Examples C in Lines

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Examples C in Lines

Graph each of the equations below.

0. $\boldsymbol{y = \sqrt{x^2}}$.

1. $\boldsymbol{y = \frac{\sqrt{x^2}}{x}}$.

2. $\boldsymbol{y = \frac{\sqrt{x^2}}{|x|}}$.

3. $\boldsymbol{y = \sqrt{x^2 - 5x + \frac{25}{4}}}$.

4. $\boldsymbol{y = 2x\frac{|x^2-1|}{x^2-1}}$.

5. $\boldsymbol{y = \sqrt{(2x+1)^2} + \sqrt{(2x-1)^2}}$.

6. $\boldsymbol{||2x| - |3y|| = 1}$.

7. $\boldsymbol{y = |2x - 1| + |3x + 2|}$.

8. $\boldsymbol{y = ||2x - 1| - |3x - 1||}$.

Suggestions or Solutions
To the Problem in the Example 0

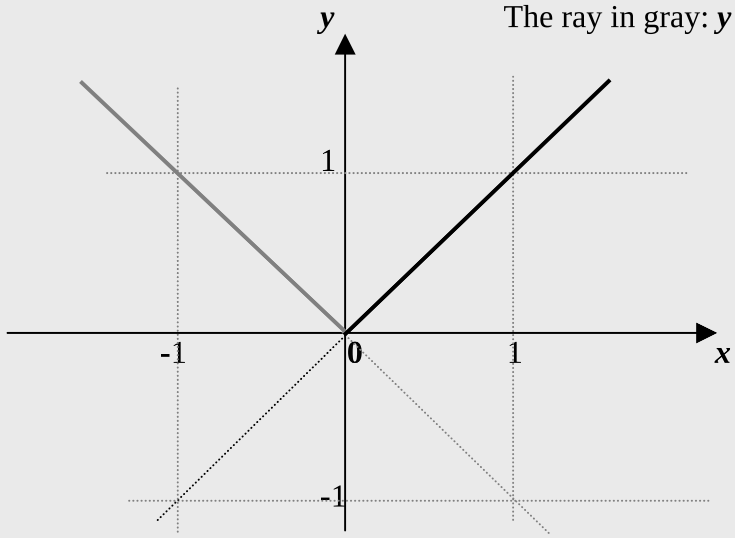
We have $y = \sqrt{x^2}$. In this case, neither of x and y has the absolute sign.
Removing however, a square root sign is as good as the removal of the absolute sign.
Removing in fact, such a root sign, we may want to use the absolute sign. How?

Can $\sqrt{x^2}$ be negative?

Of course, not, and it can only be 0 or positive.
Thus, we have $\sqrt{x^2} \geq 0$. So if $x \geq 0$, we get $\sqrt{x^2} = x$, but if $x < 0$, we get $\sqrt{x^2} = -x$.
For instance, $\sqrt{(x-2)^2} = x - 2$ if $x - 2 \geq 0$, but $\sqrt{(x-2)^2} = -(x-2)$ if $x - 2 < 0$.
Therefore, we can set $\sqrt{(x-2)^2} = |x-2|$.
So in this example, we can begin with setting $y = \sqrt{x^2} = |x|$. That is, we get $y = |x|$.
Then, we get $y = x$ for $x \geq 0$, and $y = -x$ for $x < 0$. So the graph of $y = \sqrt{x^2}$ is as follows.

Fig. 0

The ray in black: $y = x$ for $x \geq 0$
The ray in gray: $y = -x$ for $x < 0$



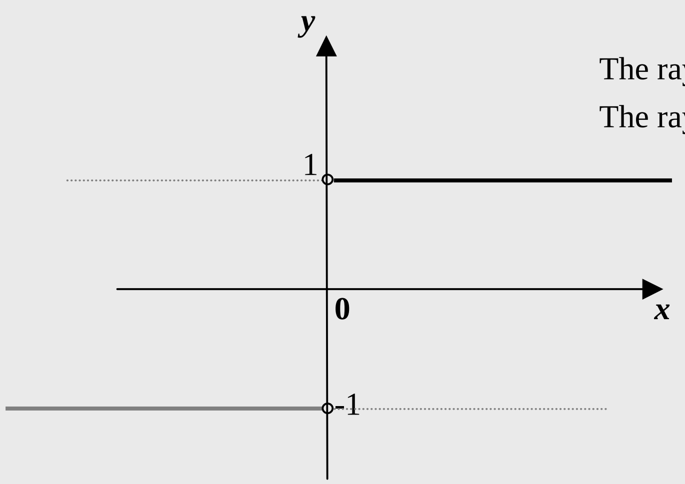
The ray in black includes the origin, but the one in gray doesn't.

Suggestions or Solutions
To the Problem in the Example 1

We have $\boldsymbol{y} = \frac{\sqrt{x^2}}{x}$. And we can graph it the way as follows.

$\boldsymbol{x > 0 \Rightarrow y = 1}$, and $\boldsymbol{x < 0 \Rightarrow y = -1}$.

Fig. 0



The ray in black: $\boldsymbol{y = 1}$ for $\boldsymbol{x > 0}$

The ray in gray: $\boldsymbol{y = 1}$ for $\boldsymbol{x < 0}$

If not quite sure of the idea behind the solution above, follow the steps below.

In this case also, neither of $\boldsymbol{x}$ and $\boldsymbol{y}$ has the absolute sign.
And the removal of the square root sign is as good as removing the absolute sign.
And we have $\sqrt{\boldsymbol{x}^2} \geq \boldsymbol{0}$, and thus, can have $\sqrt{\boldsymbol{x}^2} = |\boldsymbol{x}|$, too.

In $\boldsymbol{y} = \frac{\sqrt{x^2}}{x}$ though, we need to have $\boldsymbol{x \neq 0}$ since a denominator can never be 0, so we get

$\boldsymbol{x > 0 \Rightarrow y} = \frac{\sqrt{x^2}}{x} = \frac{x}{x} = \boldsymbol{1}$ for $\boldsymbol{x > 0}$, and not $\boldsymbol{x \geq 0}$.

And of course, we get $\boldsymbol{x < 0 \Rightarrow y} = \frac{\sqrt{x^2}}{x} = \frac{-x}{x} = \boldsymbol{-1}$ for $\boldsymbol{x < 0}$.

Thus, the equation $\boldsymbol{y} = \frac{\sqrt{x^2}}{x}$ can be graphed the way above.

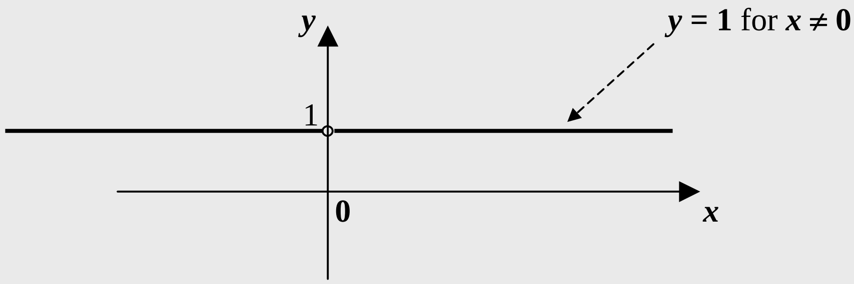
And note that the ray in gray doesn't include the point **(0, 1)**, and the one in black doesn't include the point **(0, -1)**, either.

Suggestions or Solutions
To the Problem in the Example 2

We have $y = \frac{\sqrt{x^2}}{|x|}$. And we can graph it the way below.

$x > 0 \Rightarrow y = 1$, and $x < 0 \Rightarrow y = 1$.

Fig. 0



If not quite sure of the idea behind the solution above, follow the steps below.

Unlike the previous example, x has the absolute sign. So we need to remove two signs, one is the absolute sign, and the other is the square root sign, the removal of which is as good as the removal of the absolute sign. And in fact, removing a square root sign, we often want to replace it with the absolute value sign.

So we can have $\sqrt{x^2} = |x|$. So can we get $y = \frac{\sqrt{x^2}}{|x|} = \frac{|x|}{|x|}$, or $y = \frac{\sqrt{x^2}}{|x|} = \frac{\sqrt{x^2}}{\sqrt{x^2}}$, so $y = 1$?

No, it's not quite the case. We have two cases for $|x|$, in one, we get $x \geq 0$, and in the other, we get $x < 0$, but in $y = \frac{\sqrt{x^2}}{|x|}$, x cannot be 0, since it's the denominator. So we get

$x > 0 \Rightarrow y = \frac{\sqrt{x^2}}{|x|} = \frac{x}{x} = 1$ for $x > 0$, and $x < 0 \Rightarrow y = \frac{\sqrt{x^2}}{|x|} = \frac{-x}{-x} = 1$ for $x < 0$.

Thus, we get $y = 1$ for $x \neq 0$, that is, when x is any of all real numbers but 0.

So the equation $y = \frac{\sqrt{x^2}}{|x|}$ can be graphed the way above.

And the line above does not include the point at **(0, 1)**, of course.

Suggestions or Solutions
To the Problem in the Example 3

We have $y = \sqrt{x^2 - 5x + \frac{25}{4}}$.

What's inside the radical sign above can be put in a complete square, and the radical is a square root. The removal of the square root sign is as good as that of the absolute sign. In fact, removing the square root sign, we often need to replace it with an absolute sign.

So we can have $\sqrt{A^2} = |A|$. That is, $\sqrt{A^2} = A$ if $A \geq 0$, but $\sqrt{A^2} = -A$ if $A < 0$.

Now, converting what's inside the radical sign into a complete square, we get

$x^2 - 5x + \frac{25}{4} \Rightarrow x^2 - \frac{5}{2}x + (\frac{5}{2})^2 = (x - \frac{5}{2})^2$. So we can set $y = |\, x - \frac{5}{2} \,|$.

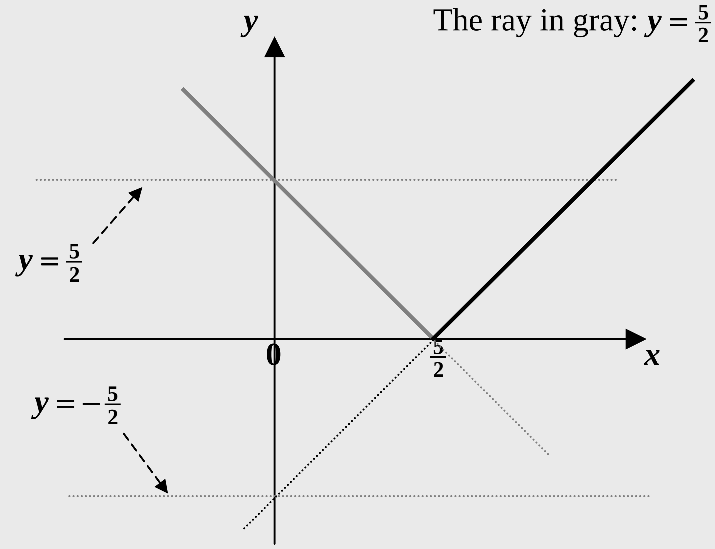
Next, removing the absolute sign, too, we get

$x - \frac{5}{2} \geq 0 \Rightarrow x \geq \frac{5}{2} \Rightarrow y = x - \frac{5}{2}$ for $x \geq \frac{5}{2}$, and $x - \frac{5}{2} < 0 \Rightarrow x < \frac{5}{2} \Rightarrow y = \frac{5}{2} - x$ for $x < \frac{5}{2}$.

Thus, putting the given equation in a graph, we get

Fig. 0

The ray in black: $y = x - \frac{5}{2}$ for $x \geq \frac{5}{2}$
The ray in gray: $y = \frac{5}{2} - x$ for $x < \frac{5}{2}$



The two rays are symmetric about a (vertical) line $x = \frac{5}{2}$.

The ray in gray doesn't include the point at $(\frac{5}{2}, 0)$, but the one in black does.

Suggestions or Solutions
To the Problem in the Example 4

We have $\boldsymbol{y = 2x\frac{|x^2-1|}{x^2-1}}$, which is not just $\mathbf{2}\boldsymbol{x}$, of course.

Removing the absolute sign, we get

First, $\boldsymbol{x^2 - 1 \ge 0 \Rightarrow (x + 1)(x - 1) \ge 0 \Rightarrow x \ge 1}$ or $\boldsymbol{x \le -1}$

$\boldsymbol{\Rightarrow y = 2x \cdot \frac{|x^2-1|}{x^2-1} = 2x \cdot \frac{x^2-1}{x^2-1} = 2x}$, so $\boldsymbol{y = 2x}$ for $\boldsymbol{x \ge 1}$ or $\boldsymbol{x \le -1}$.

Next, $\boldsymbol{x^2 - 1 < 0 \Rightarrow (x + 1)(x - 1) < 0 \Rightarrow -1 < x < 1}$

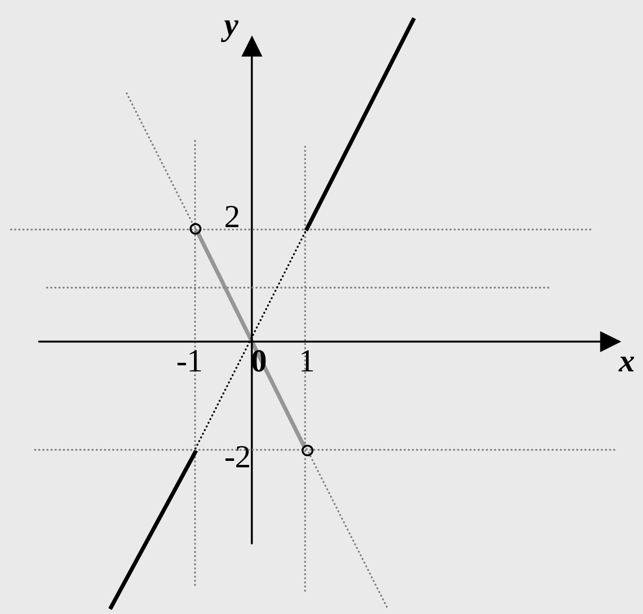
$\boldsymbol{\Rightarrow y = 2x \cdot \frac{|x^2-1|}{x^2-1} = 2x \cdot \frac{-(x^2-1)}{x^2-1} = -2x}$, so $\boldsymbol{y = -2x}$ for $\boldsymbol{-1 < x < 1}$.

Thus, putting in a graph $\boldsymbol{y = 2x\frac{|x^2-1|}{x^2-1}}$, we get

Fig. 0

The rays in black: $\boldsymbol{y = 2x}$ for $\boldsymbol{x \ge 1}$ or $\boldsymbol{x \le -1}$

The line segment in gray: $\boldsymbol{y = -2x}$ for $\boldsymbol{-1 < x < 1}$



The black rays include two points **(-1, -2)** and **(1, 2)**, but the line segment in gray doesn't include both of the end points **(-1, 2)** and **(1, -2)**.

Suggestions or Solutions
To the Problem in the Example 5

We have $y = \sqrt{(2x+1)^2} + \sqrt{(2x-1)^2}$.

Removal of a square root sign is as good as the removal of the absolute sign.
Since we have two such roots, we need to consider 4 cases as follows.

$(2x + 1 \geq 0$ and $2x - 1 \geq 0) \Rightarrow (x \geq -\frac{1}{2}$ and $x \geq \frac{1}{2}) \Rightarrow x \geq \frac{1}{2}$
$\Rightarrow y = 2x + 1 + 2x - 1 = 4x$ for $x \geq \frac{1}{2}$.

$(2x + 1 \geq 0$ and $2x - 1 < 0) \Rightarrow (x \geq -\frac{1}{2}$ and $x < \frac{1}{2}) \Rightarrow -\frac{1}{2} \leq x < \frac{1}{2}$
$\Rightarrow y = 2x + 1 + \{-(2x - 1)\} = 2x + 1 - 2x + 1 = 2$ for $-\frac{1}{2} \leq x < \frac{1}{2}$.

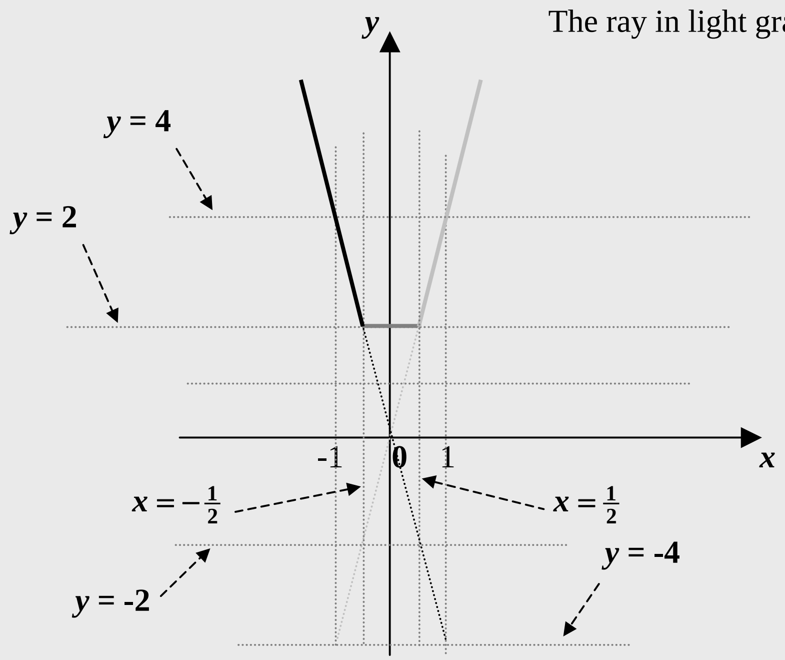
$(2x + 1 < 0$ and $2x - 1 \geq 0) \Rightarrow (x < -\frac{1}{2}$ and $x \geq \frac{1}{2}) \Rightarrow$ No existence

$(2x + 1 < 0$ and $2x - 1 < 0) \Rightarrow (x < -\frac{1}{2}$ and $x < \frac{1}{2}) \Rightarrow x < -\frac{1}{2}$
$\Rightarrow y = -(2x + 1) + \{-(2x - 1)\} = -2x - 1 - 2x + 1 = -4x$ for $x < -\frac{1}{2}$.

Thus, putting the given equation in a graph, we get

Fig. 0

The ray in black: $y = 4x$ for $x \geq \frac{1}{2}$
The segment in gray: $y = 2$ for $-\frac{1}{2} \leq x < \frac{1}{2}$
The ray in light gray: $y = -4x$ for $x < -\frac{1}{2}$



Suggestions or Solutions
To the Problem in the Example 6

The equation given is $||2x| - |3y|| = 1$. We need to consider the case where what's inside the absolute signs is negative, and also, the case where it is greater than or equal to 0. So we need to consider not only $|x|$ and $|y|$, but $|2x| - |3y| \geq 0$ and $|2x| - |3y| < 0$, too. Thus, we need to consider 8 cases as follows.

$(x \geq 0, y \geq 0,$ and $2x \geq 3y) \Rightarrow (x \geq 0, y \geq 0,$ and $y \leq \frac{2}{3}x) \Rightarrow 2x - 3y = 1$.
$(x \geq 0, y \geq 0,$ and $2x < 3y) \Rightarrow (x \geq 0, y \geq 0,$ and $y > \frac{2}{3}x) \Rightarrow -(2x - 3y) = 3y - 2x = 1$.

$(x \geq 0, y < 0,$ and $2x \geq -3y) \Rightarrow (x \geq 0, y < 0,$ and $y \geq -\frac{2}{3}x) \Rightarrow 2x - (-3y) = 2x + 3y = 1$.

Why do we have $-3y$ in "$2x \geq -3y$" above?

That's because $|3y|$ becomes $-3y$, since $y < 0$ when the absolute sign is removed.

$(x \geq 0, y < 0,$ and $2x < -3y) \Rightarrow (x \geq 0, y < 0,$ and $y < \frac{-2}{3}x) \Rightarrow -\{2x - (-3y)\} = -2x - 3y = 1.$

$(x < 0, y \geq 0,$ and $-2x \geq 3y) \Rightarrow (x < 0, y \geq 0,$ and $y \leq -\frac{2}{3}x) \Rightarrow -2x - 3y = 1.$
$(x < 0, y \geq 0,$ and $-2x < 3y) \Rightarrow (x < 0, y \geq 0,$ and $y > -\frac{2}{3}x) \Rightarrow -(-2x - 3y) = 2x + 3y = 1.$

$(x < 0, y < 0,$ and $-2x \geq -3y) \Rightarrow (x < 0, y < 0,$ and $y \geq \frac{2}{3}x) \Rightarrow -2x - (-3y) = 3y - 2x = 1.$
$(x < 0, y < 0,$ and $-2x < -3y) \Rightarrow (x < 0, y < 0,$ and $y < \frac{2}{3}x) \Rightarrow -\{-2x - (-3y)\} = 2x - 3y = 1.$

So we get

$(x \geq 0, y \geq 0,$ and $y \leq \frac{2}{3}x) \Rightarrow 2x - 3y = 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3}$.

$(x \geq 0, y \geq 0,$ and $y > \frac{2}{3}x) \Rightarrow 3y - 2x = 1 \Rightarrow y = \frac{2}{3}x + \frac{1}{3}$.

$(x \geq 0, y < 0,$ and $y \geq -\frac{2}{3}x) \Rightarrow 2x + 3y = 1 \Rightarrow y = -\frac{2}{3}x + \frac{1}{3}$.

$(x \geq 0, y < 0,$ and $y < -\frac{2}{3}x) \Rightarrow -2x - 3y = 1 \Rightarrow y = -\frac{2}{3}x - \frac{1}{3}$.

$(x < 0, y \geq 0,$ and $y \leq -\frac{2}{3}x) \Rightarrow -2x - 3y = 1 \Rightarrow y = -\frac{2}{3}x - \frac{1}{3}$.

$(x < 0, y \geq 0,$ and $y > -\frac{2}{3}x) \Rightarrow 2x + 3y = 1 \Rightarrow y = -\frac{2}{3}x + \frac{1}{3}$.

$(x < 0, y < 0,$ and $y \geq \frac{2}{3}x) \Rightarrow 3y - 2x = 1 \Rightarrow y = \frac{2}{3}x + \frac{1}{3}$.

$(x < 0, y < 0,$ and $y < \frac{2}{3}x) \Rightarrow 2x - 3y = 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3}$.

Therefore, we get

$y = \frac{2}{3}x - \frac{1}{3}$ when ($x \geq 0$, $y \geq 0$, and $y \leq \frac{2}{3}x$) or when ($x < 0$, $y < 0$, and $y < \frac{2}{3}x$).

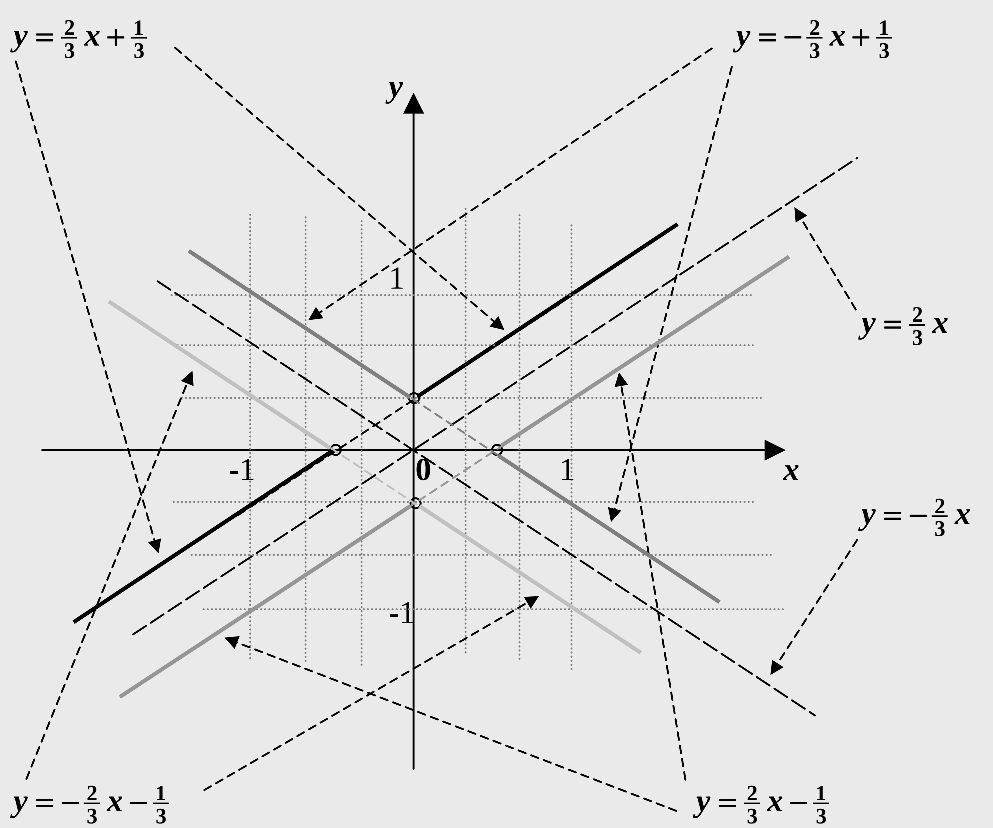
$y = \frac{2}{3}x + \frac{1}{3}$ when ($x \geq 0$, $y \geq 0$, and $y > \frac{2}{3}x$) or when ($x < 0$, $y < 0$, and $y \geq \frac{2}{3}x$).

$y = -\frac{2}{3}x + \frac{1}{3}$ when ($x \geq 0$, $y < 0$, and $y \geq -\frac{2}{3}x$) or when ($x < 0$, $y \geq 0$, and $y > -\frac{2}{3}x$).

$y = -\frac{2}{3}x - \frac{1}{3}$ when ($x \geq 0$, $y < 0$, and $y < -\frac{2}{3}x$) or when ($x < 0$, $y \geq 0$, and $y \leq -\frac{2}{3}x$).

And thus, putting in a graph, the equation $||2x| - |3y|| = 1$, we get

Fig. 0



Suggestions or Solutions
To the Problem in the Example 7

The equation given is $y = |2x - 1| + |3x + 2|$.
Since we have two sets of absolute signs, we need to consider 4 cases.

To begin with, $(2x - 1 \geq 0$ and $3x + 2 \geq 0) \Rightarrow (x \geq \frac{1}{2}$ and $x \geq -\frac{2}{3}) \Rightarrow x \geq \frac{1}{2}$.

So we get $y = 2x - 1 + 3x + 2 = 5x + 1$ for $x \geq \frac{1}{2} \Rightarrow y = 5x + 1$ for $x \geq \frac{1}{2}$.

Next, $(2x - 1 \geq 0$ and $3x + 2 < 0) \Rightarrow (x \geq \frac{1}{2}$ and $x < -\frac{2}{3}) \Rightarrow$ Nonexistence. Why?

There is no extent or value of x common to $x \geq \frac{1}{2}$, and $x < -\frac{2}{3}$, so no value for x exists in the case where $(2x - 1 \geq 0$ and $3x + 2 < 0)$. Thus, the equation gives no curve in this case.

Next, $(2x - 1 < 0$ and $3x + 2 \geq 0) \Rightarrow (x < \frac{1}{2}$ and $x \geq -\frac{2}{3}) \Rightarrow -\frac{2}{3} \leq x < \frac{1}{2}$.

So we get $y = -(2x - 1) + 3x + 2 = -2x + 1 + 3x + 2 = x + 3$ for $-\frac{2}{3} \leq x < \frac{1}{2}$

$\Rightarrow y = x + 3$ for $-\frac{2}{3} \leq x < \frac{1}{2}$.

Next, $(2x - 1 < 0$ and $3x + 2 < 0) \Rightarrow (x < \frac{1}{2}$ and $x < -\frac{2}{3}) \Rightarrow x < -\frac{2}{3}$.

So we get $y = -(2x - 1) + \{-(3x + 2)\} = -2x + 1 - 3x - 2 = -5x - 1$ for $x < -\frac{2}{3}$

$\Rightarrow y = -5x - 1$ for $x < -\frac{2}{3}$.

Thus, we get two rays and a line segment as follows.

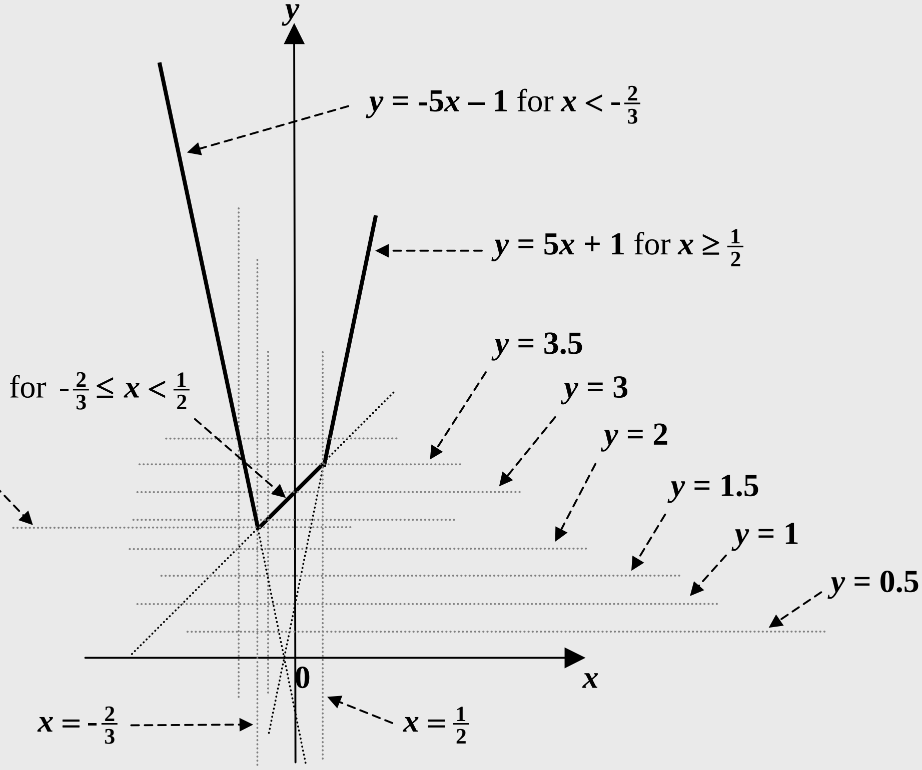
$y = 5x + 1$ for $x \geq \frac{1}{2}$.

$y = x + 3$ for $-\frac{2}{3} \leq x < \frac{1}{2}$.

$y = -5x - 1$ for $x < -\frac{2}{3}$.

Now, let's put in a graph, the equation $\boldsymbol{y = |2x - 1| + |3x + 2|}$.

Fig. 0



$\boldsymbol{y = -5x - 1}$ for $\boldsymbol{x < -\frac{2}{3}}$ does not include the point $\boldsymbol{(-\frac{2}{3}, \frac{7}{3})}$.

$\boldsymbol{y = x + 3}$ for $\boldsymbol{-\frac{2}{3} \leq x < \frac{1}{2}}$ includes the point $\boldsymbol{(-\frac{2}{3}, \frac{7}{3})}$, but not the point $\boldsymbol{(\frac{1}{2}, \frac{7}{2})}$.

$\boldsymbol{y = 5x + 1}$ for $\boldsymbol{x \geq \frac{1}{2}}$ includes the point $\boldsymbol{(\frac{1}{2}, \frac{7}{2})}$.

Suggestions or Solutions
To the Problem in the Example 8

The equation given is $y = ||2x - 1| - |3x - 1||$, which has three absolute signs.
One is $|2x - 1|$, another is $|3x - 1|$, and the other is $||2x - 1| - |3x - 1||$.

That is, setting $a = 2x - 1$, $b = 3x - 1$, $A = |a|$, and $B = |b|$, we get $y = |A - B|$.

So we want to take care of three cases $|a|$, $|b|$, and $|A - B|$, each of which has two case, and therefore, we need to consider eight cases in total.   Why?

For each of two cases of $|a|$, we need to process $|b|$ and $|A - B|$.

Processing $|b|$ and $|A - B|$, we need to process $|A - B|$ for each of two cases of $|b|$.
We have two cases for $|A - B|$, one is $A - B \geq 0$, and the other is $A - B < 0$.

In other words, we want to take care of these two cases: $A \geq B$, and $A < B$.
So processing $|b|$ and $|A - B|$, we have to take care of four cases.

Now, for each of two cases of $|a|$, we get to process $|b|$ and $|A - B|$, which has four cases, and therefore, we need to consider eight cases in total.

Though it sounds quite natural, we may want to keep in mind that an absolute value cannot be negative, that is, it has to be greater than or equal to 0.

In short, $|V| \geq 0$.

Besides, for the very same reason, we have $|V| = v$ if $v \geq 0$, and $|V| = -v$ if $v < 0$.

If things or ideas are too obvious or too close to us, we tend to ignore them.

So at least sometimes, we may want to check to see if we are not ignoring or forgetting those we need to respect and care about, which are often called basics, which is essential.
So let's get started now.

To begin with, we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, $\mathbf{3}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, and $\mathbf{(2}\boldsymbol{x} - \mathbf{1)} - \mathbf{(3}\boldsymbol{x} - \mathbf{1)} \geq \mathbf{0}$.

So we get $(\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} \geq \frac{1}{3}, \text{ and } \mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{3}\boldsymbol{x} - \mathbf{1}) \Rightarrow (\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} \geq \frac{1}{3}, \text{ and } \boldsymbol{x} \leq \mathbf{0})$.

Thus, no curve of the equation exists in this case.   Why?

There is nothing common to $(\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} \geq \frac{1}{3}, \text{ and } \boldsymbol{x} \leq \mathbf{0})$, and therefore, no value for $\boldsymbol{x}$ exists in the case where $(\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}, \mathbf{3}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}, \text{ and } \mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{3}\boldsymbol{x} - \mathbf{1})$.
We know what *and* means, don't we? Thus, the $\boldsymbol{x}$-value has to belong to all those extents, and at the same time, of course. So the equation given produces no curve in this case.

Next, we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, $\mathbf{3}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, and $\mathbf{(2}\boldsymbol{x} - \mathbf{1)} - \mathbf{(3}\boldsymbol{x} - \mathbf{1)} < \mathbf{0}$.

Thus, we get $(\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} \geq \frac{1}{3}, \text{ and } \mathbf{2}\boldsymbol{x} - \mathbf{1} < \mathbf{3}\boldsymbol{x} - \mathbf{1}) \Rightarrow (\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} \geq \frac{1}{3}, \text{ and } \boldsymbol{x} > \mathbf{0}) \Rightarrow \boldsymbol{x} \geq \frac{1}{2}$.

So we get $\boldsymbol{y} = \text{-}\{\mathbf{2}\boldsymbol{x} - \mathbf{1} - (\mathbf{3}\boldsymbol{x} - \mathbf{1})\} = \text{-}\mathbf{2}\boldsymbol{x} + \mathbf{1} + \mathbf{3}\boldsymbol{x} - \mathbf{1} = \boldsymbol{x} \Rightarrow \boldsymbol{y} = \boldsymbol{x}$ for $\boldsymbol{x} \geq \frac{1}{2}$.

Next, we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, $\mathbf{3}\boldsymbol{x} - \mathbf{1} < \mathbf{0}$, and $\mathbf{2}\boldsymbol{x} - \mathbf{1} - \{\text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1})\} \geq \mathbf{0}$.
Why do we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} - \{\text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1})\} \geq \mathbf{0}$, and not $\mathbf{(2}\boldsymbol{x} - \mathbf{1)} - \mathbf{(3}\boldsymbol{x} - \mathbf{1)} \geq \mathbf{0}$?

That's because we have $\mathbf{3}\boldsymbol{x} - \mathbf{1} < \mathbf{0}$, so we get $|\mathbf{3}\boldsymbol{x} - \mathbf{1}| = \text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1})$ in this case.
So again, we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, $\mathbf{3}\boldsymbol{x} - \mathbf{1} < \mathbf{0}$, and $\mathbf{2}\boldsymbol{x} - \mathbf{1} - \{\text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1})\} \geq \mathbf{0}$.

Thus, we get $\boldsymbol{x} \geq \frac{1}{2}$, $\boldsymbol{x} < \frac{1}{3}$, and $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1}) = \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1} \Rightarrow \mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1}$.

Meanwhile, $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1} \Rightarrow \mathbf{5}\boldsymbol{x} \geq \mathbf{2}$.
So we get $\boldsymbol{x} \geq \frac{1}{2}$, $\boldsymbol{x} < \frac{1}{3}$, and $\boldsymbol{x} \geq \frac{2}{5}$.
Thus, nothing is common to the three extents above. Hence, nonexistence.

Next, we have $\mathbf{2}\boldsymbol{x} - \mathbf{1} \geq \mathbf{0}$, $\mathbf{3}\boldsymbol{x} - \mathbf{1} < \mathbf{0}$, and $\mathbf{2}\boldsymbol{x} - \mathbf{1} - \{\text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1})\} < \mathbf{0}$.

Thus, we get $\boldsymbol{x} \geq \frac{1}{2}$, $\boldsymbol{x} < \frac{1}{3}$, and $\mathbf{2}\boldsymbol{x} - \mathbf{1} < \text{-}(\mathbf{3}\boldsymbol{x} - \mathbf{1}) = \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1} \Rightarrow \mathbf{2}\boldsymbol{x} - \mathbf{1} < \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1}$.

Meanwhile, $\mathbf{2}\boldsymbol{x} - \mathbf{1} < \text{-}\mathbf{3}\boldsymbol{x} + \mathbf{1} \Rightarrow \mathbf{5}\boldsymbol{x} < \mathbf{2}$.

So we get $(\boldsymbol{x} \geq \frac{1}{2}, \boldsymbol{x} < \frac{1}{3}, \text{ and } \boldsymbol{x} < \frac{2}{5}) \Rightarrow$ Nonexistence.

Next, we have $\mathbf{2}\boldsymbol{x}\mathbf{ - 1 < 0}$, $\mathbf{3}\boldsymbol{x}\mathbf{ - 1 \geq 0}$, and $\mathbf{-(2}\boldsymbol{x}\mathbf{ - 1) - (3}\boldsymbol{x}\mathbf{ - 1) \geq 0}$.

That's because we have $\mathbf{2}\boldsymbol{x}\mathbf{ - 1 < 0}$, so we get $\mathbf{|2}\boldsymbol{x}\mathbf{ - 1| = -(2}\boldsymbol{x}\mathbf{ - 1)}$ in this case.

Thus, we get $x < \frac{1}{2}$, $x \geq \frac{1}{3}$, and $-(2x - 1) \geq (3x - 1)$.

Meanwhile, $-(2x - 1) = -2x + 1 \geq 3x - 1 \Rightarrow 5x \leq 2$.

So we get $(x < \frac{1}{2}$, $x \geq \frac{1}{3}$, and $x \leq \frac{2}{5}) \Rightarrow \frac{1}{3} \leq x \leq \frac{2}{5}$. Thus, we get

$y = -(2x - 1) - (3x - 1) = -2x + 1 - 3x + 1 = -5x + 2 \Rightarrow y = -5x + 2$ for $\frac{1}{3} \leq x \leq \frac{2}{5}$.

Next, we have $2x - 1 < 0$, $3x - 1 \geq 0$, and $-(2x - 1) - (3x - 1) < 0$.

So we get $x < \frac{1}{2}$, $x \geq \frac{1}{3}$, and $-(2x - 1) < (3x - 1)$.

Meanwhile, $-(2x - 1) = -2x + 1 < 3x - 1 \Rightarrow 5x > 2$.

Thus, we get $(x < \frac{1}{2}$, $x \geq \frac{1}{3}$, and $x > \frac{2}{5}) \Rightarrow \frac{2}{5} < x < \frac{1}{2}$. So we get

$y = -\{-(2x - 1) - (3x - 1)\} = 2x - 1 + 3x - 1 = 5x - 2 \Rightarrow y = 5x - 2$ for $\frac{2}{5} < x < \frac{1}{2}$.

Next, we have $2x - 1 < 0$, $3x - 1 < 0$, and $-(2x - 1) - \{-(3x - 1)\} \geq 0$.

That's because we have $2x - 1 < 0$, and $3x - 1 < 0$, so we get in this case,

$|2x - 1| = -(2x - 1)$, and $|3x - 1| = -(3x - 1)$.

Thus, we get $x < \frac{1}{2}$, $x < \frac{1}{3}$, and $-(2x - 1) \geq -(3x - 1)$.

Meanwhile, $-(2x - 1) = -2x + 1 \geq -(3x - 1) = -3x + 1 \Rightarrow 2x + 1 \geq -3x + 1 \Rightarrow 5x > 0$.

Thus, we get $(x < \frac{1}{2}$, $x < \frac{1}{3}$, and $x \geq 0) \Rightarrow 0 < x < \frac{1}{3}$. So we get

$\Rightarrow y = -(2x - 1) - \{-(3x - 1)\} = -2x + 1 + 3x - 1 = x \Rightarrow y = x$ for $0 \leq x < \frac{1}{3}$.

Next, we have $2x - 1 < 0$, $3x - 1 < 0$, and $-(2x - 1) - \{-(3x - 1)\} < 0$.

Thus, we get $x < \frac{1}{2}$, $x < \frac{1}{3}$, and $-(2x - 1) < -(3x - 1)$.

Meanwhile, $-(2x - 1) = -2x + 1 < -(3x - 1) = -3x + 1 \Rightarrow -2x + 1 < -3x + 1 \Rightarrow x < 0$.

So we get $x < \frac{1}{2}$, $x < \frac{1}{3}$, and $x < 0$. Thus, we get $x < 0$.

So we get $y = -[-(2x - 1) - \{-(3x - 1)\}]$ for $x < 0$.

Why not just $y = -(2x - 1) - \{-(3x - 1)\}$, though?

That's because we have $\boldsymbol{y = ||2x - 1| - |3x - 1||}$, which cannot be negative.

We know $\boldsymbol{||2x - 1| - |3x - 1||}$ is an absolute value.

That is, we have $\boldsymbol{y = ||2x - 1| - |3x - 1|| \geq 0}$, which means, greater than *or* equal to 0.

However, we have $\boldsymbol{|2x - 1| - |3x - 1| = -(2x - 1) - \{-(3x - 1)\} < 0}$.

So we need to get $\boldsymbol{y = ||2x - 1| - |3x - 1|| = -[-(2x - 1) - \{-(3x - 1)\}]}$, which is $> \mathbf{0}$.

Meanwhile, $\boldsymbol{y = -[-(2x - 1) - \{-(3x - 1)\}] = (2x - 1) + \{-(3x - 1)\} = 2x - 1 - 3x + 1 = -x}$.

Thus, we get $\boldsymbol{y = -x}$ for $\boldsymbol{x < 0}$. Now, putting threads together, we get

$\boldsymbol{y = x}$ for $\boldsymbol{x \geq \frac{1}{2}}$, or $\boldsymbol{0 \leq x < \frac{1}{3}}$. $\boldsymbol{y = -5x + 2}$ for $\boldsymbol{\frac{1}{3} \leq x \leq \frac{2}{5}}$. $\boldsymbol{y = 5x - 2}$ for $\boldsymbol{\frac{2}{5} < x < \frac{1}{2}}$.

$\boldsymbol{y = -x}$ for $\boldsymbol{x < 0}$. Thus, putting in a graph $\boldsymbol{y = ||2x - 1| - |3x - 1||}$, we get

Fig. 0

$\boldsymbol{y = x}$ for $\boldsymbol{x \geq \frac{1}{2}}$, or $\boldsymbol{0 \leq x < \frac{1}{3}}$

$\boldsymbol{y = -x}$ for $\boldsymbol{x < 0}$

$\boldsymbol{y = 5x - 2}$ for $\boldsymbol{\frac{2}{5} < x < \frac{1}{2}}$

$\boldsymbol{y = -5x + 2}$ for $\boldsymbol{\frac{1}{3} \leq x \leq \frac{2}{5}}$