# Examples L in Lines



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# Examples L in Lines

The set of examples here is for advanced students as college students or high school students taking courses in calculus. All the examples in this set are just about the same. A minute adjustment has been made to each example in a successive manner. So you can see how a tiny change can affect the situation where a line is put.

Assuming now, $t$ is constant, determine the value of $t$ so that $t$ satisfies each case below.

0. $y = (\frac{t}{4} + 1)x + t + 4$, and $y > 0$ when $x > -3$.

1. $y = (\frac{t}{4} + 1)x + t + 5$, and $y > 0$ when $x > -3$.

2. $y = (\frac{t}{4} + 1)x + t + 3$, and $y < 0$ when $x > -3$.

3. $y = (\frac{t}{4} + 1)x + t + 4$, and $y \geq 0$ when $x > -3$.

4. $y = (\frac{t}{4} + 1)x + t + 5$, and $y \geq 0$ when $x > -3$.

5. $y = (\frac{t}{4} + 1)x + t + 3$, and $y \geq 0$ when $x > -3$.

6. $y = (\frac{t}{4} + 1)x + t + 4$, and $y > 0$ when $-2 < x < 3$.

7. $y = (\frac{t}{4} + 1)x + t + 4$, and $y > 0$ when $-5 < x < -3$.

8. $y = (\frac{t}{4} + 1)x + t + 4$, and $y > 0$ when $-5 \leq x \leq -3$.

9. $y = (\frac{t}{4} + 1)x + t + 4$, and $y \geq 0$ when $-5 < x < -3$.

A. $y = (\frac{t}{4} + 1)x + t + 4$, and $y \geq 0$ when $-5 \leq x \leq -3$.

B. $y = (\frac{t}{4} + 2)x + t + 3$, and $y$ can be positive, 0, or negative when $-5 \leq x \leq -3$.

Assuming next, $t$ is still constant, determine this time, the values of $x$ so that $x$ satisfies each case below.

C. $y = (\frac{t}{4} + 2)x + t + 3$, and $y > 0$ when $1 \leq t \leq 5$.

D. $y = 3(t - 1)x + 3 - 2t$, and $y > 0$ when $0 \leq t \leq 2$.

## Suggestions or Solutions
## To the Problem in the Example 0

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x + t + 4$, determine the value of $t$ so that $y > 0$ when $x > -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $x > -3 \Rightarrow y = f(x) > 0$.

Then, if the slope < **0**, even if $y = f(-3) > 0$, there is no value for $t$ satisfying the condition, because the line eventually goes below the $x$-axis as $x$ increases, so $y$ can be negative.

Next, if the slope = **0**, we need to have $y = f(-3) > 0$.

$\frac{t}{4} + 1 = 0 \Rightarrow t = -4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x + t + 4 = 0 \cdot x - 4 + 4 = 0$.

So the slope cannot be 0, and thus, we need to have $t \neq -4$.

Next, if the slope > **0**, we need to have $y = f(-3) \geq 0$.

First, we get $\frac{t}{4} + 1 > 0 \Rightarrow t > -4$.

And next, we get $y = f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{-3t+4t}{4} - 3 + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we have $t > -4$, and $t \geq -4$, so we get $t > -4$.

Now, we have $t \neq -4$ and $t > -4$. Therefore, we get $t > -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

To begin with, the equation indicates a line. And it is in the slope-intercept form, and in fact, can indicate a line for each value of the constant $t$.

And the form is $y = ax + b$, where $a$ is the slope, and $b$ is the $y$-intercept.

Next, since $t$ is constant, $(\frac{t}{4}+1)$ is constant, too. So for each value of $t$, we can take $(\frac{t}{4}+1)$ as a slope of a line, and take $(t + 4)$ as an $y$-intercept.

And let's next, find out about what line the equation is talking about.

The problem is telling us a condition as follows. When $x > -3$, $y > 0$.

In this case, we can call -3 in the expression $x > -3$, the critical value of $x$.

What do we mean by the critical value though?

A critical value can be said to be a value we can use when we explain the boundary of a particular state. Then, isn't it better to just call it a boundary value?

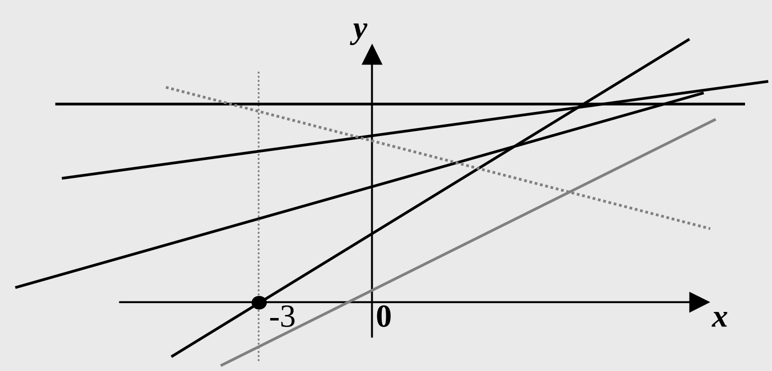
We normally use the word, 'boundary value' for some other purpose in math.
And by the same token, we can call a point **(-3, 0)**, the critical point in the line.
A critical point is like a turning point, and in this case, we can take it as a point that indicates the boundary of a particular state.

Now, the condition stated above is that the value of ***y*** is positive when the value of ***x*** is greater than -3. So what?

So let's first, put some lines in the ***x-y*** plane, and see what happens.
While putting some lines, we can see better what lines are appropriate for the condition of the problem. That is, we can see better whereabouts of the solution.

Constructing a graph, we may want to show first, such a crucial item as the critical value mentioned above. The critical value is -3, which is in the ***x***-axis, but cannot be a value of ***x*** in the equation, because we have "***x* > -3**."

**Fig. 0.0**




Now, if we want ***y*** to be only positive when ***x*** is greater than -3, the line has to be one of the black ones in the graph above. Notice that the one horizontal is one of them, too.

Let's now, call the point **(-3, 0)** in the graph above, a critical point.

Then, we can see that the line itself can include the critical point.

The line dotted in gray seems to meet the condition of the problem, yet it will eventually go below the ***x***-axis as ***x*** increases, and therefore, is not allowed.

What then, can we say about the solution?

• The solution depends on the slope. In fact, what matters most in a line is the slope.

If a slope is put in terms of variables or constants, we should consider three cases below. “The slope is positive.” “The slope is negative.” “The slope is 0.”

Then first, we can see in this problem, the slope cannot be negative.
That is, the slope has to be positive or 0.

That is because if the slope $< 0$, even if $y > 0$ when $x = -3$, we can have $y < 0$, too, since the line eventually goes below the ***x***-axis as ***x*** increases.

Now, using a function, we can easily keep track of values of variables.
So setting now, $y = f(x) = (\frac{t}{4} + 1)x + t + 4$, we need to get $x > -3 \Rightarrow y = f(x) > 0$.

Meanwhile, we should follow the fundamental principle on lines. The principle says

• Once set, the slope (or direction) of a line doesn’t change. That is, the slope is constant.

So a line can be in either of the four cases below.

- A line keeps going up (slope > 0) steadily.
- A line keeps going down (slope < 0) steadily.
- A line keeps being horizontal (slope is 0).
- A line keeps being vertical (no slope).

If the slope is positive, the line slants to the left, and if negative, it slants to the right. If the slope is 0, the line is horizontal. What then, about a vertical line?

In math, a vertical line is said to have no slope. So no slope does not mean the slope of 0.

Suppose now, $c$ is constant, and in $y = ax + b$, $y = 0$ for $x = c$, and $y > 0$ when $x > c$.
What then, is the sign of the slope $a$?

The line has to go up. So we need to have $\boldsymbol{a} > \mathbf{0}$.

Suppose next, $\boldsymbol{d}$ is constant, and in $\boldsymbol{y = ax + b}$, $\boldsymbol{y = d}$ for all $\boldsymbol{x}$.
That is, $\boldsymbol{y}$ gets the same value for every value of $\boldsymbol{x}$. What then, is the sign of the slope $\boldsymbol{a}$?

The line has to be horizontal. So we need to have $\boldsymbol{a} = \mathbf{0}$.

Now, what then, can we practically mean by the condition that if $\boldsymbol{x} > \mathbf{-3}$, $\boldsymbol{y = f(x)} > \mathbf{0}$?

The condition's practical meaning can be as follows.
"The slope > $\mathbf{0}$, and $\boldsymbol{y = f}\mathbf{(-3)} \geq \mathbf{0}$." or "The slope = $\mathbf{0}$, and $\boldsymbol{y = f}\mathbf{(-3)} > \mathbf{0}$."

The meaning above is the basis we can build the solution. So we can now start working on the meaning above. First, we have $\boldsymbol{y = (\frac{t}{4}+1)x + t + 4}$. So the slope is $\boldsymbol{\frac{t}{4}+1}$.

Thus, being based on the practical meaning above, we can see two cases are possible.
One is "$\boldsymbol{\frac{t}{4}+1=0}$, and $\boldsymbol{y = f}\mathbf{(-3)} > \mathbf{0}$.", and the other is "$\boldsymbol{\frac{t}{4}+1>0}$, and $\boldsymbol{y = f}\mathbf{(-3)} \geq \mathbf{0}$."

Then, can we put the two together, and simply say that $\boldsymbol{\frac{t}{4}+1 \geq 0}$, and $\boldsymbol{y = f}\mathbf{(-3)} \geq \mathbf{0}$?

No, we can't say that. The two cases above are separate from each other. That's because "$\boldsymbol{y = f}\mathbf{(-3)} > \mathbf{0}$." is subject to "$\boldsymbol{\frac{t}{4}+1=0}$.", but "$\boldsymbol{y = f}\mathbf{(-3)} \geq \mathbf{0}$." is subject to "$\boldsymbol{\frac{t}{4}+1>0}$."

Let's check to see if the first of the two cases is valid, that is, if the slope $\boldsymbol{(\frac{t}{4}+1)}$ can be 0.

$\boldsymbol{\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x + t + 4 = 0 \cdot x - 4 + 4 = 0 \Rightarrow y = f(x) = 0}$.

Thus, $\boldsymbol{y}$ is $\mathbf{0}$ for all $\boldsymbol{x}$, so we get $\boldsymbol{y} = \mathbf{0}$ for $\boldsymbol{x} = \mathbf{-3}$.
That is, $\boldsymbol{y = f}\mathbf{(-3) = 0}$, which is not positive, but we need to have $\boldsymbol{y = f}\mathbf{(-3)} > \mathbf{0}$.
So the slope cannot be 0. In other words, we need to have $\boldsymbol{t} \neq \mathbf{-4}$.
Now, that's one, so let's move on to the other case where $\boldsymbol{\frac{t}{4}+1>0}$, and $\boldsymbol{y = f}\mathbf{(-3)} \geq \mathbf{0}$.

First, we get $\frac{t}{4}+1>0 \Rightarrow t>-4$.

Next, we get $y=f(-3)=(\frac{t}{4}+1)(-3)+t+4=\frac{-3t+4t}{4}-3+4=\frac{t}{4}+1\geq 0 \Rightarrow t\geq -4$.

Thus, we get $t > -4$, and $t \geq -4$, at the same time, and therefore, we get $t > -4$.

Besides, in the first case, we have found that $t \neq -4$.
However, '$t > -4$' includes the case where $t \neq -4$, too.

Thus, choosing for the value of $t$ a value greater than **-4**, we get $y > 0$ when $x > -3$.

**In short:**

Setting first, $y=f(x)=(\frac{t}{4}+1)x+t+4$, we need to have $x > -3 \Rightarrow y = f(x) > 0$.
Then, if the slope < **0**, even if $y = f(-3) > 0$, there is no value for $t$ satisfying the condition, because the line eventually goes below the $x$-axis as $x$ increases, so $y$ can be negative.

Next, if the slope = **0**, we need to have $y = f(-3) > 0$.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y=f(x)=(\frac{t}{4}+1)x+t+4=0\cdot x-4+4=0$.

So the slope cannot be 0, and thus, we need to have $t \neq -4$.

Next, if the slope > **0**, we need to have $y = f(-3) \geq 0$.
First, we get $\frac{t}{4}+1>0 \Rightarrow t>-4$.
And next, we get $y=f(-3)=(\frac{t}{4}+1)(-3)+t+4=\frac{-3t+4t}{4}-3+4=\frac{t}{4}+1\geq 0 \Rightarrow t\geq -4$.

Thus, we have $t > -4$, and $t \geq -4$, so we get $t > -4$.
Now, we have $t \neq -4$ and $t > -4$. Therefore, we get $t > -4$.

## Suggestions or Solutions
## To the Problem in the Example 1

**Assuming $t$ is constant, and $y=(\frac{t}{4}+1)x+t+5$, determine the value of $t$ so that $y > 0$ when $x > -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+5$, we need to have $x > -3 \Rightarrow y = f(x) > 0$.

To begin with, if the slope < **0**, even if $y = f(-3) > 0$, there is no value for $t$ satisfying the condition, because the line eventually goes below the $x$-axis as $x$ increases, and thus, $y$ can be negative.

Next, if the slope = **0**, we need to have $y = f(-3) > 0$.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x+t+5 = 0\cdot x-4+5=1$, which is positive.

So the slope can be 0, and thus, $t$ can be -4.

Next, if the slope > **0**, we need to have $y = f(-3) \geq 0$.

So first, we get $\frac{t}{4}+1>0 \Rightarrow t>-4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+5 = \frac{-3t+4t}{4}-3+5 = \frac{t}{4}+2 \geq 0 \Rightarrow t \geq -8$.

Thus, we get $t > -4$, and $t \geq -8$, so we get $t > -4$.

Besides, $t$ can be -4, too, and thus, we need to have $t \geq -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

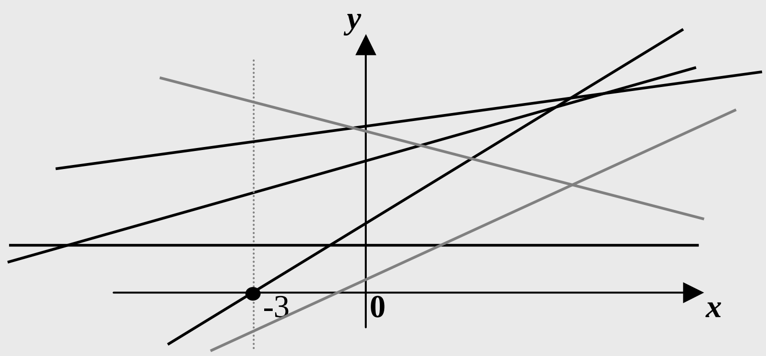
A line depends on the slope. A line is no other than a slope, and a slope is no other than a line. Going up, a line keeps going up, but going down, it keeps going down, and being horizontal, it stays horizontal, but being vertical, it stays so, and it does at a constant rate.

Setting now, $y = f(x) = (\frac{t}{4}+1)x+t+5$, we need to have $y = f(x) > 0$ when $x > -3$.

To begin with, let's put some lines in the $x$-$y$ plane. Then, we can make visible what the problem is talking about. So don't just think, and start graphing. While adding lines to a graph, we can see better what lines can satisfy the problem, so we can see quicker the solution's whereabouts.

Constructing a graph, we may want to show first, such a crucial item as the critical point mentioned in the previous example. The critical point is **(-3, 0)**, which is in the ***x***-axis.

Fig. 1.0




Now, there is a condition, which is "***y* = *f*(*x*) > 0** when ***x* > -3**.", which means, for any value of ***x* > -3**, the value of ***y*** has to be positive, which is however, no more than a literal meaning. We want to see what it practically means.

The situation in this example is not quite different from that of the previous example.

Only one thing different is the ***y***-intercept, which was ***t* + 4** in the previous example.

Thus, what the condition, "When ***x* > -3**, ***y* = *f*(*x*) > 0**." practically means is as follows.

"The slope > **0**, and ***y* = *f*(-3) ≥ 0**." or "The slope **= 0**, and ***y* = *f*(-3) > 0**."

So we should be able to build the solution from the practical meaning above.

To begin with, the slope of the line is $\frac{t}{4}+1$. Thus, we need to have

"$\frac{t}{4}+1>0$, and ***y* = *f*(-3) ≥ 0**." or "$\frac{t}{4}+1=0$, and ***y* = *f*(-3) > 0**."

So let's first, check to see if the slope, $\frac{t}{4}+1$ can be 0.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y=f(x)=(\frac{t}{4}+1)x+t+5=0\cdot x-4+5=1$.

Thus, we get ***y* = *f*(*x*) = 1**, which means ***y*** is **1** for all ***x*** if ***t* = -4**, that is, if the slope is 0.

So we get ***y* = *f*(-3) = 1**, which is positive, that is, ***y* = *f*(-3) > 0**. Thus, the slope can be 0.
In other words, we can have ***t* = -4**.

Next, let's move on to the case where $\frac{t}{4}+1>0$, and $y = f(-3) \geq 0$.

First, we get $\frac{t}{4}+1>0 \Rightarrow t>-4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+5 = \frac{-3t+4t}{4} - 3+5 = \frac{t}{4}+2 \geq 0 \Rightarrow t \geq -8$.

Thus, we have $t > -4$, and $t \geq -8$ at the same time, so we get $t > -4$.

However, since the slope can be 0, $t$ can be -4, too. So we get $t \geq -4$.

Thus, choosing for the value of $t$ a value $\geq -4$, we get $y > 0$ when $x > -3$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+5$, we need to have $x > -3 \Rightarrow y = f(x) > 0$.

To begin with, if the slope < **0**, even if $y = f(-3) > 0$, there is no value for $t$ satisfying the condition, because the line eventually goes below the $x$-axis as $x$ increases, and thus, $y$ can be negative.

Next, if the slope = **0**, we need to have $y = f(-3) > 0$.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x+t+5 = 0 \cdot x - 4+5 = 1$, which is positive.

So the slope can be 0, and thus, $t$ can be -4.

Next, if the slope > **0**, we need to have $y = f(-3) \geq 0$.

So first, we get $\frac{t}{4}+1>0 \Rightarrow t > -4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+5 = \frac{-3t+4t}{4} - 3+5 = \frac{t}{4}+2 \geq 0 \Rightarrow t \geq -8$.

Thus, we get $t > -4$, and $t \geq -8$, so we get $t > -4$.

Besides, $t$ can be -4, too, and thus, we need to have $t \geq -4$.

## Suggestions or Solutions
## To the Problem in the Example 2

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x + t + 3$, determine the value of $t$ so that $y < 0$ when $x > -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 3$, we need to have $x > -3 \Rightarrow y = f(x) < 0$.

To begin with, if the slope > **0**, even if $y = f(-3) < 0$, there is no value for $t$ satisfying the condition, because the line eventually goes above the $x$-axis as $x$ increases, and thus, $y$ can be positive, too. Next, if the slope **= 0**, we need to have $y = f(-3) < 0$.

$\frac{t}{4}+1 = 0 \Rightarrow t = -4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x + t + 3 = 0 \cdot x - 4 + 3 = -1$.

So the slope can be 0, and thus, $t$ can be -4.

Next, if the slope < **0**, we need to have $y = f(-3) \le 0$.

First, we get $\frac{t}{4}+1 < 0 \Rightarrow t < -4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3) + t + 3 = \frac{-3t+4t}{4} - 3 + 3 = \frac{t}{4} \le 0 \Rightarrow t \le 0$.

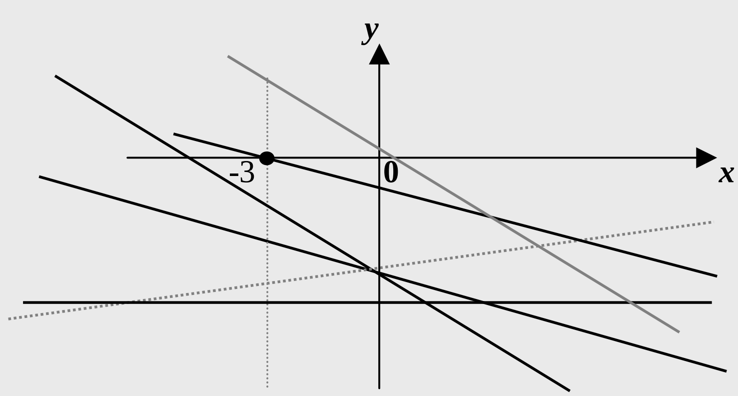
Thus, we get $t < -4$, and $t \le 0$, so we get $t < -4$. Besides, $t$ can be –4, and thus, $t \le -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 3$, we need to have $x > -3 \Rightarrow y = f(x) < 0$.

And in this problem, the critical point is **(-3, 0)**, which is in the $x$-axis.

Fig. 2.0



Now, the condition is "$y = f(x) < 0$ when $x > -3$.", and means that for any value of $x > -3$, the value of $y$ has to be negative, which is however, just a literal meaning.
We want to see its practical meaning, from which we can build the solution.

Suppose now, $A$ is a line that satisfies the condition above.

Then, when $x$ is greater than -3, if we want $y$ to get a value positive only, $A$ has to be one of the lines in black in the graph above. And we shouldn't neglect the one horizontal.

Also, we can see that the line $A$ itself can include the critical point **(-3, 0)**.
The line dotted in gray seems to satisfy the condition, yet it eventually goes above the $x$-axis as $x$ increases, and therefore, is not allowed.
What then, can we say about the solution?

First of all, we can see in this case, the slope cannot be positive.
That is, the slope has to be 0 or negative.

That's because if the slope > **0**, even if $y = f(-3) < 0$, there is no value for $t$ satisfying the condition, because the line eventually goes above the $x$-axis as $x$ increases, and thus, $y$ can be positive, too.

Thus, of all the curves of $f$ for all values of $t$, the ones satisfying the condition,
"When $x > -3$, $y = f(x) < 0$." are lines like the ones in black in the graph above.

Suppose now, $c$ is constant, and in $y = ax + b$, $y = 0$ for $x = c$, and $y < 0$ when $x > c$.
Then, what is the sign of the slope $a$?

The line has to go down. So we need to have $a < 0$.

Suppose next, $d$ is constant, and in $y = ax + b$, $y = d$ for all $x$.
That is, $y$ gets the same value for any value of $x$.

Then, what is the sign of the slope $a$?

The line has to be horizontal. So we need to have $a = 0$.

Thus, what the condition, “When $x > -3$, $y = f(x) < 0$.” practically means is as follows.
“The slope < **0**, and $y = f(-3) \le 0$.” or “The slope = **0**, and $y = f(-3) < 0$.”

So we can now start building the solution from the practical meaning above.

To begin with, we have $y = (\frac{t}{4}+1)x + t + 3$.
So the slope is $\frac{t}{4}+1$, and two cases are possible.

One is “$\frac{t}{4}+1=0$, and $y = f(-3) < 0$.”, and the other is “$\frac{t}{4}+1<0$, and $y = f(-3) \le 0$.”.

Then, can we put the two together, and simply say that $\frac{t}{4}+1 \le 0$, and $y = f(-3) \le 0$?

No, we can’t say that. That’s because
“$y = f(-3) < 0$.” is subject to “$\frac{t}{4}+1=0$.”, and “$y = f(-3) \le 0$.” is subject to “$\frac{t}{4}+1<0$.”

So they are two separate cases. Let’s first, check to see if the slope, $\frac{t}{4}+1$ can be 0.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow y = f(x) = (\frac{t}{4}+1)x + t + 3 = 0 \cdot x - 4 + 3 = -1$.

Thus, we get $y = f(x) = -1$ for all $x$, which means $y$ is $-1$ for all $x$.
So we get $y = f(-3) = -1$, which means, $y = f(-3) < 0$. Thus, the slope can be 0.
That is, we can have $t = -4$.

Next, let’s move on to the case where $\frac{t}{4}+1<0$, and $y = f(-3) \le 0$.

First, we get $\frac{t}{4}+1<0 \Rightarrow t<-4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3) + t + 3 = \frac{-3t+4t}{4} - 3 + 3 = \frac{t}{4} \le 0 \Rightarrow t \le 0$.

Thus, we get $t < -4$, and $t \le 0$ at the same time, so we get $t < -4$.
Besides, since the slope can be 0, $t$ can be -4, too.
Therefore, we get $t \le -4$.
Thus, choosing for the value of $t$ a value $\le -4$, we get $y < 0$ when $x > -3$.

## Suggestions or Solutions
## To the Problem in the Example 3

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x+t+4$, determine the value of $t$ so that $y \geq 0$ when $x$ > -3.**

Setting first, $\boldsymbol{y = f(x) = (\frac{t}{4}+1)x+t+4}$, we need to have $\boldsymbol{x > -3 \Rightarrow y = f(x) \geq 0}$.

So to begin with, if the slope < **0**, even if ***f*(-3) > 0**, we can have ***y* < 0**, too, because the line eventually goes below the ***x***-axis as ***x*** increases.

We can have two cases where the condition can be met.

One is that the slope = **0**, and ***y* = *f*(-3) ≥ 0**.

The other is that the slope > **0**, and ***y* = *f*(-3) ≥ 0**.

So after all, the two cases can be put into one where the slope ≥ **0** and ***y* = *f*(-3) ≥ 0**.

So first, we get $\boldsymbol{\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4}$.

Next, $\boldsymbol{y = f(-3) = (\frac{t}{4}+1)(-3)+t+4 = \frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4}$.

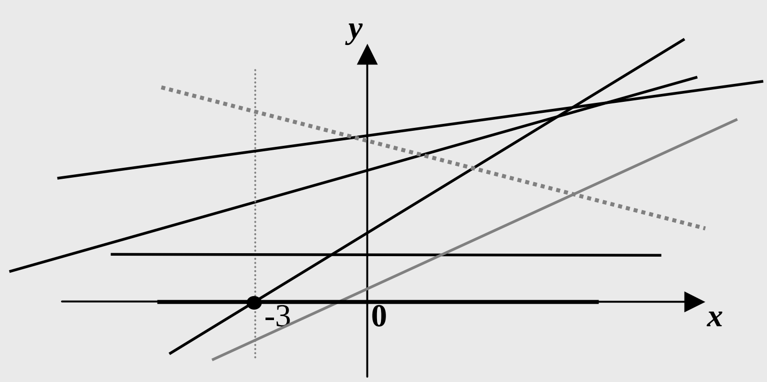
So in both cases, we have ***t* ≥ -4**, and thus, we get ***t* ≥ -4**.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $\boldsymbol{y = f(x) = (\frac{t}{4}+1)x+t+4}$, we need to have $\boldsymbol{x > -3 \Rightarrow y = f(x) \geq 0}$.

And in this problem, the critical point is **(-3, 0)**, which is in the ***x***-axis.

Fig. 3.0



Now, "When $x > -3$, $y = f(x) \geq 0$." is the condition to be met, and means that for any value of $x$ greater than -3, the value of $y$ is positive or 0, which is however, nothing but a literal meaning.

So putting the idea more practically, we can say that we want to see what value $y = f(-3)$ has to be if $x > -3$ and $y = f(x) \geq 0$, and also, how the value of $y = f(x)$ should change when $x > -3$. That is, we want to see the value of $y = f(-3)$ and the sign of the slope.

Suppose now, $A$ is a line that can satisfy the condition above.

Then, if we want $y$ to be $\geq 0$ when $x$ is $> -3$, the line $A$ has to be one of the black ones in the graph above. In this case of course, we don't want to neglect the ones horizontal. In fact, the line horizontal and passing through the origin looks pretty significant in this example. That is, the $x$-axis itself seems to be quite important in this case.

Anyway, coordinate axes are not trivial at all, and are quite often in fact, very crucial.

Also, we can see that the line $A$ itself can include the critical point **(-3, 0)**.
The line dotted in gray seems to meet the condition, yet it will eventually go below the $x$-axis as $x$ increases, and therefore, is not allowed.

Now, what then, can we say about the solution?

First of all, we can see in this case, the slope cannot be negative.
That is, the slope has to be positive or 0.
That is because if the slope $< 0$, even if $y = f(-3) > 0$, we can have $y < 0$ since the line eventually goes below the $x$-axis as $x$ increases.

Therefore, of all the lines of $f$ for all values of $t$, the ones that can satisfy the condition, "When $x > -3$, $y = f(x) \geq 0$." are lines like the ones in black in the graph above.

Suppose now, $c$ is constant, and in $y = ax + b$, $y = 0$ when $x = c$, and $y > 0$ when $x > c$.
Then, what is the sign of the slope $a$?

The line has to go up. So we need to have $a > 0$.

Suppose next, $\boldsymbol{d}$ is constant, too, and in $\boldsymbol{y = ax + b}$, $\boldsymbol{y = d}$ for all $\boldsymbol{x}$. That is, $\boldsymbol{y}$ gets the same value for any value of $\boldsymbol{x}$. Then, what is the sign of the slope $\boldsymbol{a}$?

The line has to be horizontal. So we need to have $\boldsymbol{a = 0}$.
Then first, if the slope = **0**, we need to have $\boldsymbol{y = f(-3) \geq 0}$.

Next, if the slope > **0**, we need to have $\boldsymbol{y = f(-3) \geq 0}$. Then, we can put the two together, and simply say that the slope ≥ **0**, and $\boldsymbol{y = f(-3) \geq 0}$. How?

“ $\boldsymbol{y = f(-3) \geq 0}$.” is subject to “the slope = **0**.” or “the slope > **0**.”

The slope ≥ **0** means that the slope = **0**, or that the slope > **0**.

Thus, “ $\boldsymbol{y = f(-3) \geq 0}$.” is subject to “the slope ≥ **0**.”

So after all, we have only to have “The slope ≥ **0**, and $\boldsymbol{y = f(-3) \geq 0}$.”

Thus, what the condition, “When $\boldsymbol{x > -3}$, $\boldsymbol{y = f(x) \geq 0}$.” practically means is as follows.
“The slope ≥ **0**, and $\boldsymbol{y = f(-3) \geq 0}$.”

So we can now practically start finding $\boldsymbol{t}$ from the practical meaning above.

To begin with, the slope is $\frac{t}{4}+1$, so we get $\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.
Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+4 = \frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.

So in both cases, we have $\boldsymbol{t \geq -4}$, and thus, we get $\boldsymbol{t \geq -4}$.
Therefore, choosing for the value of $\boldsymbol{t}$ a value ≥ **-4**, we get $\boldsymbol{y \geq 0}$ when $\boldsymbol{x > -3}$.

By the way, let’s take a look at the case where the slope is 0.

$\frac{t}{4}+1 = 0 \Rightarrow t = -4 \Rightarrow y = f(x) = 0 \cdot x + 0 = 0$. So we get $\boldsymbol{y = f(x) = 0}$ for all $\boldsymbol{x}$.

Therefore, if the slope is 0, the line $\boldsymbol{A}$ needs to be the $\boldsymbol{x}$-axis itself.
If the curve of a function is the $\boldsymbol{x}$-axis itself, the function is often called a **0-function**.
It’s not quite rare to see such a function in calculus.

## Suggestions or Solutions
## To the Problem in the Example 4

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x + t + 5$, determine the value of $t$ so that $y \geq 0$ when $x$ > -3.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 5$, we need to have $x > -3 \Rightarrow y = f(x) \geq 0$.

So to begin with, if the slope < **0**, even if ***f*(-3)** > **0**, we can have ***y*** < **0**, too, because the line eventually goes below the ***x***-axis as ***x*** increases. And we can have two cases where the condition can be met.

One is that the slope = **0**, and $y = f(-3) \geq 0$.

The other is that the slope > **0**, and $y = f(-3) \geq 0$.

So after all, we have only to have the slope ≥ **0** and $y = f(-3) \geq 0$.

So first, we get $\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.

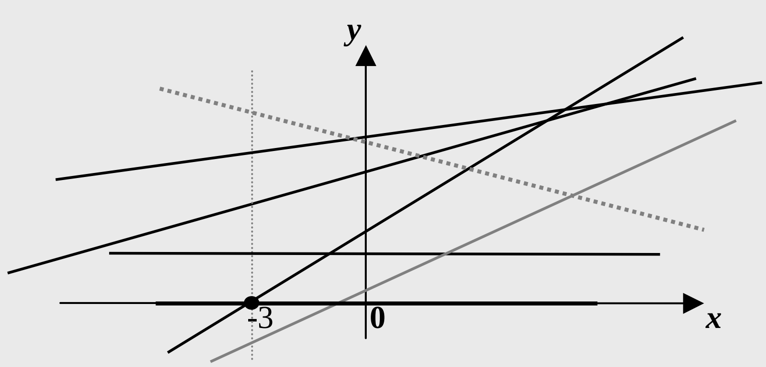
Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3) + t + 5 = \frac{t}{4} + 2 \geq 0 \Rightarrow t \geq -8$.

So we have $t \geq -4$, and $t \geq -8$, and thus, we get $t \geq -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 5$, we need to have $x > -3 \Rightarrow y = f(x) \geq 0$.
Thus, the situation is just about the same as that of the previous example. Only one thing different is the ***y***-intercept. Now, let's begin with putting some lines in a graph. Then of course, we may want to keep in mind that the critical point is at **(-3, 0)**.

Fig. 4.0



Now, examining the graph above, we can see what lines can satisfy this problem.

First, if the slope = **0**, we need to have $\boldsymbol{y = f(-3) \geq 0}$.

Next, if the slope > **0**, we need to have $\boldsymbol{y = f(-3) \geq 0}$.

Thus, "$\boldsymbol{y = f(-3) \geq 0}$." is common in both cases. So after all, the two cases above can be put into one where the slope ≥ **0**, and $\boldsymbol{y = f(-3) \geq 0}$. What if the slope < **0**, though?

Then, even if $\boldsymbol{y = f(-3) > 0}$, we can have $\boldsymbol{y < 0}$, too, since the line eventually goes below the $\boldsymbol{x}$-axis as $\boldsymbol{x}$ increases. So a line where the slope < **0** cannot satisfy this problem.

Now, let's find $\boldsymbol{t}$. The slope is $\boldsymbol{\frac{t}{4}+1}$. So first, we get $\boldsymbol{\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4}$.

Next, we get $\boldsymbol{y = f(-3) = (\frac{t}{4}+1)(-3)+t+5 = \frac{t}{4}+2 \geq 0 \Rightarrow t \geq -8}$.

Thus, we have $\boldsymbol{t \geq -4}$, and $\boldsymbol{t \geq -8}$ at the same time. So we get $\boldsymbol{t \geq -4}$.

Thus, choosing for $\boldsymbol{t}$ a value ≥ **-4**, we get $\boldsymbol{y \geq 0}$ when $\boldsymbol{x > -3}$.

## Suggestions or Solutions
## To the Problem in the Example 5

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x+t+3$, determine the value of $t$ so that $y \geq 0$ when $x > -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+3$, we need to have $x > -3 \Rightarrow y = f(x) \geq 0$.

So to begin with, if the slope < **0**, even if $f(-3) > 0$, we can have $y < 0$, too, because the line eventually goes below the $x$-axis as $x$ increases. So we can have two cases where the condition can be met.
In one, the slope = **0**, and $y = f(-3) \geq 0$. And in the other, the slope > **0**, and $y = f(-3) \geq 0$.

So after all, we have only to have the slope ≥ **0** and $y = f(-3) \geq 0$.

So first, we get $\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+3 = \frac{t}{4} \geq 0 \Rightarrow t \geq 0$.

So we have $t \geq -4$, and $t \geq 0$, and thus, we get $t \geq 0$.

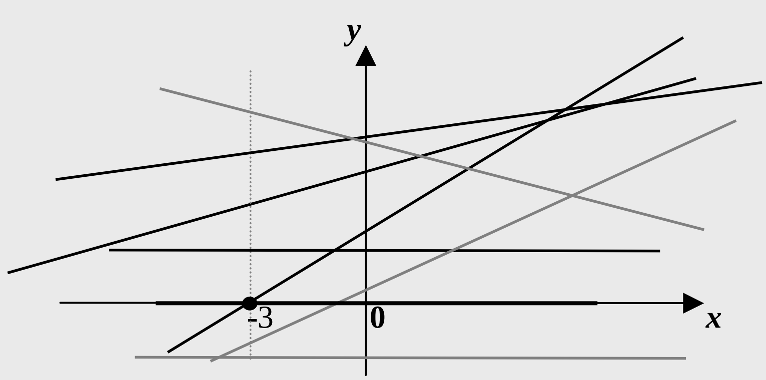
*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+3$, we need to have $x > -3 \Rightarrow y = f(x) \geq 0$.

Thus, the situation is just about the same as that of the previous example.

Only one thing different is in the $y$-intercept. Now, let's begin with putting some lines in a graph keeping in mind of course, **(-3, 0)** is the critical point.

Fig. 5.0



Then, examining the graph above, we can see what lines can satisfy the problem.

First, if the slope = **0**, we need to have $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$.

Next, if the slope > **0**, we need to have $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$.

And if the slope < **0**, even if $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) > \mathbf{0}$, we can have $\boldsymbol{y} < \mathbf{0}$, too, since the line will eventually go below the $\boldsymbol{x}$-axis as $\boldsymbol{x}$ increases. So a line of slope negative cannot satisfy the problem.

So we can notice that "$\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$." is common in the two cases we need to have.
So after all, we only have to have "The slope ≥ **0**, and $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$.", from which we can start finding $\boldsymbol{t}$.

To begin with, the slope is $\frac{t}{4}+1$, so we get $\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+3 = \frac{t}{4} \geq 0 \Rightarrow t \geq 0$.

Thus, we have $\boldsymbol{t} \geq \mathbf{-4}$, and $\boldsymbol{t} \geq \mathbf{0}$ at the same time. So we get $\boldsymbol{t} \geq \mathbf{0}$.

Thus, choosing for $\boldsymbol{t}$ a value ≥ **0**, we get $\boldsymbol{y} \geq \mathbf{0}$ when $\boldsymbol{x} > \mathbf{-3}$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+3$, we need to have $\boldsymbol{x} > \mathbf{-3} \Rightarrow \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) \geq \mathbf{0}$.

So to begin with, if the slope < **0**, even if $\boldsymbol{f}(\mathbf{-3}) > \mathbf{0}$, we can have $\boldsymbol{y} < \mathbf{0}$, too, because the line eventually goes below the $\boldsymbol{x}$-axis as $\boldsymbol{x}$ increases. So we can have two cases where the condition can be met.
In one, the slope = **0**, and $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$. And in the other, the slope > **0**, and $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$.

So after all, we have only to have the slope ≥ **0** and $\boldsymbol{y} = \boldsymbol{f}(\mathbf{-3}) \geq \mathbf{0}$.

So first, we get $\frac{t}{4}+1 \geq 0 \Rightarrow t \geq -4$.

Next, we get $y = f(-3) = (\frac{t}{4}+1)(-3)+t+3 = \frac{t}{4} \geq 0 \Rightarrow t \geq 0$.

So we have $\boldsymbol{t} \geq \mathbf{-4}$, and $\boldsymbol{t} \geq \mathbf{0}$, and thus, we get $\boldsymbol{t} \geq \mathbf{0}$.

## Suggestions or Solutions
## To the Problem in the Example 6

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x+t+4$, determine the value of $t$ so that $y > 0$ when $-2 < x < 3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+4$, we need to have $-2 < x < 3 \Rightarrow y = f(x) > 0$.

Then, when $x$ is between –2 and 3, the line needs to be above the $x$-axis.
So we need to have $f(-2) \geq 0$, and $f(3) \geq 0$. Let's first, see if $f$ can be a 0-function.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow f(x)=0 \cdot x-4+4=0$.

So $f$ is a 0-function if $t = -4$, and thus, we need to have $t \neq -4$.

Now first, we get $f(-2)=(\frac{t}{4}+1)(-2)+t+4=\frac{t}{2}+2 \geq 0 \Rightarrow t \geq -4$.

Next, we get $f(3)=(\frac{t}{4}+1)(3)+t+4=\frac{7t}{4}+7 \geq 0 \Rightarrow t \geq -4$.

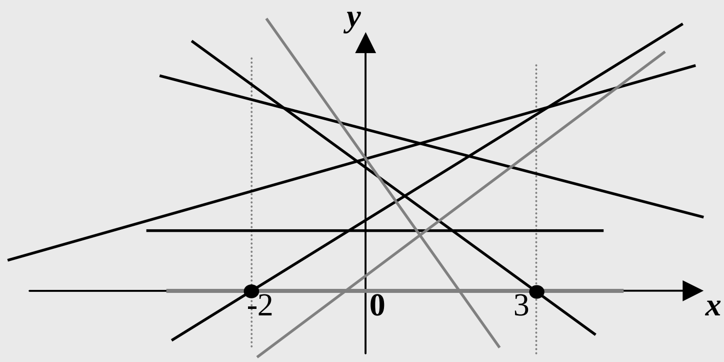
So we get $t \geq -4$, but we have $t \neq -4$, too. Thus, $t > -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x+t+4$, we need to have $-2 < x < 3 \Rightarrow y = f(x) > 0$.

So "When $-2 < x < 3$, $y = f(x) > 0$." is the condition we need to meet, and it means that when $x$ is between –2 and 3, $y = f(x)$ needs to be always positive, which is however, just a literal meaning of the condition. So let's now begin with putting some lines in a graph.

**Fig. 6.0**




The critical points are two in this case, and are **(–2, 0)** and **(3, 0)**, which are in the $x$-axis.

Examining the graph above, we can notice that we want to know about $f(-2)$ and $f(3)$ in the case where $f(x) > 0$ when $-2 < x < 3$, and also, how the value of $f(x)$ should change as $x$ changes from **-2** to **3**.

That is, we want to see the slope, together with information on $f(-2)$ and $f(3)$.

While constructing the graph however, we probably have noticed that the slope doesn't really matter in this particular case. Haven't noticed yet?

Examining the lines in black in the graph, we can see that the slope can be not only positive or 0, but negative, too. So we don't have to reason too hard on the slope. What we need to do is to consider what's going on around the critical points.

The critical points are **(-2, 0)** and **(3, 0)**, that is, $(-2, f(-2))$ and $(3, f(3))$.

Still not clear though, why the slope doesn't really matter in this case?

First, we can have $f(-2) \geq 0$, and the slope > **0**.
In this case, while $x$ changes from –2 to 3, $y = f(x)$ begins with a value positive or 0, and then, keeps increasing.

Next, we can have $f(-2) > 0$, $f(3) > 0$, and the slope **< 0**.
In this case, while $x$ changes from –2 to 3, $y = f(x)$ begins with a positive value, keeps decreasing, but can stay above the $x$-axis, which means, positive until $x$ is 3.

Finally, we can have $f(x) = c > 0$ where $c$ is constant, and the slope **= 0**.
In this case, while $x$ changes from –2 to 3, $y = f(x)$ begins with a value positive, and then, stays unchanged, that is, constant.

So the slope can be negative as well as positive or 0.
Calculations will be onerous though, if all the three cases are taken care of individually.

Suppose now, $A$ is a line that can satisfy the problem.

Then, even examining the graph only, we can see how the line $A$ has to be put.

That is, we can get the practical meaning as follows.

"When $x$ is between –2 and 3, the line has to be above the $x$-axis."

Thus, we need to have $f(-2) \geq 0$ and $f(3) \geq 0$, for now. Why 'for now', though?

The statement that $f(-2) \geq 0$ and $f(3) \geq 0$ can mean four case as follows.

$f(-2) > 0$ and $f(3) > 0$. $f(-2) = 0$ and $f(3) > 0$. $f(-2) > 0$ and $f(3) = 0$.
$f(-2) = 0$ and $f(3) = 0$.

So "$f(-2) \geq 0$ and $f(3) \geq 0$." includes a case where "$f(-2) = 0$ and $f(3) = 0$."
That is to say that the function $f$ could be a 0-function.

Now, if there is a value of $t$ for which $y = f(x) = 0$, the value makes $f$ a 0-function.

We don't want however, the function $f$ to be a 0-function in this problem. So we don't want $t$ to have such a value as described above. How can $f$ be a 0-function, though?

If the slope is 0, $f$ is a constant function, and if the constant is 0, $f$ is a 0-function.
That's because in that case, the curve of $f$, that is, the line is the $x$-axis itself.
The $y$-coordinate at every point in the $x$-axis is 0.

Now, let's find $t$.

We have $y = f(x) = (\frac{t}{4}+1)x + t + 4$. To begin with, let's see if $f$ can be a 0-function.

$\frac{t}{4}+1 = 0 \Rightarrow t = -4 \Rightarrow f(x) = 0 \cdot x - 4 + 4 = 0$.

So $f$ is a 0-function if $t = -4$, and thus, we don't want $t$ to be -4.

Now, we have "$f(-2) \geq 0$ and $f(3) \geq 0$".

So first, we get $f(-2) \geq 0 \Rightarrow (\frac{t}{4}+1)(-2) + t + 4 = \frac{t}{2} + 2 \geq 0 \Rightarrow t \geq -4$.

Next, we get $f(3) \geq 0 \Rightarrow (\frac{t}{4}+1)(3) + t + 4 = \frac{7t}{4} + 7 \geq 0 \Rightarrow t \geq -4$.

So we get $t \geq -4$ and $t \geq -4$, and thus, we get $t \geq -4$.

However, we have $t \neq -4$, too, because if $t = -4$, $f$ is a 0-function, that is, the line is the $x$-axis itself, so we get $y = 0$ when $-2 < x < 3$, which we don't want.
So we need to have $t > -4$.
Therefore, choosing for $t$ a value $> -4$, we get $y = f(x) > 0$ when $-2 < x < 3$.

## Suggestions or Solutions
## To the Problem in the Example 7

**Assuming $t$ is constant, and $y = (\frac{t}{4}+1)x + t + 4$, determine the value of $t$ so that $y$ is positive when -5 < $x$ < -3.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have **-5 < $x$ < -3 ⇒ $y$ = $f(x)$ > 0**.

Then, when $x$ is between -5 and -3, the line needs to be above the $x$-axis.
So we need to have **$f$(-5) ≥ 0**, and **$f$(-3) ≥ 0**.

To begin with, let's see if $f$ can be a 0-function.

$\frac{t}{4}+1=0 \Rightarrow t=-4 \Rightarrow f(x)=0\cdot x-4+4=0$.

So $f$ is a 0-function if **$t$ = -4**, and thus, we need to have **$t \neq$ -4**.

Now first, we get $f(-5)=(\frac{t}{4}+1)(-5)+t+4=\frac{-t}{4}-1\geq 0 \Rightarrow t\leq -4$.

Next, we get $f(-3)=(\frac{t}{4}+1)(-3)+t+4=\frac{t}{4}+1\geq 0 \Rightarrow t\geq -4$.

So we get **$t$ = -4**, but we have **$t \neq$ -4**, too.

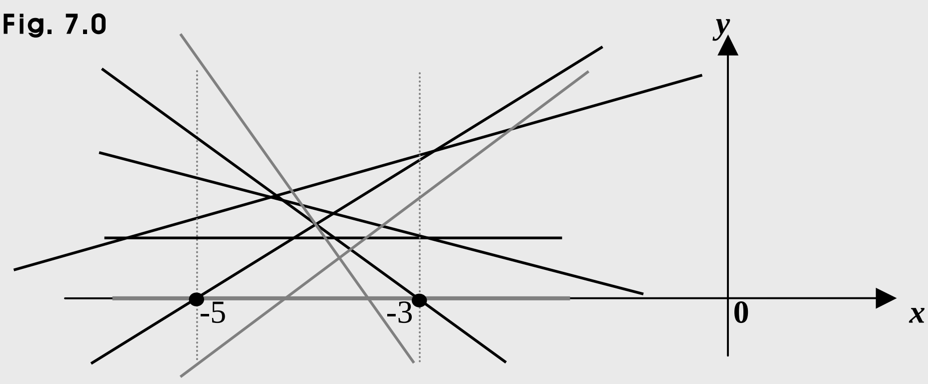
Thus, there is no value of $t$ for which $y$ is positive when **-5 < $x$ < -3**.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have **$y$ = $f(x)$ > 0** when **-5 < $x$ < -3**.

So the critical points are **(-5, 0)** and **(-3, 0)**, which are in the $x$-axis. And putting some lines in a graph, together with the critical points above, we can have

Fig. 7.0



Examining the graph above, we can see that the slope doesn't really matter. The slope can be negative as well as positive or 0, so we don't have to reason too hard on the slope.

What we need to do is to consider what's going on around the critical points.

The critical points are **(-5, 0)** and **(-3, 0)**, that is, **(-5, *f* (-5))** and **(-3, *f*(-3))**.

Now, even examining the graph only, we can see the practical meaning as follows.
"When ***x*** is between -5 and -3, the line has to be above the ***x***-axis."

Thus, we need to have $f(-5) \geq 0$ and $f(-3) \geq 0$, for now.

The statement that $f(-5) \geq 0$ and $f(-3) \geq 0$ includes a case where $f(-2) = 0$ and $f(3) = 0$, which indicates that the curve of ***f***, that is, the line is the ***x***-axis itself.

That is to say that the function ***f*** could be a 0-function, which however, doesn't satisfy this problem, so we want to give it a check.

Now that we've got the practical meaning, let's find ***t***.

We have $y = f(x) = (\frac{t}{4}+1)x + t + 4$.

We may want to first, see if ***f*** can be a 0-function.

$\frac{t}{4}+1 = 0 \Rightarrow t = -4 \Rightarrow f(x) = 0 \cdot x - 4 + 4 = 0$.

So ***f*** is a 0-function if $t = -4$, and thus, we don't want to let ***t*** be -4.

Now first, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

So we get $t = -4$, but we have $t \neq -4$, too, because if $t = -4$, ***f*** is a 0-function, that is, the line is the ***x***-axis itself. Thus, there is no value for ***t*** in this problem.

Therefore, for any ***t***, we cannot get $y = f(x) > 0$ when $-5 < x < -3$.

## Suggestions or Solutions
## To the Problem in the Example 8

**Assuming $y = (\frac{t}{4}+1)x + t + 4$ where $t$ is constant, determine the value of $t$ so that $y$ is positive when -5 ≤ $x$ ≤ -3.**

Setting first $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 \le x \le -3$. Then, when $x$ changes from -5 to -3, the line needs to be above the $x$-axis.

So we need to have $f(-5) > 0$, and $f(-3) > 0$.

First, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 > 0 \Rightarrow t < -4$.

Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 > 0 \Rightarrow t > -4$.

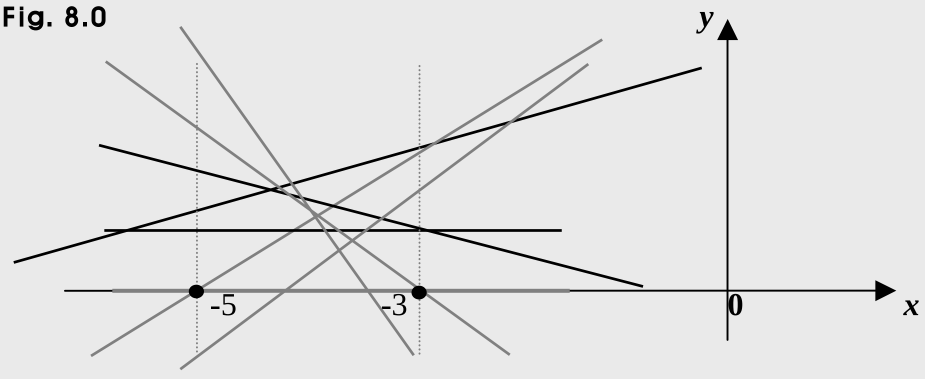
Thus, there is no value of $t$ for which $y$ is positive when $-5 \le x \le -3$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 \le x \le -3$.

So we need to meet the condition, "When $-5 \le x \le -3$, $y = f(x) > 0$.", and its literal meaning is that when $x$ changes from -5 to -3, $y = f(x)$ has to be always positive.

Fig. 8.0




Examining the graph above, we can see that the slope doesn't really matter.

What we need to do is to consider what's going on around the critical points, which are **(-5, 0)** and **(-3, 0)**, that is, **(-5, *f* (-5))** and **(-3, *f*(-3))**.

Then, getting back to the graph above, we can see the practical meaning as follows.

"When $x$ changes from -5 to -3, the line has to be above the $x$-axis."

So we need to have $f(-5) > 0$ and $f(-3) > 0$.

Thus, unlike the previous example, $f(-5)$ and $f(-3)$ cannot be 0 at the same time, so we don't have to check to see if $f$ can be a 0-function.
Now, let's find $t$. We have $y = f(x) = (\frac{t}{4} + 1)x + t + 4$.

Then first, we get $f(-5) = (\frac{t}{4} + 1)(-5) + t + 4 = \frac{-t}{4} - 1 > 0 \Rightarrow t < -4$.

Next, we get $f(-3) = (\frac{t}{4} + 1)(-3) + t + 4 = \frac{t}{4} + 1 > 0 \Rightarrow t > -4$.

Thus, we get $t < -4$, and $t > -4$, so there is no value for $t$.

Therefore, for any $t$, we cannot get $y = f(x) > 0$ when $-5 \le x \le -3$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4} + 1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 \le x \le -3$.
Then, when $x$ changes from -5 to -3, the line needs to be above the $x$-axis.

So we need to have $f(-5) > 0$, and $f(-3) > 0$.

First, we get $f(-5) = (\frac{t}{4} + 1)(-5) + t + 4 = \frac{-t}{4} - 1 > 0 \Rightarrow t < -4$.

Next, we get $f(-3) = (\frac{t}{4} + 1)(-3) + t + 4 = \frac{t}{4} + 1 > 0 \Rightarrow t > -4$.

Thus, there is no value of $t$ for which $y$ is positive when $-5 \le x \le -3$.

## Suggestions or Solutions
## To the Problem in the Example 9

**Assuming $y = (\frac{t}{4}+1)x + t + 4$ where $t$ is constant, determine the value of $t$ so that $y \geq 0$ when $-5 < x < -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 < x < -3$.

Then, when $x$ is between -5 and -3, the line needs to be above or on the $x$-axis.

So we need to have $f(-5) \geq 0$, and $f(-3) \geq 0$.

Then first, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

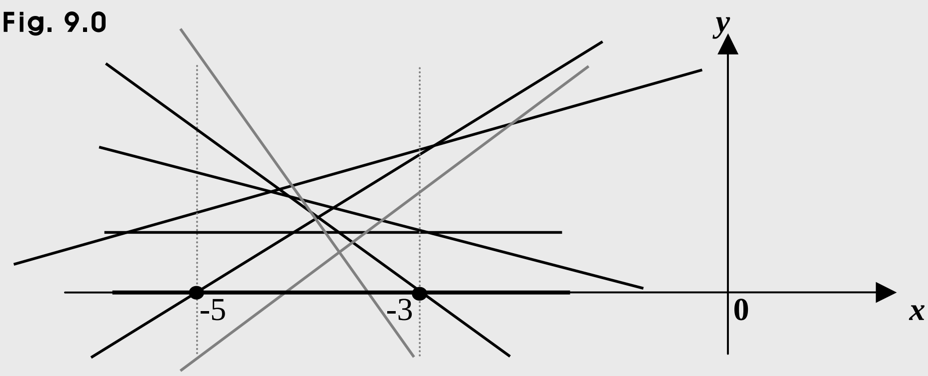
Thus, we get $t = -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) \geq 0$ when $-5 < x < -3$.

So we need to meet a condition that $y = f(x) \geq 0$ when $-5 < x < -3$, and its literal meaning is that when $x$ is between -5 and -3, $y = f(x)$ has to be greater than or equal to 0, which is however, merely a reiteration. So putting some lines in a graph, we get

Fig. 9.0




Examining the graph above, we can see that the slope doesn't really matter since it can be negative as well as positive or 0.

So what we need to do is to consider what's going on around the critical points, which are **(-5, 0)** and **(-3, 0)**, that is, **(-5, *f* (-5))** and **(-3, *f*(-3))**.

Now, getting back to the graph above, we can see the practical meaning as follows.

"When $x$ is between -5 and -3, the line has to be above or on the $x$-axis."

So we need to have $f(-5) \geq 0$ and $f(-3) \geq 0$.

What difference then, do we see in this example comparing the ones covered earlier?

The line can be the $x$-axis itself, that is, $f$ is allowed to be a 0-function.
So we don't have to bother to check to see if it can be a 0-funciton.

Now, let's find $t$. We have $y = f(x) = (\frac{t}{4}+1)x + t + 4$.

Then, first, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we get $t \leq -4$, and $t \leq -4$, so we get $t = -4$.

When $t = -4$, we get $y = f(x) = (\frac{t}{4}+1)x + t + 4 = (\frac{-4}{4}+1)x - 4 + 4 = 0$.

So we get $y = f(x) = 0$. That is, $f$ can be a 0-function only.

Therefore, for $t = -4$ only, we can get $y = f(x) \geq 0$ when $-5 < x < -3$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 < x < -3$.

Then, when $x$ is between -5 and -3, the line needs to be above or on the $x$-axis.

So we need to have $f(-5) \geq 0$, and $f(-3) \geq 0$.

Then first, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we get $t = -4$.

## Suggestions or Solutions
## To the Problem in the Example A

**Assuming $y = (\frac{t}{4}+1)x + t + 4$, and $t$ is constant, determine the value of $t$ so that $y \geq 0$ when $-5 \leq x \leq -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 \leq x \leq -3$.

Then, when $x$ changes from -5 to -3, the line needs to be above or on the $x$-axis.

So we need to have $f(-5) \geq 0$, and $f(-3) \geq 0$.

First, we get $f(-5) = (\frac{t}{4}+1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

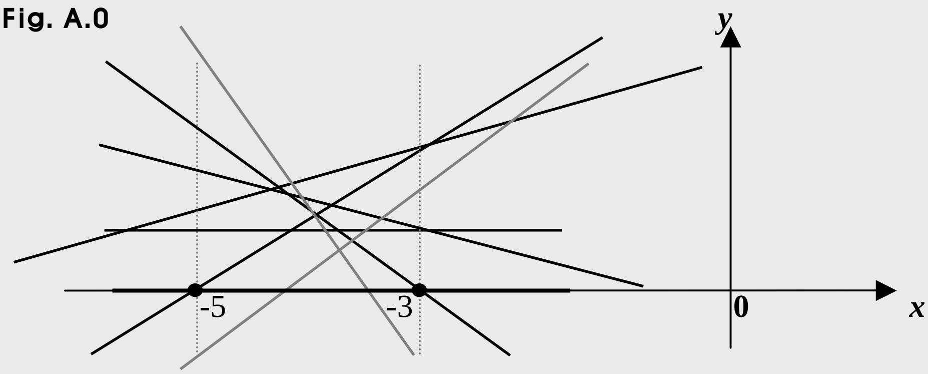
Next, we get $f(-3) = (\frac{t}{4}+1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we get $t = -4$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Setting first, $y = f(x) = (\frac{t}{4}+1)x + t + 4$, we need to have $y = f(x) \geq 0$ when $-5 \leq x \leq -3$, and its literal meaning is that when $x$ changes from -5 to -3, $y = f(x)$ has to be greater than or equal to 0, which is however, nothing but a reiteration. So putting some lines in a graph, together with the critical points, we get

Fig. A.0




Examining the graph above, we can see that the slope doesn't really matter.
So what we need to do is to consider what's going on around the critical points, which are **(-5, 0)** and **(-3, 0)**, that is, **(-5, $f$ (-5))** and **(-3, $f$(-3))**.

Now, getting back to the graph above, we can see the practical meaning as follows.
"When $x$ changes from -5 to -3, the line has to be above or on the $x$-axis."

So we need to have $f(-5) \geq 0$ and $f(-3) \geq 0$.

Since we have $y \geq 0$ when $-5 \leq x \leq -3$, we can have $y = f(x) = 0$ when $-5 \leq x \leq -3$.

Then, as in the case of the previous example, the line can be the $x$-axis itself, that is, $f$ is allowed to be a 0-function.

Now, let's find $t$. We have $y = f(x) = (\frac{t}{4} + 1)x + t + 4$.

Then first, we get $f(-5) = (\frac{t}{4} + 1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4} + 1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we get $t \leq -4$, and $t \leq -4$, so we get $t = -4$.

When $t = -4$, we get $y = f(x) = (\frac{t}{4} + 1)x + t + 4 = (\frac{-4}{4} + 1)x - 4 + 4 = 0$.

So we get $y = f(x) = 0$. That is, $f$ can be a 0-function only.

Therefore, for $t = -4$ only, we can get $y = f(x) \geq 0$ when $-5 \leq x \leq -3$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4} + 1)x + t + 4$, we need to have $y = f(x) > 0$ when $-5 \leq x \leq -3$.
Then, when $x$ changes from -5 to -3, the line needs to be above or on the $x$-axis.
So we need to have $f(-5) \geq 0$, and $f(-3) \geq 0$.

First, we get $f(-5) = (\frac{t}{4} + 1)(-5) + t + 4 = \frac{-t}{4} - 1 \geq 0 \Rightarrow t \leq -4$.

Next, we get $f(-3) = (\frac{t}{4} + 1)(-3) + t + 4 = \frac{t}{4} + 1 \geq 0 \Rightarrow t \geq -4$.

Thus, we get $t = -4$.

## Suggestions or Solutions
## To the Problem in the Example B

**Assuming that $y=(\frac{t}{4}+2)x+t+3$ where $t$ is constant, determine the value of $t$ so that $y$ can be negative as well as positive or 0 when $-5 \le x \le -3$.**

Setting first, $y = f(x) = (\frac{t}{4}+2)x+t+3$, we need to have

$f(x) \ge 0$ or $f(x) < 0$ when $-5 \le x \le -3$.

Then, the line needs to cross the $x$-axis between -5 and -3.
So we have two cases as follows.

One is that $f(-5) < 0$ and $f(-3) > 0$, and the other is that $f(-5) > 0$ and $f(-3) < 0$.

Putting both cases together, we get $f(-5)f(-3) < 0$. So first, we get

$f(-5)=(\frac{t}{4}+2)(-5)+t+3=\frac{-t}{4}-7$, and $f(-3)=(\frac{t}{4}+2)(-3)+t+3=\frac{t}{4}-3$.

Next, we get $f(-5)f(-3)=(\frac{-t}{4}-7)(\frac{t}{4}-3)=-(\frac{t}{4}+7)(\frac{t}{4}-3)<0 \Rightarrow (\frac{t}{4}+7)(\frac{t}{4}-3)>0$

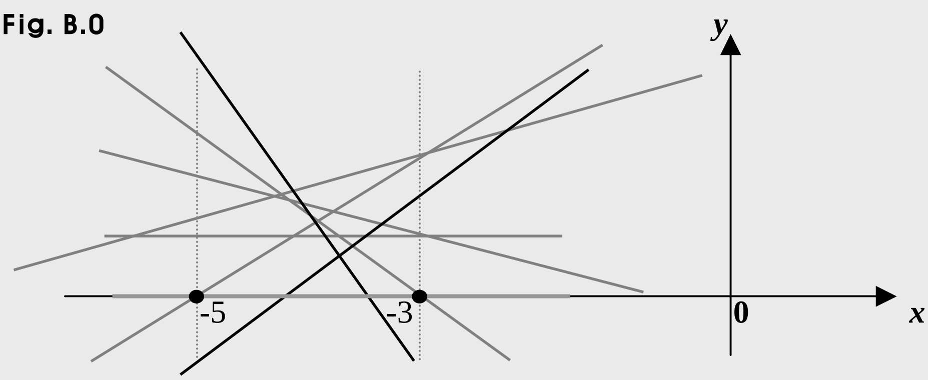
$\Rightarrow (t+28)(t-12) > 0$.

Therefore, $t > 12$ or $t < -28$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

Putting first, some lines in a graph, together with the critical points, we get

Fig. B.0



This example, too, might look similar to the previous ones, but is quite different.

We can set $\boldsymbol{y = f(x) = (\frac{t}{4} + 2)x + t + 3}$ for $\boldsymbol{x}$ and $\boldsymbol{t}$ real, but $\boldsymbol{t}$ is constant.

Then, the curve of $\boldsymbol{f}$ is a line, and when $\mathbf{-5 \le \boldsymbol{x} \le -3}$, $\boldsymbol{y = f(x)}$ can get all kinds of values if you will. That is, the condition is $\boldsymbol{f(x) \ge 0}$ or $\boldsymbol{f(x) < 0}$ when $\mathbf{-5 \le \boldsymbol{x} \le -3}$.

Then first, the literal meaning is that when $\boldsymbol{x}$ changes from -5 to -3, $\boldsymbol{y = f(x)}$ gets values, and the values can be positive, 0, and negative. So let's next, get the practical meaning. How does the curve of $\boldsymbol{f}$ have to be put though, if it needs to meet the condition?

The curve of $\boldsymbol{f}$ has to cross the $\boldsymbol{x}$-axis *between* -5 and –3. In other words, the curve has to cross the $\boldsymbol{x}$-axis in the interval of $\mathbf{-5 < \boldsymbol{x} < -3}$, and not in the interval of $\mathbf{-5 \le \boldsymbol{x} \le -3}$.

The curve is a line, so when $\mathbf{-5 \le \boldsymbol{x} \le -3}$, one of the values of $\boldsymbol{y}$ has to be 0, at least one of the others has to be either positive or negative, and all the others have to be values with the opposite sign. That is, the sign of $\boldsymbol{y}$–value has to change once only as the line crosses the $\boldsymbol{x}$-axis in that interval. When the line meets the $\boldsymbol{x}$-axis, the $\boldsymbol{y}$-value is 0, of course. Besides, even examining the graph only, we can readily notice the fact above, too.

Then, we have two cases as follows, and they form the practical meaning.
One is that $\boldsymbol{f(-5) < 0}$ and $\boldsymbol{f(-3) > 0}$, and the other is that $\boldsymbol{f(-5) > 0}$ and $\boldsymbol{f(-3) < 0}$.

Now, let's begin with the first case, $\boldsymbol{f(-5) < 0}$ and $\boldsymbol{f(-3) > 0}$.

Then first, we get $\boldsymbol{f(-5) = (\frac{t}{4} + 2)(-5) + t + 3 = \frac{-t}{4} - 7 < 0 \Rightarrow t > -28}$.

Next, we get $\boldsymbol{f(-3) = (\frac{t}{4} + 2)(-3) + t + 3 = \frac{t}{4} - 3 > 0 \Rightarrow t > 12}$.

Thus, we have $\boldsymbol{t > -28}$, and $\boldsymbol{t > 12}$, so we get $\boldsymbol{t > 12}$.

Next, let's move on to the second case, $\boldsymbol{f(-5) > 0}$ and $\boldsymbol{f(-3) < 0}$.

Then first, we get $\boldsymbol{f(-5) = \frac{-t}{4} - 7 > 0 \Rightarrow t < -28}$.

Next, we get $\boldsymbol{f(-3) = \frac{t}{4} - 3 < 0 \Rightarrow t < 12}$.

Thus, we have $t < -28$, and $t < 12$, so we get $t < -28$.

Putting the two cases together, we get $t > 12$ or $t < -28$.

• We have another method.

We have two cases as follows.
One is that $f(-5) < 0$ and $f(-3) > 0$, and the other is that $f(-5) > 0$ and $f(-3) < 0$.

In both cases, $f(-5)$ and $f(-3)$ have signs opposite of each other.
So we can put both cases in such a single expression as follows. $f(-5)f(-3) < 0$.

We have $f(-5) = \frac{-t}{4} - 7$, and $f(-3) = \frac{t}{4} - 3$.

So we get $f(-5)f(-3) = (\frac{-t}{4} - 7)(\frac{t}{4} - 3) = -(\frac{t}{4} + 7)(\frac{t}{4} - 3) < 0 \Rightarrow (\frac{t}{4} + 7)(\frac{t}{4} - 3) > 0$

$\Rightarrow (t + 28)(t - 12) > 0 \Rightarrow t > 12$ or $t < -28$.

**In short:**

Setting first, $y = f(x) = (\frac{t}{4} + 2)x + t + 3$, we need to have

$f(x) \geq 0$ or $f(x) < 0$ when $-5 \leq x \leq -3$.

Then, the line needs to cross the $x$-axis between -5 and -3.
So we have two cases as follows.
One is that $f(-5) < 0$ and $f(-3) > 0$, and the other is that $f(-5) > 0$ and $f(-3) < 0$.

Putting both cases together, we get $f(-5)f(-3) < 0$. So first, we get

$f(-5) = (\frac{t}{4} + 2)(-5) + t + 3 = \frac{-t}{4} - 7$, and $f(-3) = (\frac{t}{4} + 2)(-3) + t + 3 = \frac{t}{4} - 3$.

Next, we get $f(-5)f(-3) = (\frac{-t}{4} - 7)(\frac{t}{4} - 3) = -(\frac{t}{4} + 7)(\frac{t}{4} - 3) < 0 \Rightarrow (\frac{t}{4} + 7)(\frac{t}{4} - 3) > 0$

$\Rightarrow (t + 28)(t - 12) > 0$.

Therefore, $t > 12$ or $t < -28$.

## Suggestions or Solutions
## To the Problem in the Example C

**Assuming $y=(\frac{t}{4}+2)x+t+3$, and $t$ is constant, determine the values of $x$ so that we get $y>0$ when $1\le t\le 5$.**

We have an equation of a line, $\boldsymbol{y=(\frac{t}{4}+2)x+t+3}$ where $\boldsymbol{t}$ is constant.
Suppose $\boldsymbol{a}$ is the slope, and $\boldsymbol{s}$ is the $\boldsymbol{x}$-intercept. Then, we get first, $\boldsymbol{a=\frac{t}{4}+2}$.
And finding $\boldsymbol{s}$ next, we get

$\boldsymbol{0=(\frac{t}{4}+2)x+t+3 \Rightarrow x=\frac{-(t+3)}{\frac{t}{4}+2}=\frac{-4(t+3)}{t+8}}$, and $\boldsymbol{\frac{t+3}{t+8}=\frac{t+3+5-5}{t+8}=\frac{t+8-5}{t+8}=1-\frac{5}{t+8}}$

$\boldsymbol{\Rightarrow x=-4(1-\frac{5}{t+8})=-4+\frac{20}{t+8}}$. Thus, the $\boldsymbol{x}$-intercept $\boldsymbol{s=\frac{20}{t+8}-4}$.

Next, finding the extent of the slope $\boldsymbol{a}$, and the extent of the $\boldsymbol{x}$-intercept $\boldsymbol{s}$, we get

$\boldsymbol{1\le t\le 5 \Rightarrow \frac{1}{4}\le\frac{t}{4}\le\frac{5}{4} \Rightarrow \frac{9}{4}\le\frac{t}{4}+2\le\frac{13}{4} \Rightarrow \frac{9}{4}\le a\le\frac{13}{4}}$.

When $\boldsymbol{t=1}$, $\boldsymbol{s=\frac{20}{t+8}-4=\frac{20}{9}-4=-\frac{16}{9}}$, and when $\boldsymbol{t=5}$, $\boldsymbol{s=\frac{20}{t+8}-4=\frac{20}{13}-4=-\frac{32}{13}}$.

So we get $\boldsymbol{-\frac{32}{13}\le s\le -\frac{16}{9}}$.

When the slope is positive, we get $\boldsymbol{y>0}$ if $\boldsymbol{x>}$ the $\boldsymbol{x}$-intercept.

Since $\boldsymbol{\frac{9}{4}\le a\le\frac{13}{4}}$ when $\boldsymbol{1\le t\le 5}$, the slope is positive when $\boldsymbol{1\le t\le 5}$.

Thus, when $\boldsymbol{1\le t\le 5}$ and $\boldsymbol{x>-\frac{16}{9}}$, we get $\boldsymbol{y=(\frac{t}{4}+2)x+t+3>0}$.
Therefore, $\boldsymbol{x>-\frac{16}{9}}$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

A constant is not a variable, of course, but both can share some properties.

Each can hold one value at a time. We can set an extent where a variable can take its value, for instance, we can set $\boldsymbol{x>0}$, and the same is true for a constant, too. So in terms of an extent, a constant can be used like a variable. What then, is the difference between the two? For details on constants and variables, refer to **BASIC FUNCTIONS**.

Now in this problem, we are given an extent of a constant, and need to find the value of a variable, which is used in an equation of a line, and the equation is as follows.

$y=(\frac{t}{4}+2)x+t+3$ where $t$ is constant.

Reiterating the problem, we want to find the value of $x$ for which $y=(\frac{t}{4}+2)x+t+3>0$ when $1 \le t \le 5$. If exists anyway, is there only one value of $x$ that can make $y$ positive?

If any, there is more than one, and in fact, infinitely many values can be the value of $x$.

Basically therefore, the solution to this problem is the interval in the $x$-axis where $y$ is positive, no matter how small (or large, of course) the interval may be. Such an interval is an extent of a variable or constant, of course. How then, can we get the interval?

When a line crosses the $x$-axis, a change in sign happens. That is, the sign of $y$-value changes from + to –, or from – to +. Thus, knowing the $x$-intercept, we can see where the sign changes. It is not enough however, to know the $x$-intercept only. Why not?

That is because knowing the $x$-intercept only, we cannot be sure if the $y$-value is positive or not when $x$ is greater than the $x$-intercept. What else then, do we need to know?

We need the slope of the line, too. If the slope is positive, the $y$-value is positive when $x$ is greater than the $x$-intercept.
In this example though, the slope depends on the constant $t$, which has an extent.
So the slope, too, gets determined depending on the extent of the constant $t$.
In other words, we want to get not just a value of the slope but the extent of it, too.
That's not it though, of course. We need the extent of the $x$-intercept, too.

Now, let's begin with the slope, which is $\frac{t}{4}+2$. So assuming now, $a=\frac{t}{4}+2$, we can say that since we have $1 \le t \le 5$, as $t$ increases, the slope $a$ increases, too.

That is, as $\boldsymbol{t}$ increases, the line gets steeper. Thus, the extent of the slope $\boldsymbol{a}$ is as follows.

$\mathbf{1 \le t \le 5 \Rightarrow \frac{1}{4} \le \frac{t}{4} \le \frac{5}{4} \Rightarrow \frac{9}{4} \le \frac{t}{4} + 2 \le \frac{13}{4} \Rightarrow \frac{9}{4} \le a \le \frac{13}{4}}$.

Next, let's find the extent of the $\boldsymbol{x}$-intercept. We can use two kinds of methods.

- In the first method, we directly find the $\boldsymbol{x}$-intercept to find the extent.
- In the second, we find the extent of the $\boldsymbol{y}$-intercept, then the extent of the $\boldsymbol{x}$-intercept.

- Let's begin with the second method.

First, the equation is $\mathbf{y = (\frac{t}{4} + 2)x + t + 3}$. And suppose next, $\mathbf{b = t + 3}$.

Then, $\boldsymbol{b}$ is the $\boldsymbol{y}$-intercept, and since we have $\mathbf{1 \le t \le 5}$, as $\boldsymbol{t}$ increases, the $\boldsymbol{y}$-intercept $\boldsymbol{b}$ increases, too. That is, as $\boldsymbol{t}$ increases, the line looks moving up along the $\boldsymbol{y}$-axis.

Thus, the extent of the $\boldsymbol{y}$-intercept $\boldsymbol{b}$ is as follows. $\mathbf{1 \le t \le 5 \Rightarrow 4 \le t + 3 \le 8 \Rightarrow 4 \le b \le 8}$, and we can now, go ahead and get the $\boldsymbol{x}$-intercept.

Since the $\boldsymbol{x}$-intercept is the value of $\boldsymbol{x}$ when $\mathbf{y = 0}$, setting $\mathbf{y = 0}$ in the equation of the line, and solving the equation for $\boldsymbol{x}$, we can take the solution for the $\boldsymbol{x}$-intercept.

Suppose now, that the $\boldsymbol{x}$-intercept is $\boldsymbol{s}$.
Then first, when $\mathbf{t = 1}$, we get $\mathbf{y = \frac{9}{4}x + 4}$, so we get $\mathbf{0 = \frac{9}{4}x + 4 \Rightarrow x = -\frac{16}{9} \Rightarrow s = -\frac{16}{9}}$.

Next, when $\mathbf{t = 5}$, we get $\mathbf{y = \frac{13}{4}x + 8}$. Then, $\mathbf{0 = \frac{13}{4}x + 8 \Rightarrow x = -\frac{32}{13} \Rightarrow s = -\frac{32}{13}}$.

Therefore, we get $\mathbf{-\frac{32}{13} \le s \le -\frac{16}{9}}$, which is the extent of the $\boldsymbol{x}$-intercept.

- Next, let's try the first method stated above.

The equation is $\mathbf{y = (\frac{t}{4} + 2)x + t + 3}$.

So finding the $\boldsymbol{x}$-intercept, we get $\mathbf{0 = (\frac{t}{4} + 2)x + t + 3 \Rightarrow x = \frac{-(t+3)}{\frac{t}{4}+2} = \frac{-4(t+3)}{t+8}}$.

Meanwhile, $\mathbf{\frac{t+3}{t+8} = \frac{t+3+5-5}{t+8} = \frac{t+8-5}{t+8} = 1 - \frac{5}{t+8}}$. So we get $\mathbf{x = -4(1 - \frac{5}{t+8}) = -4 + \frac{20}{t+8}}$.

Thus, we can see that the $\boldsymbol{x}$-intercept $\mathbf{s = \frac{20}{t+8} - 4}$.

So let's next, move on to the extent of the $\boldsymbol{x}$-intercept $\boldsymbol{s}$.

Then, since we have $\mathbf{1} \le \boldsymbol{t} \le \mathbf{5}$, as $\boldsymbol{t}$ increases, the $\boldsymbol{x}$-intercept $\boldsymbol{s}$ keeps decreasing since the denominator keeps increasing.

Now, actually finding the extent of the $\boldsymbol{x}$-intercept $\boldsymbol{s}$, we get

When $\boldsymbol{t} = \mathbf{1}$, we get $\boldsymbol{s} = \frac{20}{t+8} - 4 = \frac{20}{9} - 4 = -\frac{16}{9}$.

When $\boldsymbol{t} = \mathbf{5}$, we get $\boldsymbol{s} = \frac{20}{t+8} - 4 = \frac{20}{13} - 4 = -\frac{32}{13}$. So we get $-\frac{32}{13} \le s \le -\frac{16}{9}$.

Now, putting threads together, we get

When $\boldsymbol{t} = \mathbf{1}$, we have $\boldsymbol{a} = \frac{9}{4}$ and $\boldsymbol{s} = -\frac{16}{9}$, and when $\boldsymbol{t} = \mathbf{5}$, we have $\boldsymbol{a} = \frac{13}{4}$ and $\boldsymbol{s} = -\frac{32}{13}$.

Then first of all, the slope is always positive when $\mathbf{1} \le \boldsymbol{t} \le \mathbf{5}$.
So if the value of $\boldsymbol{x}$ is greater than the $\boldsymbol{x}$-intercept $\boldsymbol{s}$, $\boldsymbol{y}$ is always positive.

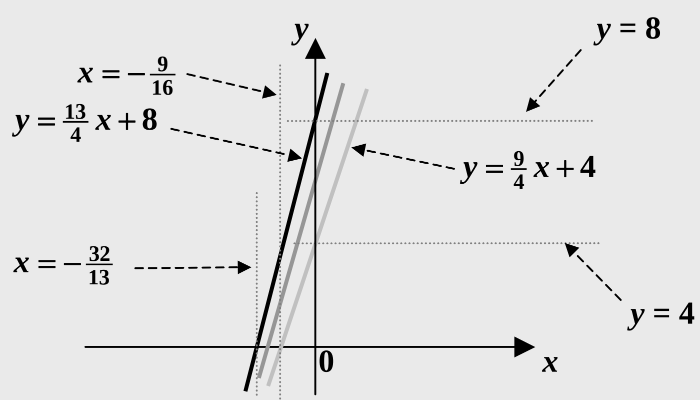
Next, as $\boldsymbol{t}$ changes from 1 to 5, the $\boldsymbol{x}$-intercept $\boldsymbol{s}$ changes from $-\frac{16}{9}$ to $-\frac{32}{13}$.

Thus, the biggest $\boldsymbol{x}$-intercept is $-\frac{16}{9}$ when $\mathbf{1} \le \boldsymbol{t} \le \mathbf{5}$, so we get $\boldsymbol{y} > \mathbf{0}$ if $x > -\frac{16}{9}$.

In sum, if $x > -\frac{16}{9}$, we get $\boldsymbol{y} > \mathbf{0}$ when $\mathbf{1} \le \boldsymbol{t} \le \mathbf{5}$.

Now, putting in a graph $y = (\frac{t}{4} + 2)x + t + 3$ where $\boldsymbol{t}$ is constant, and $\mathbf{1} \le \boldsymbol{t} \le \mathbf{5}$, we get

Fig. C.0 $y = \frac{13}{4}x + 8$ when $\boldsymbol{t} = \mathbf{5}$, and $y = \frac{9}{4}x + 4$ when $\boldsymbol{t} = \mathbf{1}$




When $\boldsymbol{t}$ is 1, the line is in light gray, and the equation of the line is $y = \frac{9}{4}x + 4$, where the $\boldsymbol{x}$-intercept is $-\frac{16}{9}$.

When $t$ is 5, the line is in black, and the equation of the line is $y = \frac{13}{4}x + 8$, where the $x$-intercept is $-\frac{32}{13}$.

Thus, as $t$ changes from 1 to 5, the line changes from the one in light gray to the one in black. So we can see this:

If we want $y > 0$ when $1 \le t \le 5$, we need $x > -\frac{16}{9}$.

Therefore, we get $y = (\frac{t}{4} + 2)x + t + 3 > 0$ when $1 \le t \le 5$ and $x > -\frac{16}{9}$.

If however, we want $y < 0$ when $1 \le t \le 5$, we need $x < -\frac{32}{13}$.
Now, what if $1 < t \le 5$, though?

Then, the value of $x$ can be $-\frac{16}{9}$. That is, we get $x \ge -\frac{16}{9}$. That's because the line in light gray gets made when $t = 1$, and in that case, we get $x = -\frac{16}{9} \Rightarrow y = 0$ since $x = -\frac{16}{9}$ is the $x$-intercept in the line light gray.

If however, $1 < t \le 5$, $t$ cannot be 1, so the line in light gray cannot be made, and we get $x = -\frac{16}{9} \Rightarrow y > 0$ since $x = -\frac{16}{9}$ is the $x$-intercept in the line light gray.

Therefore, if $1 < t \le 5$, we get $y > 0$ if $x \ge -\frac{16}{9}$.

Now, let's approach the solution above with the method used in the previous examples. That is, we are going to find the value of $x$ for which $y > 0$ when $1 < t \le 5$.

First, we take $t$ for a variable and $x$ for a constant in the given equation, and then, change the form of the equation. Then, we find the value of $x$.

Of course, a constant is different from a variable. We just take a constant for a variable in terms of extents only, and take a variable for a constant in terms of extents only, too.

Now, we can set $y = f(x) = (\frac{t}{4} + 2)x + t + 3$ for $x$ and $t$ real, but $t$ is constant.

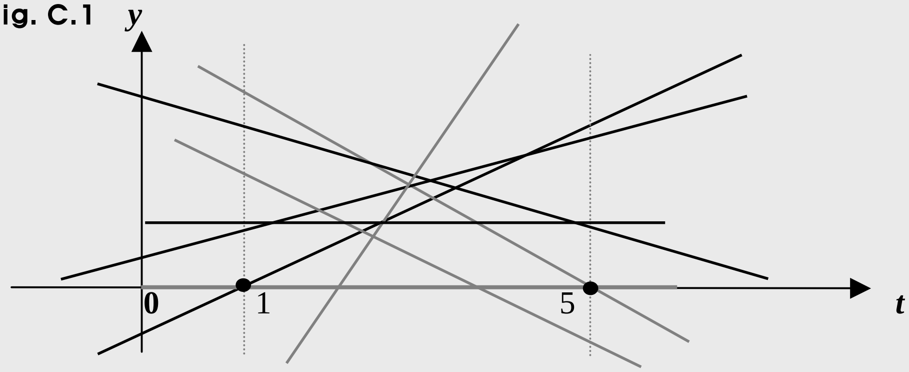
Let's this time, change the function $f(x)$ to a function $g(t)$.
Then, the only difference will be the coordinate-system names, so both functions will be actually the same.

Then, we get $y = f(x) = (\frac{t}{4} + 2)x + t + 3 = \frac{xt}{4} + 2x + t + 3 = (\frac{x}{4} + 1)t + 2x + 3 = g(t)$.

So suppose now, that $y = g(t) = (\frac{x}{4} + 1)t + 2x + 3$ where $x$ and $t$ real, but $x$ is a constant.

Then, we need to find the value of $x$ for which $g(t) > 0$ when $1 < t \leq 5$.

Fig. C.1




Suppose now, $A$ is the curve of the function $g$. Then, $A$ is a line, and when the value of $t$ is between 1 and 5, that is, when $1 < t < 5$, the line $A$ needs to be above the $t$-axis.

In this case however, we want to have $g(1) = 0$, but need to have $g(5) \neq 0$.

Therefore, we need to have $g(1) \geq 0$ and $g(5) > 0$.

So first, we get $g(1) = (\frac{x}{4} + 1)(1) + 2x + 3 = \frac{9x}{4} + 4 \geq 0 \Rightarrow x \geq -\frac{16}{9}$.

Next, we get $g(5) = (\frac{x}{4} + 1)(5) + 2x + 3 = \frac{13x}{4} + 8 > 0 \Rightarrow x > -\frac{32}{13}$.

Thus, we have $x \geq -\frac{16}{9}$ and $x > -\frac{32}{13}$, so we get $x \geq -\frac{16}{9}$.

So if the value of $x \geq -\frac{16}{9}$, we get $y = g(t) > 0$ when $1 < t \leq 5$.

That is, if $x \geq -\frac{16}{9}$, we get $y > 0$ when $1 < t \leq 5$.
And we know $y = f(x)$, too.
Thus, if the value of $x \geq -\frac{16}{9}$, we get $y = f(x) > 0$ when $1 < t \leq 5$.

**In short:**

Setting first, $\boldsymbol{y = f(x) = (\frac{t}{4}+2)x+t+3}$, and changing the function $\boldsymbol{f(x)}$ to a function $\boldsymbol{g(t)}$, we get $\boldsymbol{y = f(x) = (\frac{t}{4}+2)x+t+3 = \frac{xt}{4}+2x+t+3 = (\frac{x}{4}+1)t+2x+3 = g(t)}$.

So assuming next, $\boldsymbol{y = g(t) = (\frac{x}{4}+1)t+2x+3}$, we want to find the value of $\boldsymbol{x}$ for which $\boldsymbol{g(t) > 0}$ when $\boldsymbol{1 \le t \le 5}$.

Then, when the value of $\boldsymbol{t}$ changes from 1 to 5, the line needs to be above the $\boldsymbol{t}$-axis.

Therefore, we need to have $\boldsymbol{g(1) > 0}$, and $\boldsymbol{g(5) > 0}$. Thus, we get

$\boldsymbol{g(1) = (\frac{x}{4}+1)(1)+2x+3 = \frac{9x}{4}+4 > 0 \Rightarrow x > -\frac{16}{9}}$, and also,

$\boldsymbol{g(5) = (\frac{x}{4}+1)(5)+2x+3 = \frac{13x}{4}+8 > 0 \Rightarrow x > -\frac{32}{13}}$.

Thus, we have $\boldsymbol{x > -\frac{16}{9}}$ and $\boldsymbol{x > -\frac{32}{13}}$, so we get $\boldsymbol{x > -\frac{16}{9}}$.

So $\boldsymbol{x > -\frac{16}{9} \Rightarrow y = g(t) > 0}$ when $\boldsymbol{1 \le t \le 5}$.

That is, if $\boldsymbol{x > -\frac{16}{9}}$, we get $\boldsymbol{y > 0}$ when $\boldsymbol{1 \le t \le 5}$. And we have $\boldsymbol{y = f(x)}$, too.

And thus, if $\boldsymbol{x > -\frac{16}{9}}$, we get $\boldsymbol{y = f(x) > 0}$ when $\boldsymbol{1 \le t \le 5}$.

## Suggestions or Solutions
## To the Problem in the Example D

**Assuming $y = 3(t - 1)x + 3 - 2t$, where $t$ is constant, determine the values of $x$ so that we get $y > 0$ when $0 \le t \le 2$.**

First, assuming $a$ is the slope, we get $a = 3(t - 1)$.

Next, assuming $s$ is the $x$-intercept, and finding $s$, we get

$$y = 3(t-1)x + 3 - 2t \Rightarrow 0 = 3(t-1)x + 3 - 2t \Rightarrow 3x = \frac{2t-3}{t-1} = \frac{2t-2-1}{t-1} = 2 - \frac{1}{t-1} = 2 + \frac{1}{1-t}$$

$\Rightarrow x = \frac{1}{3}(2 + \frac{1}{1-t})$, and thus, we get the $x$-intercept $s = \frac{1}{3}(2 + \frac{1}{1-t})$.

Next, assuming $c = \frac{1}{1-t}$, we get $s = \frac{1}{3}(2 + \frac{1}{1-t}) = \frac{1}{3}(2 + c)$.

So first, when $t = 0$, we get $c = \frac{1}{1-t} = 1 \Rightarrow s = \frac{1}{3}(2+1) = 1$.

Thus, as $t$ nears 1, $c$ goes to infinity, and so does $s$.

If $t = 1$, the line is horizontal, because the slope $a = 3(t - 1) = 0$.

Thus, when $0 \le t < 1$, we get the $x$-intercept $s \ge 1$.

Next, when $t = 2$, we get $c = \frac{1}{1-t} = \frac{1}{1-2} = -1 \Rightarrow s = \frac{1}{3}(2+c) = \frac{1}{3}(2-1) = \frac{1}{3}$.

Thus, as $t$ approaches to 1, $c$ goes to negative infinity, and so does $s$.

So we get $s \le \frac{1}{3}$ when $1 < t \le 2$.

Next, finding the extent of the slope $a = 3(t - 1)$, we get

$0 \le t < 1 \Rightarrow -1 \le t - 1 < 0 \Rightarrow -3 \le 3(t - 1) < 0 \Rightarrow -3 \le a < 0$.

$1 < t \le 2 \Rightarrow 0 < t - 1 \le 1 \Rightarrow 0 < 3(t - 1) \le 3 \Rightarrow 0 < a \le 3$.

Now, putting threads together, we get

$0 \le t < 1 \Rightarrow -3 \le a < 0$, and $s \ge 1$.  $1 < t \le 2 \Rightarrow 0 < a \le 3$, and $s \le \frac{1}{3}$.

If the slope is negative, and $x$ is less than the $x$-intercept $s$, $y$ is positive.

So when $0 \le t < 1$, and $x < 1$, we get $y > 0$.

If the slope is positive, and $x$ is greater than the $x$-intercept $s$, $y$ is positive.

So when $1 < t \le 2$, and $x > \frac{1}{3}$, we get $y > 0$.

Also, $t = 1 \Rightarrow y = 3(t - 1)x + 3 - 2t = 1 \Rightarrow y = 1$, and thus, $y$ is positive for all $x$.

Therefore, when $0 \le t \le 2$, if $\frac{1}{3} < x < 1$, we get $y > 0$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

This time, too, given the extent of a constant, we find a particular extent of a variable.

A constant is different from a variable, but has properties of a variable, too. A variable can have its extent where it can take a value. The same is true for a constant, also. Thus, we sometimes use a constant like a variable, yet we do so in terms of extent only.

The equation given is an equation of a line.
After all, the extent of the constant $t$ gets to determine the extent of the slope, that of the $x$-intercept, and that of the $y$-intercept.

So let's suppose $a$ is the slope, $s$ is the $x$-intercept, and $b$ is the $y$-intercept, and then, find the extent of each.

Now, let's begin with the slope $a$. We have $a = 3(t - 1)$, and $0 \le t \le 2$.
Thus, as $t$ increases, the slope $a$ keeps increasing, too. That is, as $t$ increases, the line gets steeper. So the extent of $a$ is as follows.

$0 \le t \le 2 \Rightarrow -1 \le t - 1 \le 1 \Rightarrow -3 \le 3(t - 1) \le 3 \Rightarrow -3 \le a \le 3$, that is, $|a| \le 3$.

Next, let's find the extent of the $x$-intercept $s$.
However, we will find first, the extent of the $y$-intercept $b$, then find the extent of $s$.

We have $b = 3 - 2t$, and $0 \le t \le 2$.
Thus, as $t$ increases, the $y$-intercept $b$ keeps decreasing. That is, as $t$ increases, the line looks moving down along the $y$-axis. And the extent of $b$ is as follows.

$0 \le t \le 2 \Rightarrow 0 \ge -2t \ge -4 \Rightarrow 3 \ge 3 - 2t \ge -1 \Rightarrow -1 \le b \le 3$.

Let's now, find the extent of the $x$-intercept. Since the $x$-intercept is the value of $x$ when $y = 0$, setting $y = 0$ in the equation for lines, and solving the equation for $x$, we can take the solution for the $x$-intercept.

First, when $t = 0$, we get $y = -3x + 3$. So $0 = -3x + 3 \Rightarrow x = 1 \Rightarrow s = 1$.

Next, when $t = 2$, we get $y = 3x - 1$. So $0 = 3x - 1 \Rightarrow x = \frac{1}{3} \Rightarrow s = \frac{1}{3}$.

Therefore, it looks like we get $\frac{1}{3} \le s \le 1$. Well then, is that so, indeed?

Meanwhile, finding the $x$-intercept $s$, we get

$$y = 3(t-1)x + 3 - 2t \Rightarrow 0 = 3(t-1)x + 3 - 2t \Rightarrow 3x = \frac{2t-3}{t-1} = \frac{2t-2-1}{t-1} = 2 - \frac{1}{t-1} = 2 + \frac{1}{1-t}.$$

Thus, we get $x = \frac{1}{3}(2 + \frac{1}{1-t})$, so we get the $x$-intercept $s = \frac{1}{3}(2 + \frac{1}{1-t})$.

Next, let's see how $s$ changes as $t$ increases. We have $0 \le t \le 2$.

Suppose $c = \frac{1}{1-t}$. Then, we get $s = \frac{1}{3}(2 + \frac{1}{1-t}) = \frac{1}{3}(2 + c)$.

Thus, as $t$ increases from 0 toward 1, $c$ increases, too, and so does $s$.
Up to what is $c$ going to increase, though?

As $t$ nears 1, the denominator $(1 - t)$ gets close to 0. So $c$ goes to infinity.

Besides, we have $s = \frac{1}{3}(2 + c)$, and thus, $s$ goes to infinity, too.

However, $t$ cannot be 1 in $c$, because $(1 - t)$ is the denominator.
In fact, if $t = 1$, the line is horizontal, because the slope $a = 3(t - 1) = 0$, and in this case, the line doesn't have an $x$-intercept. Now, putting threads together, we have

When $t = 0$, we get $c = \frac{1}{1-t} = 1$, and thus, we get $s = \frac{1}{3}(2 + c) = \frac{1}{3}(2 + 1) = 1$.

As $t$ approaches 1, $c$ goes to infinity, and so does $s$.
Thus, when $0 \le t < 1$, we get the $x$-intercept $s \ge 1$.

Next, when $t = 2$, we get $c = \frac{1}{1-t} = \frac{1}{1-2} = -1$, and thus, we get $s = \frac{1}{3}(2 + c) = \frac{1}{3}(2 - 1) = \frac{1}{3}$.

Thus, as $t$ decreases from 2 toward 1, $c$ decreases, too, and so does $s$.
Up to what is $c$ going to decrease, though?

As $t$ approaches 1, the denominator $(1 - t)$ gets closer to 0, and is negative, so $c$ goes to negative infinity. And as $c$ goes to negative infinity, $s$ goes to negative infinity, too.

So putting threads together, we get $s \le \frac{1}{3}$ when $1 < t \le 2$.

Thus, when $0 \le t \le 2$, the proper extent of the $x$-intercept $s$ is as follows. $s \le \frac{1}{3}$ or $s \ge 1$.

Therefore, when $0 \le t \le 2$, $s$ cannot have a value between $\frac{1}{3}$ and 1.

So it is not the case where $\frac{1}{3} \le s \le 1$, and it is not a good idea to find the extent of the $x$-intercept by means of the extent of the $y$-intercept only.

Now, we have two cases, one of which is $0 \le t < 1$, and the other is $1 < t \le 2$.
For each of the two cases above, let's find the extent of the slope $a = 3(t - 1)$. Then,

$0 \le t < 1 \Rightarrow -1 \le t - 1 < 0 \Rightarrow -3 \le 3(t - 1) < 0 \Rightarrow -3 \le a < 0$.

$1 < t \le 2 \Rightarrow 0 < t - 1 \le 1 \Rightarrow 0 < 3(t - 1) \le 3 \Rightarrow 0 < a \le 3$.

Thus, putting threads together, we get

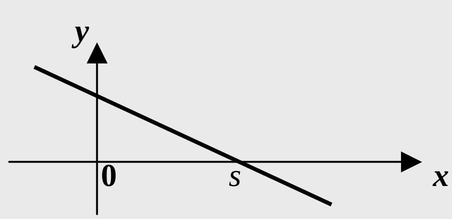
$0 \le t < 1 \Rightarrow -3 \le a < 0$, and $s \ge 1$, that is, **-3 ≤ the slope < 0**, and **the $x$-intercept ≥ 1**.

$1 < t \le 2 \Rightarrow 0 < a \le 3$, and $s \le \frac{1}{3}$, that is, **0 < the slope ≤ 3**, and **the $x$-intercept** ≤ $\frac{1}{3}$.

Now, we have all the things we need to decide the extent of $x$, so that $y$ is positive when $0 \le t \le 2$. And we have three cases where $y$ is always positive.

• First, if the slope is negative, and $x$ is less than the $x$-intercept $s$, $y$ is positive.

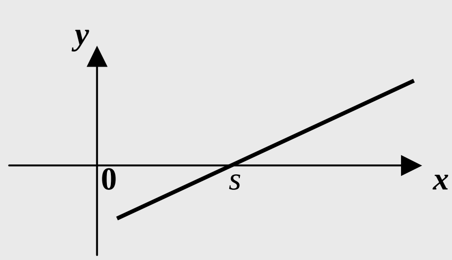
Fig. D.0




We know when $0 \le t < 1$, **-3 ≤ the slope < 0**, and **the $x$-intercept ≥ 1**.
So $x$ has to be less than 1 if $y$ is always positive when $0 \le t < 1$.
Thus, when $0 \le t < 1$, and $x < 1$, we always get $y > 0$.

• Next, if the slope is positive, and $x$ is greater than the $x$-intercept $s$, $y$ is positive.

Fig. D.1



We know when $\mathbf{1} < \boldsymbol{t} \leq \mathbf{2}$, $\mathbf{0} <$ **the slope** $\leq \mathbf{3}$, and **the $\boldsymbol{x}$-intercept** $\leq \frac{1}{3}$.

So $\boldsymbol{x}$ has to be greater than $\frac{1}{3}$ if $\boldsymbol{y}$ is always positive when $\mathbf{1} < \boldsymbol{t} \leq \mathbf{2}$.

Thus, when $\mathbf{1} < \boldsymbol{t} \leq \mathbf{2}$, and $\boldsymbol{x} > \frac{1}{3}$, we always get $\boldsymbol{y} > \mathbf{0}$.

• Also, $\boldsymbol{t} = \mathbf{1} \Rightarrow \boldsymbol{y} = \mathbf{3}(\boldsymbol{t} - \mathbf{1})\boldsymbol{x} + \mathbf{3} - \mathbf{2}\boldsymbol{t} = \mathbf{1} \Rightarrow \boldsymbol{y} = \mathbf{1}$, and thus, $\boldsymbol{y}$ is positive for all $\boldsymbol{x}$.

In sum, we have three cases as below.

When $\mathbf{0} \leq \boldsymbol{t} < \mathbf{1}$, and $\boldsymbol{x} < \mathbf{1}$, we get $\boldsymbol{y} > \mathbf{0}$.

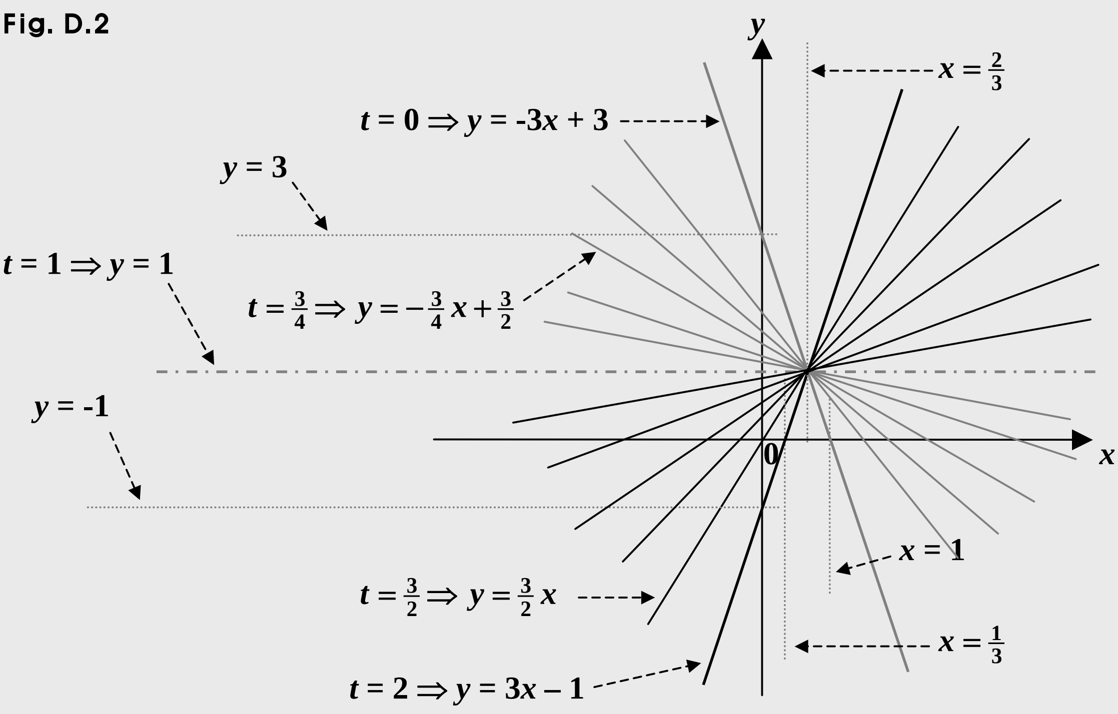
When $\boldsymbol{t} = \mathbf{1}$, for all $\boldsymbol{x}$, we get $\boldsymbol{y} > \mathbf{0}$, and in fact, is 1.

When $\mathbf{1} < \boldsymbol{t} \leq \mathbf{2}$, and $\boldsymbol{x} > \frac{1}{3}$, we get $\boldsymbol{y} > \mathbf{0}$.

So whether $\mathbf{0} \leq \boldsymbol{t} < \mathbf{1}$, or $\mathbf{1} < \boldsymbol{t} \leq \mathbf{2}$, if $\frac{1}{3} < \boldsymbol{x} < \mathbf{1}$, we get $\boldsymbol{y} > \mathbf{0}$.
Besides, when $\boldsymbol{t} = \mathbf{1}$, for all $\boldsymbol{x}$, we get $\boldsymbol{y} > \mathbf{0}$.

Therefore, when $\mathbf{0} \leq \boldsymbol{t} \leq \mathbf{2}$, if $\frac{1}{3} < \boldsymbol{x} < \mathbf{1}$, we get $\boldsymbol{y} > \mathbf{0}$.
Now, putting in a graph lines for some values of $\boldsymbol{t}$, we can get

**Fig. D.2**



Let's find the point where the lines in gray and black meet each other.

Beginning with the $x$-coordinate, we get

$$-3x+3=3x-1 \Rightarrow 6x=4 \Rightarrow x=\tfrac{2}{3} \Rightarrow y=3x-1=2\cdot\tfrac{2}{3}-1=1.$$

Therefore, the two lines meet each other at the point $(\tfrac{2}{3},1)$.

In fact, every line that can be expressed by $y = 3(t-1)x + 3 - 2t$ for each $t$ passes through the point $(\tfrac{2}{3},1)$.

That is, all the lines that can be indicated by the equation above for all the values of $t$ meet each other at the point $(\tfrac{2}{3},1)$ altogether. And we have covered how it is the case in one of the previous sets of examples.

Now, we can see that as $t$ changes, it is not that the line moves down along the $y$-axis but that it rotates around a point, which is at $(\tfrac{2}{3},1)$.

What if $0 < t < 2$?

Then, we get $\tfrac{1}{3} \le x \le 1$. How?

First, we get $x = 1 \Rightarrow y = 3(t-1)x + 3 - 2t = 3(t-1) + 3 - 2t = t \Rightarrow y = t$.

Next, we get $x=\tfrac{1}{3} \Rightarrow y=3(t-1)x+3-2t=3(t-1)(\tfrac{1}{3})+3-2t=2-t$.

Also, $0 < t < 2 \Rightarrow -2 < -t < 0 \Rightarrow 0 < 2 - t < 2 \Rightarrow 0 < y < 2$.

So anyway, we get $y > 0$ in both cases.

Now, let's try the case where $0 < t < 2$ by the method we used for the previous examples. Then, we begin with taking $t$ for a variable, and $x$ for a constant in the given equation.

Of course, a constant and a variable are different from each other, yet we take advantage of the fact that both a constant and a variable can have extents.

Next, we change the from only of the equation given. Then, we find the extent of $x$.

So first, we can set $y = f(x) = 3(t - 1)x + 3 - 2t$ where $x$ and $t$ real, but $t$ is a constant.

Then, we change the function $f(x)$ to a function $g(t)$. What then, is $g(t)$?

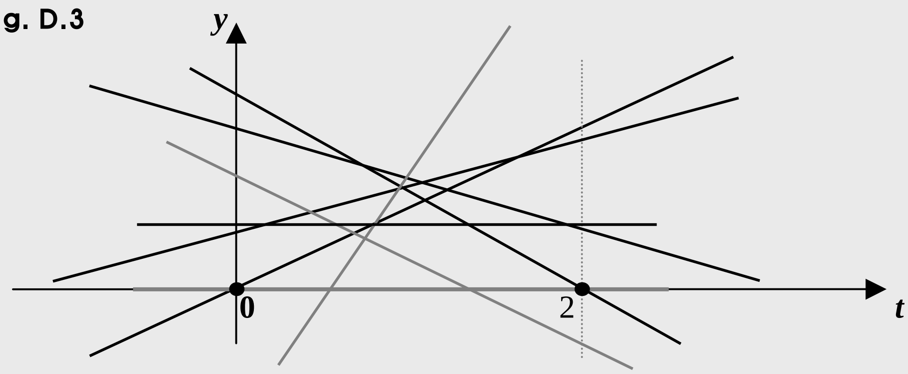
The only difference will be their coordinate-system names, and thus, $f$ and $g$ indicate actually the same function.

$y = f(x) = 3(t - 1)x + 3 - 2t = 3xt - 3x + 3 - 2t = (3x - 2)t + 3 - 3x = g(t)$.

So suppose now, that $y = g(t) = (3x - 2)t + 3 - 3x$ where $x$ is a constant.

Then, we want to get the value of $x$ for which $g(t) > 0$ when $0 < t < 2$.

Fig. D.3




Suppose now, $A$ is the curve of the function $g$.

Then, $A$ is a line, and when the value of $t$ is between 0 and 2, the line $A$ needs to be above the $t$-axis. Also, we can have either $g(0) = 0$ or $g(2) = 0$.

Then, we get $g(0) \geq 0$ or $g(2) > 0$, or $g(0) > 0$ or $g(2) \geq 0$.

We can however, set $g(0) \geq 0$ and $g(2) \geq 0$, for now.

Then of course, "$g(0) \geq 0$ and $g(2) \geq 0$." includes a case where "$g(0) = 0$ and $g(2) = 0$.".

That is, we can have a case where $g$ can be a 0-function.
We don't want however, $g$ to be a 0-function in this problem.

So let's first, check to see if $g$ can be a 0-function.

The slope is $3x - 2$. So we get

$3x - 2 = 0 \Rightarrow x = \frac{2}{3} \Rightarrow y = g(t) = (3x - 2)t + 3 - 3x = 0 \cdot t + 3 - 2 = 1 \Rightarrow y = g(t) = 1$,
which shows that $g$ cannot be a 0-function, so $x$ can be $\frac{2}{3}$.

So first, we get $g(0) \geq 0 \Rightarrow (3x - 2)(0) + 3 - 3x = 3 - 3x \geq 0 \Rightarrow x \leq 1$.

Next, we get $g(2) \geq 0 \Rightarrow (3x - 2)(2) + 3 - 3x = 6x - 4 + 3 - 3x = 3x - 1 \geq 0 \Rightarrow x \geq \frac{1}{3}$.

Thus, we have $x \leq 1$, and $x \geq \frac{1}{3}$. So we get $\frac{1}{3} \leq x \leq 1$.

Thus, if $\frac{1}{3} \leq x \leq 1$, we get $y = g(t) > 0$ when $0 < t < 2$.

That is, if $\frac{1}{3} \leq x \leq 1$, we get $y > 0$ when $0 < t < 2$. And we have $y = f(x)$, too.

Thus, if $0 < t < 2$, we get $y = f(x) > 0$ when $\frac{1}{3} \leq x \leq 1$.

**In short:**

Assuming first, $y = f(x) = 3(t - 1)x + 3 - 2t$, and changing the function $f(x)$ to a function $g(t)$, we get $y = f(x) = 3(t - 1)x + 3 - 2t = 3xt - 3x + 3 - 2t = (3x - 2)t + 3 - 3x = g(t)$.

Suppose now, that $y = g(t) = (3x - 2)t + 3 - 3x$ where $x$ and $t$ real, but $x$ is constant.
Then, we want to find the value of $x$ for which $g(t) > 0$ when $0 \leq t \leq 2$.

Then, when the value of $t$ changes from 0 to 2, the line needs to be above the $t$-axis.
Therefore, we need to have $g(0) > 0$ and $g(2) > 0$.

First, we get $g(0) = (3x - 2)(0) + 3 - 3x = 3 - 3x > 0 \Rightarrow x < 1$.

Next, we get $g(2) = (3x - 2)(2) + 3 - 3x = 6x - 4 + 3 - 3x = 3x - 1 > 0 \Rightarrow x > \frac{1}{3}$.

Thus, we have $x < 1$, and $x > \frac{1}{3}$. So we get $\frac{1}{3} < x < 1$.

So if $\frac{1}{3} < x < 1$, we get $y = g(t) > 0$ when $0 \leq t \leq 2$.

That is, if $\frac{1}{3} < x < 1$, we get $y > 0$ when $0 \leq t \leq 2$. We have $y = f(x)$, too.

Thus, if $0 \leq t \leq 2$, we get $y = f(x) > 0$ when $\frac{1}{3} < x < 1$.

Graphing matters in math.
It is particularly the case in equations and functions, so it is indispensable in calculus.

Graphing, we normally put a curve in a plane. And of course, it can be put in a space in higher math, too. Of many kinds in curves, the most frequently used are lines.

A line is the most simplest geometric object in math.
When it comes to a problem however, it is not that simple.
Understanding lines, we can pass through the threshold of calculus.
What do we do with calculus or doing it?

Basically, we break an object into pieces, and put them together.
We don't just break it though, of course. And we don't just put them together, either.

Breaking it, we want to see how it changes. More specifically, we want to measure how it changes, that is, increases or decreases, gets hotter or colder, expands or shrinks, etc.

We don't want to just measure it, of course. We want to see how it changes with respect to a change in something else as time, distance, or anything measurable.
How do we call such a measurement? More specifically, how do we call one amount of change with respect to the other?

It is called a rate. And it is embedded in a line, can be called a slope, too, and is in fact, what a line is about. It's not just a line, though. It is a line tangent. Tangent to what?

It is tangent to the behavior that the object makes while it changes.
We can look at the behavior via the curve the object makes while changing.
So the line is tangent to the curve, and has such a rate as above every moment it changes.

A line tangent to a curve matters in calculus, and is what we are after quite often doing calculus since it has such a rate as stated above. How do we get such a line or rate?

We can get it breaking the curve into pieces, each of which is very tiny.    How tiny?

Its magnitude is as good as 0 if you will. It still has the flavor of the object, though, and is called an infinitesimal displacement (or change) in the object.

Finding such a rate, we do differentiations in calculus, and doing such, we do differential calculus.

And we can add together all such changes, too.
We don’t just add them up, of course.

Doing so, we do integrations in calculus, and doing such, we do integral calculus.

Doing an integration, we often do a summation, not in a court of law, of course, but in a special fashion. The number of pieces we add up is not usually finite but infinite.

We reason to find a pattern adding them up, and the pattern shows us the sum, which is not just a number, but takes a form of a function, too. Graphing it, we can actually see it.