suggestions or Solutions
To the Problem in the Example 3

The equation given is $\mathbf{2x + |\frac{y}{2}| = 2}$. And we can get the graph the way as follows.

First, if $y \geq 0$, we get $2x + |\frac{y}{2}| = 2 \Rightarrow 2x + \frac{y}{2} = 2 \Rightarrow y = 4 - 4x$ for $y \geq 0$.

And we get $y \geq 0 \Rightarrow 4 - 4x \geq 0 \Rightarrow 4x \leq 4 \Rightarrow x \leq 1$.

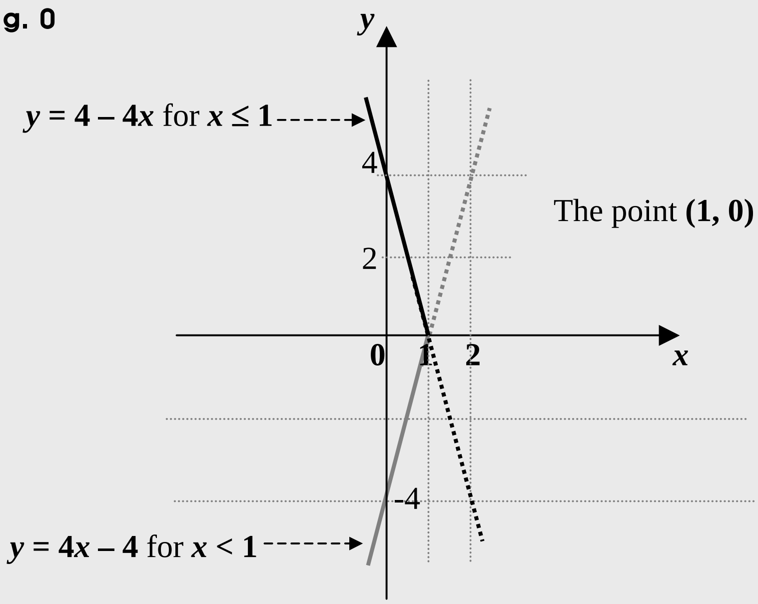
So we get $y = 4 - 4x$ for $x \leq 1$.

Next, if $y < 0$, we get $2x + |\frac{y}{2}| = 2 \Rightarrow 2x - \frac{y}{2} = 2 \Rightarrow y = 4x - 4$ for $y < 0$.

And we get $y < 0 \Rightarrow 4x - 4 < 0 \Rightarrow 4x < 4 \Rightarrow x < 1$.

Thus, we get $y = 4x - 4$ for $x < 1$.

Fig. 0



The point **(1, 0)** belongs to the ray in black.

If not quite sure of the idea behind the solution above, follow the steps below.

No matter what value y may take, $|\frac{y}{2}|$ indicates half the magnitude of the y-value.

So $\frac{|y|}{2}$ is the same as $|\frac{y}{2}|$, and thus, $2x + |\frac{y}{2}| = 2$ is the same as $2x + \frac{|y|}{2} = 2$.

And keep in mind that it's not just trivial that we have **|-*x*| = |*x*| ≥ 0**, too.
So if $x \geq 0$, we get **|-*x*| = |*x*| = *x***, and if $x < 0$, we get **|-*x*| = |*x*| = -*x***.

Now, the given equation $2x + \left|\frac{y}{2}\right| = 2$ is no other than $2x + \frac{|y|}{2} = 2$.
So we need to take care of two cases, one is $y \geq 0$, and the other is $y < 0$.

Beginning thus, with the case where $y \geq 0$, we get $2x + \left|\frac{y}{2}\right| = 2 \Rightarrow 2x + \frac{y}{2} = 2$.

So we get $y = 4 - 4x$ for $y \geq 0$. And we put its graph in the ***x*-*y*** plane.

So next, we want to get the extent of ***x***.

Finding thus, the extent of ***x***, we get $y \geq 0 \Rightarrow 4 - 4x \geq 0 \Rightarrow 4x \leq 4 \Rightarrow x \leq 1$.

So we get $y = 4 - 4x$ for $x \leq 1$.

And next, moving on to the case where $y < 0$, we get $2x + \left|\frac{y}{2}\right| = 2 \Rightarrow 2x - \frac{y}{2} = 2$.

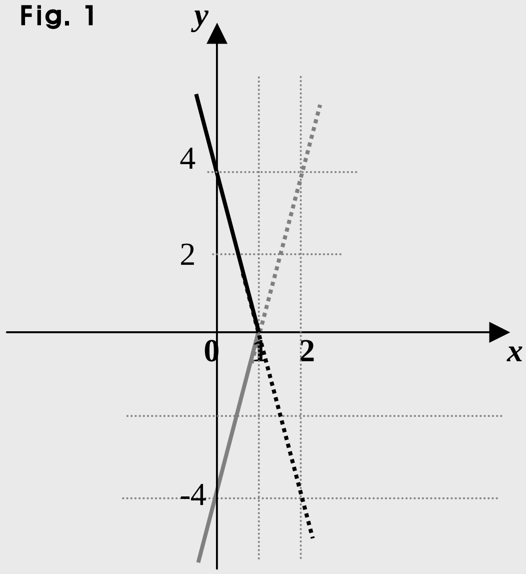
Thus, we get $y = 4x - 4$ for $y < 0$.

Next, finding the extent of ***x***, we get $y < 0 \Rightarrow 4x - 4 < 0 \Rightarrow 4x < 4 \Rightarrow x < 1$.

So we get $y = 4x - 4$ for $x < 1$.

And thus, the equation $2x + \left|\frac{y}{2}\right| = 2$ can be graphed the way below.

Fig. 1



The ray in black: $y = 4 - 4x$ for $x \leq 1$
The ray in gray: $y = 4x - 4$ for $x < 1$

The point **(1, 0)** belongs to the ray in black.

The rays in gray and black are symmetric to each other about the ***x***-axis.

What if we want to put it in the ***y*-*x*** plane, though?

We can certainly do so, if we really want to. Then, we want to put $\boldsymbol{x}$ in terms of $\boldsymbol{y}$, that is, solve the equation above for $\boldsymbol{x}$.

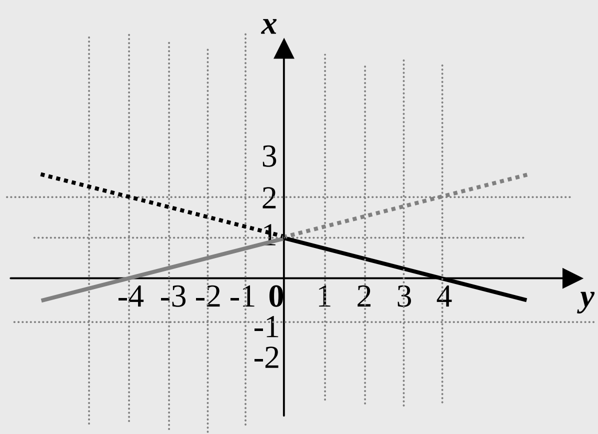
So first, we have $\boldsymbol{y = 4 - 4x}$ for $\boldsymbol{y} \geq \mathbf{0}$.
So we get $\boldsymbol{y = 4 - 4x \Rightarrow 4x = 4 - y \Rightarrow x = -\frac{y}{4} + 1}$ for $\boldsymbol{y} \geq \mathbf{0}$.

And next, we have $\boldsymbol{y = 4x - 4}$ for $\boldsymbol{y} < \mathbf{0}$.
So we get $\boldsymbol{y = 4x - 4 \Rightarrow 4x = y + 4 \Rightarrow x = \frac{y}{4} + 1}$ for $\boldsymbol{y} < \mathbf{0}$.

And thus, we can graph the equation $\boldsymbol{2x + |\frac{y}{2}| = 2}$ the way below, too.

Fig. 2

The ray in black: $\boldsymbol{x = -\frac{y}{4} + 1}$ for $\boldsymbol{y} \geq \mathbf{0}$
The ray in gray: $\boldsymbol{x = \frac{1}{4}y + 1}$ for $\boldsymbol{y} < \mathbf{0}$.



The graph above can be said to be in the $\boldsymbol{y}$-$\boldsymbol{x}$ plane, and the point **(0, 1)** belongs to the ray in black.

Suggestions or Solutions
To the Problem in the Example 4

The equation given is $|\mathbf{2}x|+\frac{y}{2}=\mathbf{2}$.

First, express the equation for each interval we can get from **|2*x*|**.
|2*x*| is the magnitude of a value twice the ***x***-value. If ***x*** is positive, **2*x*** is positive, too, if ***x*** is negative, **2*x*** is negative, also, and thus, we can get **|2*x*| = 2|*x*|**.

Thus, we get two intervals, one is $x \geq 0$, and the other is $x < 0$.
So first, we can convert the given equation in such a way as follows.

$|2x|+\frac{y}{2}=2 \Rightarrow 2|x|+\frac{y}{2}=2 \Rightarrow ?\ x|+\frac{y}{4}=1$.

If more comfortable with a form of ***y*** = bla bla bla, we can put it that way, too, so

$|x|+\frac{y}{4}=1 \Rightarrow \frac{y}{4}=1-|x| \Rightarrow y=4-4|x|$, which is similar to the standard form.

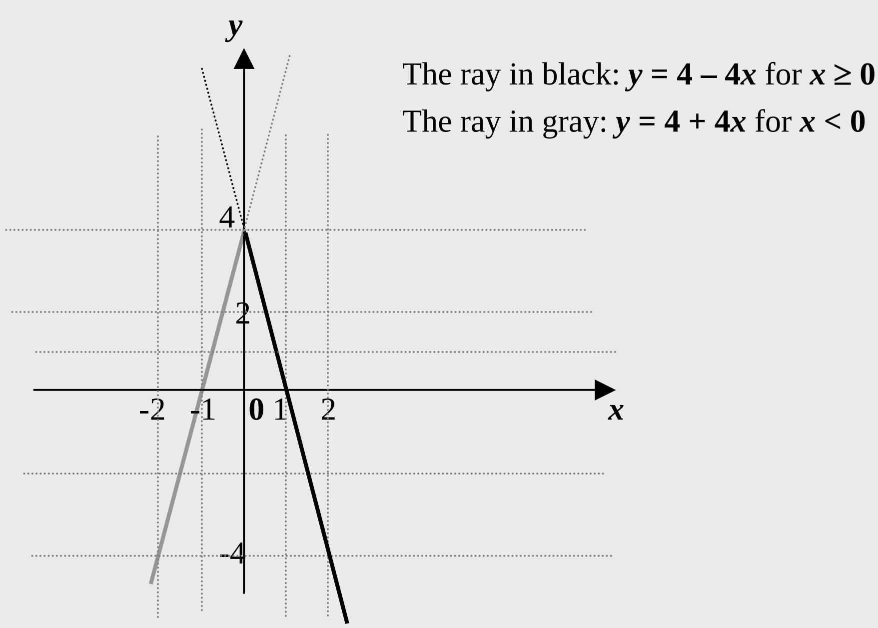
Next, removing the absolute sign, we get

$x \geq 0 \Rightarrow y = 4 - 4x$, and $x < 0 \Rightarrow y = 4 - 4(-x) = 4 + 4x$.

So we get $y = 4 - 4x$ for $x \geq 0$, and $y = 4 + 4x$ for $x < 0$.

Now, putting $|2x|+\frac{y}{2}=2$ in a graph, we get

Fig. 0



The rays in gray and black are symmetric to each other about the ***y***-axis.

Suggestions or Solutions
To the Problem in the Example 5

We have $|\,2x\,| + |\,\frac{y}{2}\,| = 2$.

To begin with, we have $|2x| = 2|x|$, and $|\,\frac{y}{2}\,| = \frac{|y|}{2}$.

So next, we can put the equation given the way below.

$|\,2x\,| + |\,\frac{y}{2}\,| = 2 \;\Rightarrow 2|x| + \frac{|y|}{2} = 2 \Rightarrow |y| = 4 - 4|x|$.

So in this case, the absolute signs are applied to both the two variables x and y.
And in that case, we need to consider four cases, and the four are as follows.

$x \geq 0$ and $y \geq 0$, $x \geq 0$ and $y < 0$, $x < 0$ and $y \geq 0$, $x < 0$ and $y < 0$.

So next, considering each of the four cases, we get

$x \geq 0$ and $y \geq 0 \Rightarrow y = 4 - 4x$ where $x \geq 0$ and $y \geq 0$.

$x \geq 0$ and $y < 0 \Rightarrow -y = 4 - 4x \Rightarrow y = 4x - 4$ where $x \geq 0$ and $y < 0$.

$x < 0$ and $y \geq 0 \Rightarrow y = 4 - 4(-x) = 4 + 4x \Rightarrow y = 4x + 4$ where $x < 0$ and $y \geq 0$.

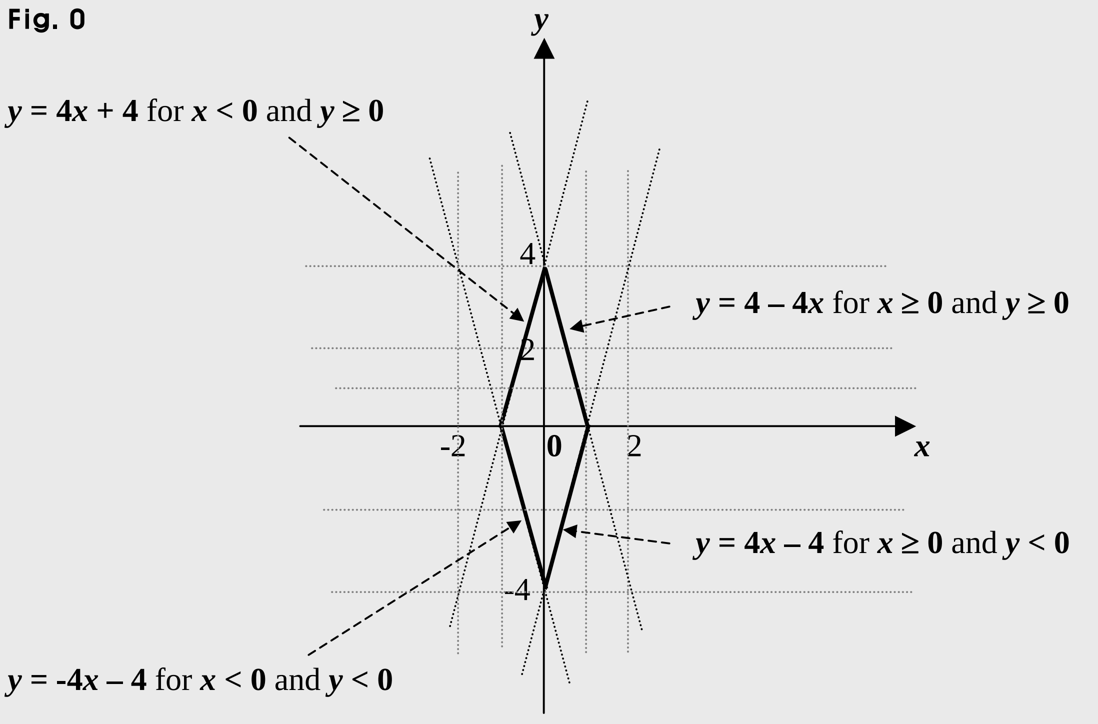
$x < 0$ and $y < 0 \Rightarrow -y = 4 - 4(-x) = 4 + 4x \Rightarrow y = -4x - 4$ where $x < 0$ and $y < 0$.

So now, putting threads together, we have

$y = 4 - 4x$ where $x \geq 0$ and $y \geq 0$.
$y = 4x - 4$ where $x \geq 0$ and $y < 0$.
$y = 4x + 4$ where $x < 0$ and $y \geq 0$.
$y = -4x - 4$ where $x < 0$ and $y < 0$.

And thus, graphing this: $|\,2x\,| + |\,\frac{y}{2}\,| = 2$, we graph this: $|y| = 4 - 4|x|$.
And graphing it, we can put it in a graph the way below.

Fig. 0



Each line segment is symmetric to each of the others about each axis and the origin.

The segment (***y* = 4 – 4*x*** for ***x* ≥ 0** and ***y* ≥ 0**) includes the points **(0, 4)** and **(1, 0)**.

The segment (***y* = 4*x* – 4** for ***x* ≥ 0** and ***y* < 0**) includes the point **(0, -4)** but not **(1, 0)**.

The segment (***y* = -4*x* – 4** for ***x* < 0** and ***y* < 0**) does not include both of the end points.

The segment (***y* = 4*x* + 4** for ***x* < 0** and ***y* ≥ 0**) includes **(-1, 0)** but not **(0, 4)**.