Examples 1 in Parabolas

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0. Find a parabola assuming the axis of symmetry is parallel to the y-axis, and the parabola is passing through three points $P_1(1, 6)$, $P_2(-1, 4)$, and $P_3(-2, 9)$.

1. Find a parabola assuming the axis of symmetry is parallel to the x-axis, and the parabola is passing through three points $P_1(1, 6)$, $P_2(-1, 4)$, and $P_3(-2, 9)$.

2. Assuming b and c are constant, and a parabola $y = 2x^2 + bx + c$ includes two points at which another parabola $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$ meets a line $y = \frac{1}{2}x - \frac{3}{4}$, find b and c.

3. Assuming a and c are constant, and a parabola $y = ax^2 - \frac{1}{2}x + c$ includes two points at which another parabola $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$ meets a line $y = \frac{1}{2}x - \frac{3}{4}$, find a and c.

4. Assuming a and c are constant, and a parabola $x = ay^2 - \frac{1}{2}y + c$ includes two points at which another parabola $y = 2x^2 + x - 3$ meets a line $y = 3x + 1$, find a and c.

Suggestions or Solutions
To the Problem in the Example 0

Find a parabola assuming the axis of symmetry is parallel to the y-axis, and the parabola is passing through three points $P_1(1, 6)$, $P_2(-1, 4)$, and $P_3(-2, 9)$.

Assuming the parabola is $y = ax^2 + bx + c$, we get

$P_1(1, 6) \Rightarrow 6 = a + b + c$

$P_2(-1, 4) \Rightarrow 4 = a - b + c$

$P_3(-2, 9) \Rightarrow 9 = 4a - 2b + c$ So we get

$(a + b + c) + (a - b + c) = 6 + 4 \Rightarrow 2a + 2c = 10 \Rightarrow a + c = 5$.

$2(a + b + c) + (4a - 2b + c) = 12 + 9 \Rightarrow 6a + 3c = 21 \Rightarrow 2a + c = 7$.

$(2a + c) - (a + c) = 7 - 5 \Rightarrow a = 2 \Rightarrow a + c = 2 + c = 5 \Rightarrow c = 3$

$\Rightarrow 6 = a + b + c = 2 + b + 3 \Rightarrow b = 1$.

Therefore, the parabola is $y = 2x^2 + x + 3$.

If not quite sure of the idea behind the processes above, follow the steps below.

We are given three points in a parabola, so we can find the parabola.
And finding it, we can use the vertex or the non-vertex form.

Using the non-vertex form, we can use $y = ax^2 + bx + c$ or $x = ay^2 + by + c$, where a, b, and c are constant, of course.

The first the two is for the case where the axis of symmetry is parallel to the y-axis.

And if the axis of the symmetry is parallel to the x-axis, we can use the second.

In this example, the axis is parallel to the y-axis. So we can use this: $y = ax^2 + bx + c$.

Finding thus, the values of the constants, we get the parabola.
How then can we get the constants?

We know that the parabola passes through the three points given. So putting into the form each point given, we can get one equation for the three constants. So we can get a system of three equations for the constants. And the rest is a matter of solving the system.

So to begin with, putting each point into the form, we get

$P_1(1, 6) \Rightarrow y = ax^2 + bx + c \Rightarrow 6 = a \cdot 1^2 + b \cdot 1 + c \Rightarrow 6 = a + b + c$.
$P_2(-1, 4) \Rightarrow 4 = a - b + c$.
$P_3(-2, 9) \Rightarrow 9 = 4a - 2b + c$.

So we now have a system of three equations, and the system is
$6 = a + b + c$, $4 = a - b + c$, and $9 = 4a - 2b + c$.   So the rest is a matter of algebra.

Eliminating b first, we get

$(a + b + c) + (a - b + c) = 6 + 4 \Rightarrow 2a + 2c = 10 \Rightarrow a + c = 5$.
$2(a + b + c) + (4a - 2b + c) = 12 + 9 \Rightarrow 6a + 3c = 21 \Rightarrow 2a + c = 7$.

So next, getting rid of c, we can find a. Then, we get $(2a + c) - (a + c) = 7 - 5 \Rightarrow a = 2$.

Then, $a + c = 5 \Rightarrow 2 + c = 5 \Rightarrow c = 3$, and thus, $6 = a + b + c \Rightarrow 6 = 2 + b + 3 \Rightarrow b = 1$.

Therefore, the parabola is $y = 2x^2 + x + 3$.

Let's now, put the parabola in a graph.

Putting the equation in the vertex form, we can get the vertex, together with the axis of symmetry. So putting it into the form now, we get

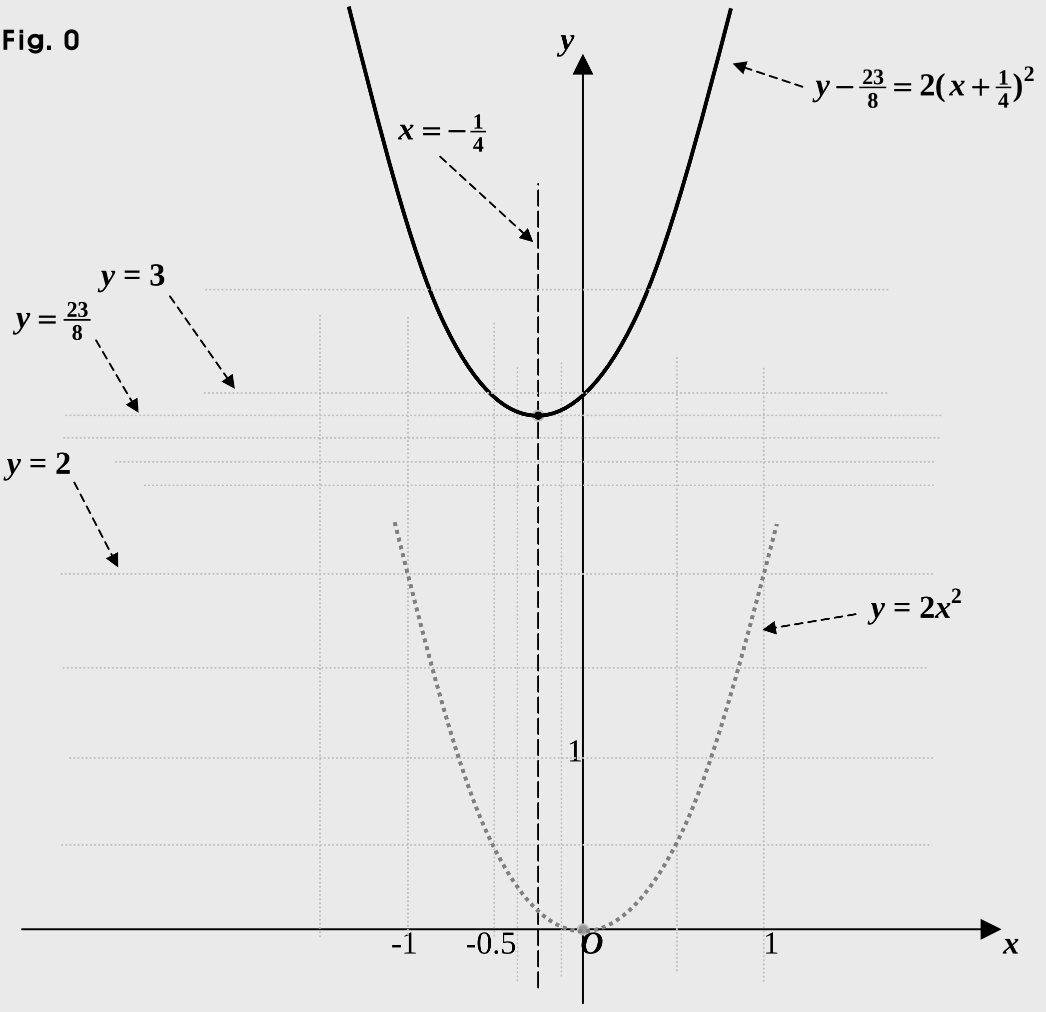
$$y = 2x^2 + x + 3 = 2(x^2 + \tfrac{1}{2}x) + 3 = 2\{x^2 + \tfrac{1}{2}x + (\tfrac{1}{4})^2 - (\tfrac{1}{4})^2\} + 3 = 2(x + \tfrac{1}{4})^2 - \tfrac{1}{8} + 3$$

$$\Rightarrow y = 2(x + \tfrac{1}{4})^2 - \tfrac{1-24}{8} = 2(x + \tfrac{1}{4})^2 + \tfrac{23}{8} \Rightarrow y = 2(x + \tfrac{1}{4})^2 + \tfrac{23}{8}.$$

And if a parabola is $y = m(x - p) + q$, and $m > 0$, the parabola is concave-up, the vertex is (p, q), and the axis of symmetry is $x = p$. And if $m < 0$, the parabola is concave-down.

So the parabola given is concave-up, the vertex is $\left(-\frac{1}{4}, \frac{23}{8}\right)$, and the axis of symmetry is a line $x = -\frac{1}{4}$.

Fig. 0



Suggestions or Solutions To the Problem in the Example 1

Find a parabola that has the directrix parallel to the *y*-axis, and is passing through three points $P_1(1, 6)$, $P_2(-1, 4)$, and $P_3(-2, 9)$.

Assuming the parabola is $x = ay^2 + by + c$, we get

$P_1(1, 6) \Rightarrow x = ay^2 + by + c \Rightarrow 1 = 36a + 6b + c$.

$P_2(-1, 4) \Rightarrow -1 = 16a + 4b + c$.

$P_3(-2, 9) \Rightarrow -2 = 81a + 9b + c$.

$(36a + 6b + c) - (16a + 4b + c) = 1 - (-1) \Rightarrow 20a + 2b = 2 \Rightarrow 10a + b = 1$.

$(81a + 9b + c) - (16a + 4b + c) = -2 - (-1) \Rightarrow 65a + 5b = -1 \Rightarrow 13a + b = -\frac{1}{5}$.

$(13a + b) - (10a + b) = -\frac{1}{5} - 1 \Rightarrow 3a = -\frac{6}{5} \Rightarrow a = -\frac{2}{5}$.

Then, $a = -\frac{2}{5} \Rightarrow 10a + b = -\frac{20}{5} + b = 1 \Rightarrow b = 1 + 4 = 5$.

So $-1 = 16a + 4b + c = -\frac{32}{5} + 20 + c \Rightarrow c = \frac{32}{5} - 21 = \frac{32-105}{5} = -\frac{73}{5}$.

Therefore, the parabola is $x = -\frac{2}{5}y^2 + 5y - \frac{73}{5}$.

If not quite sure of the idea behind the processes above, follow the steps below.

We are given three points in a parabola, so we can find the parabola.
And finding it, we can use the vertex or the non-vertex form.

Using the non-vertex form, we can use $y = ax^2 + bx + c$ or $x = ay^2 + by + c$, where a, b, and c are constant, of course.

The first the two is for the case where the axis of symmetry is parallel to the y-axis.
And if the axis of the symmetry is parallel to the x-axis, we can use the second.

In this example, the axis is parallel to the x-axis. So we can use this: $x = ay^2 + by + c$.
Finding thus, the values of the constants, we get the parabola.
How then can we get the constants?

We know that the parabola passes through the three points given. So putting into the form each point given, we can get one equation for the three constants. So we can get a system of three equations for the constants. And the rest is a matter of solving the system.

So to begin with, putting each point into the form, we get

$P_1(1, 6) \Rightarrow x = ay^2 + by + c \Rightarrow 1 = 36a + 6b + c$.
$P_2(-1, 4) \Rightarrow -1 = 16a + 4b + c$.
$P_3(-2, 9) \Rightarrow -2 = 81a + 9b + c$.

Eliminating c first, we get

$(36a + 6b + c) - (16a + 4b + c) = 1 - (-1) \Rightarrow 20a + 2b = 2 \Rightarrow 10a + b = 1$.

$(81a + 9b + c) - (16a + 4b + c) = -2 - (-1) \Rightarrow 65a + 5b = -1 \Rightarrow 13a + b = -\frac{1}{5}$.

Next, getting rid of b, we can get a. Then, we get

$(13a + b) - (10a + b) = -\frac{1}{5} - 1 \Rightarrow 3a = -\frac{6}{5} \Rightarrow a = -\frac{2}{5}$.

Next, we get $a = -\frac{2}{5} \Rightarrow 10a + b = -\frac{20}{5} + b = 1 \Rightarrow b = 1 + 4 = 5$.

And next, we get $-1 = 16a + 4b + c = -\frac{32}{5} + 20 + c \Rightarrow c = \frac{32}{5} - 21 = \frac{32-105}{5} = -\frac{73}{5}$.

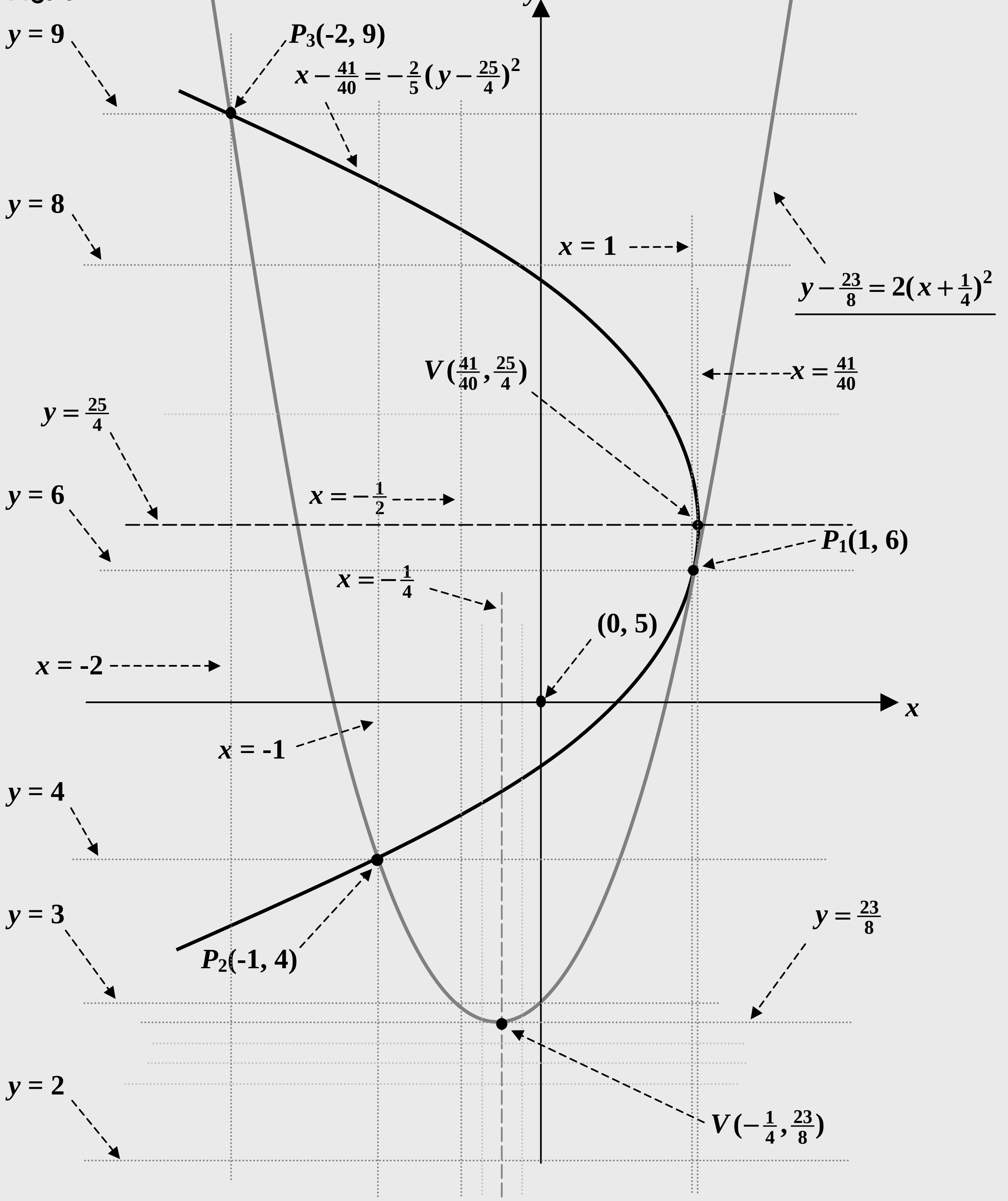
Therefore, the parabola is $x = -\frac{2}{5}y^2 + 5y - \frac{73}{5}$.

Let's now, put the parabola in a graph. Getting the vertex, we can easily get the graph. And putting the equation in the vertex form, we can put it the way below.

$$x = -\frac{2}{5}y^2 + 5y - \frac{73}{5} \Rightarrow 5x = -2y^2 + 25y - 73 = -2(y^2 - \frac{25}{2}y) - 73$$
$$= -2\{y^2 - \frac{25}{2}y + (\frac{25}{4})^2 - (\frac{25}{4})^2\} - 73 = -2(y - \frac{25}{4})^2 + \frac{625}{8} - 73 = -2(y - \frac{25}{4})^2 + \frac{625-584}{8}$$
$$\Rightarrow 5x - \frac{41}{8} = -2(y - \frac{25}{4})^2 \Rightarrow x = -\frac{2}{5}(y - \frac{25}{4})^2 + \frac{41}{40}.$$

And if a parabola is $x = m(y - q) + p$, and $m > 0$, the parabola is concave-right, the vertex is (p, q), and the axis of symmetry is $y = q$. And if $m < 0$, the parabola is concave-left.
So the parabola given is concave-left, the vertex is $(\frac{41}{40}, \frac{25}{4})$, and the axis of symmetry is a line $y = \frac{25}{4}$.

Fig. 0



Note: the origin is not shown in the graph above, but is somewhere below the line ***y* = 2**. The parabola in black is the solution to this problem, and the parabola in gray was found in the example 0, and also, includes the three given points, too.

Suggestions or Solutions
To the Problem in the Example 2

Assuming b and c are constant, and a parabola $y = 2x^2 + bx + c$ includes two points at which another parabola $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$ meets a line $y = \frac{1}{2}x - \frac{3}{4}$, find b and c.

Finding first, the $\boldsymbol{x}$-coordinates of the two points, we get

$$-2x^2 + \frac{3}{2}x + \frac{9}{4} = \frac{1}{2}x - \frac{3}{4} \Rightarrow 2x^2 - x - 3 = 0 \Rightarrow (2x-3)(x+1) = 0 \Rightarrow x = \frac{3}{2} \text{ or } -1.$$

And finding next, the $\boldsymbol{y}$-coordinates of the two points, we get

$$x = \frac{3}{2} \Rightarrow y = \frac{1}{2}x - \frac{3}{4} = \frac{1}{2} \cdot \frac{3}{2} - \frac{3}{4} = 0 \Rightarrow y = 0.$$
$$x = -1 \Rightarrow y = \frac{1}{2}x - \frac{3}{4} = -\frac{1}{2} - \frac{3}{4} = -\frac{5}{4} \Rightarrow y = -\frac{5}{4}.$$

So the other parabola and the line meet at $(\frac{3}{2}, 0)$ and $(-1, -\frac{5}{4})$.

Next, we know the parabola $y = 2x^2 + bx + c$ includes the two points above. So we get

$$0 = 2(\tfrac{3}{2})^2 + \frac{3}{2}b + c = \frac{9}{2} + \frac{3}{2}b + c \Rightarrow 3b + 2c + 9 = 0 \Rightarrow 3b + 2c = -9.$$

$$-\frac{5}{4} = 2(-1)^2 + (-1)b + c = 2 - b + c \Rightarrow b - c = \frac{13}{4}.$$

So next, solving the system of the two equations above, we get

$$(3b + 2c) + 2(b - c) = -9 + \frac{26}{4} = -\frac{10}{4} \Rightarrow 5b = -\frac{5}{2} \Rightarrow b = -\frac{1}{2}.$$
$$b = -\frac{1}{2} \Rightarrow b - c = -\frac{1}{2} - c = \frac{13}{4} \Rightarrow c = -\frac{1}{2} - \frac{13}{4} = -\frac{15}{4}.$$

If not quite sure of the idea behind the processes above, follow the steps below.

Suppose first, $\boldsymbol{A}$ is the parabola $y = 2x^2 + bx + c$.

Then, knowing two points the parabola $\boldsymbol{A}$ passes through, we can get two equations putting the two points into $\boldsymbol{A}$, and the two equations are for $\boldsymbol{b}$ and $\boldsymbol{c}$.

How then can we get the two points?

We know at the two points, the other parabola meets a line.
And the other parabola is $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$, and the line is $y = \frac{1}{2}x - \frac{3}{4}$.

So finding first, the x-coordinates at the two points, we get

$$-2x^2 + \frac{3}{2}x + \frac{9}{4} = \frac{1}{2}x - \frac{3}{4} \Rightarrow 2x^2 - x - 3 = 0 \Rightarrow (2x-3)(x+1) = 0 \Rightarrow x = \frac{3}{2} \text{ or } -1.$$

And finding next, the y-coordinates of the two points, we get

$$x = \frac{3}{2} \Rightarrow y = \frac{1}{2}x - \frac{3}{4} = \frac{1}{2}\cdot\frac{3}{2} - \frac{3}{4} = 0 \Rightarrow y = 0.$$
$$x = -1 \Rightarrow y = \frac{1}{2}x - \frac{3}{4} = -\frac{1}{2} - \frac{3}{4} = -\frac{5}{4} \Rightarrow y = -\frac{5}{4}.$$

So the other parabola and the line meet at $(\frac{3}{2}, 0)$ and $(-1, -\frac{5}{4})$.

Next, we know that the parabola $y = 2x^2 + bx + c$ includes the two points above.

So putting the two points into the equation of the parabola, we get two equations, and the two equations are for b and c. Putting thus, the two points into the equation, we get

$$(\tfrac{3}{2}, 0) \Rightarrow 0 = 2(\tfrac{3}{2})^2 + \tfrac{3}{2}b + c = \tfrac{9}{2} + \tfrac{3}{2}b + c \Rightarrow 3b + 2c + 9 = 0 \Rightarrow 3b + 2c = -9.$$
$$(-1, -\tfrac{5}{4}) \Rightarrow -\tfrac{5}{4} = 2(-1)^2 + (-1)b + c = 2 - b + c \Rightarrow b - c = \tfrac{13}{4}.$$

So we get a system of two equations, $3b + 2c = -9$, and $b - c = \frac{13}{4}$.

Thus next, solving the system above, we get

$$(3b + 2c) + 2(b - c) = -9 + \tfrac{26}{4} = -\tfrac{10}{4} \Rightarrow 5b = -\tfrac{5}{2} \Rightarrow b = -\tfrac{1}{2}.$$
$$b = -\tfrac{1}{2} \Rightarrow b - c = -\tfrac{1}{2} - c = \tfrac{13}{4} \Rightarrow c = -\tfrac{1}{2} - \tfrac{13}{4} = -\tfrac{15}{4}.$$

So the parabola is $y = 2x^2 - \frac{1}{2}x - \frac{15}{4}$.

Let’s now, put the two parabolas and the line in one graph.

Then, first, putting each parabola in the vertex form, we get

$$y = 2x^2 - \frac{1}{2}x - \frac{15}{4} \Rightarrow 4y = 8x^2 - 2x - 15 = 8(x^2 - \frac{1}{4}x) - 15$$
$$= 8\{x^2 - \frac{1}{4}x + (\frac{1}{8})^2 - (\frac{1}{8})^2\} - 15 = 8(x - \frac{1}{8})^2 - \frac{1}{8} - 15 = 8(x - \frac{1}{8})^2 - \frac{121}{8}$$
$$\Rightarrow 4y = 8(x - \frac{1}{8})^2 - \frac{121}{8} \Rightarrow y = 2(x - \frac{1}{8})^2 - \frac{121}{32}.$$

And if a parabola is $y = m(x - p) + q$, and $m > 0$, the parabola is concave-up, the vertex is (p, q), and the axis of symmetry is $x = p$. And if $m < 0$, the parabola is concave-down.

So the vertex is $(\frac{1}{8}, -\frac{121}{32}) = (0.125, -3.78125)$, the axis of symmetry is a line $x = \frac{1}{8}$, and the parabola is concave-up.

And next, moving on to the other parabola, we get

$$y = -2x^2 + \frac{3}{2}x + \frac{9}{4} = -2\{x^2 - \frac{3}{4}x + (\frac{3}{8})^2 - (\frac{3}{8})^2\} + \frac{9}{4} = -2(x - \frac{3}{8})^2 + \frac{9}{32} + \frac{9}{4}$$
$$\Rightarrow y = -2(x - \frac{3}{8})^2 - \frac{81}{32}.$$

So the vertex is $(\frac{3}{8}, \frac{81}{32}) = (0.375,\ 2.53125)$, the axis of symmetry is a line $x = \frac{3}{8}$, and the parabola is concave-down.

Now, the two parabolas have different equations. So are the parabolas themselves are different, too?

In math, we take the two parabolas as two different parabolas, because their equations are different. The parabolas themselves are however, exactly the same. They are just put in two different locations in the coordinate plane, that is, their locations are different in the x-y plane.
How do we know though, that the two curves themselves are the same?

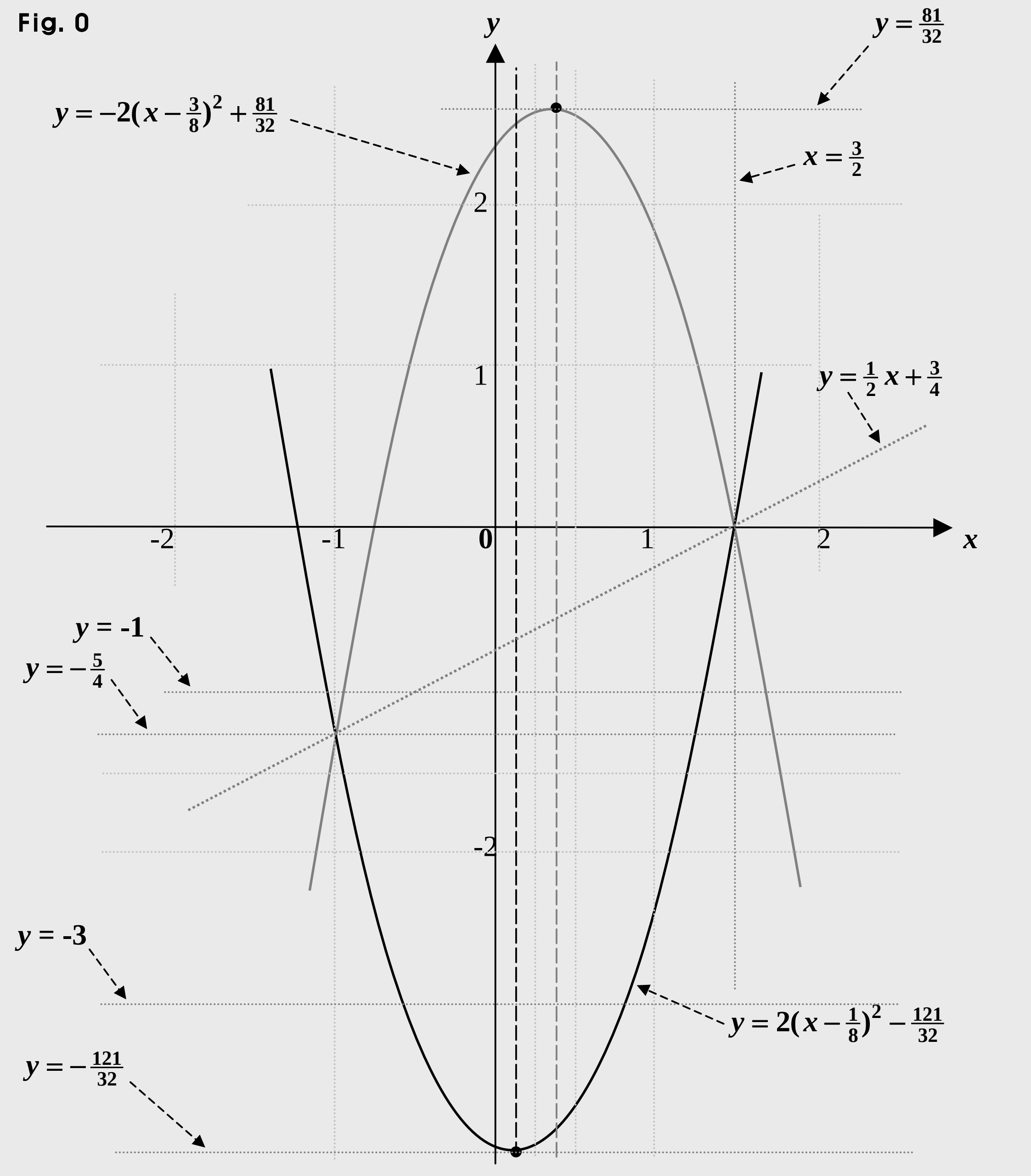
One of the two parabolas is $y = 2x^2 - \frac{1}{2}x - \frac{15}{4}$. And the other is $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$.

So we can notice that the magnitudes of the coefficients of the x^2-terms are the same.

That is, their absolute values are the same. In fact, if the absolute values are the same, the parabolas themselves are identical to each other.

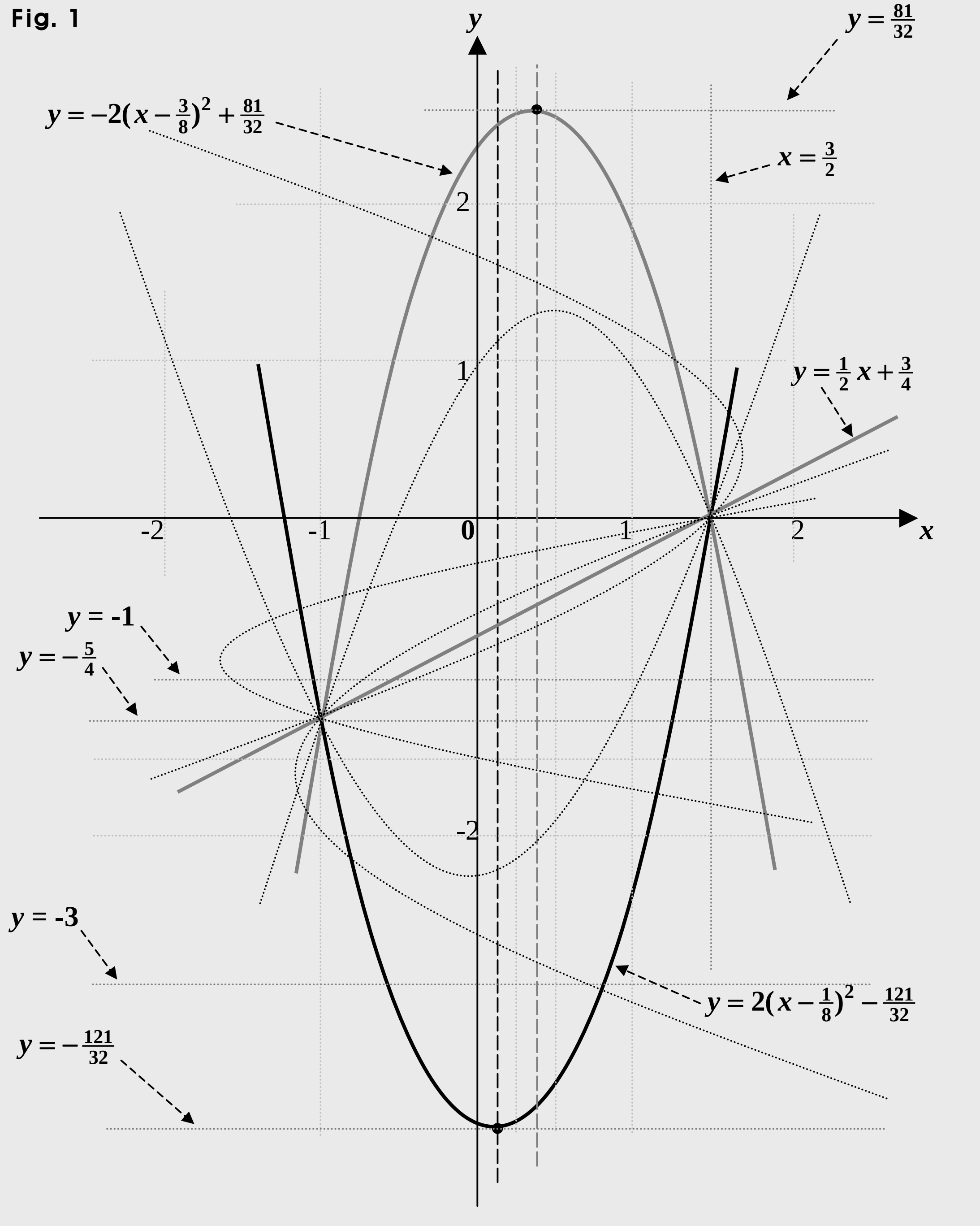
Now, putting the two parabolas and the line in one graph, we can put them the way as follows.

Fig. 0



Note that there can be infinitely many parabolas passing through the two points. And we can put them in the same graph the way below.

Fig. 1


y
$y = \frac{81}{32}$
$y = -2(x - \frac{3}{8})^2 + \frac{81}{32}$
$x = \frac{3}{2}$
2
1
$y = \frac{1}{2}x + \frac{3}{4}$
-2
-1
0
1
2
x
y = -1
$y = -\frac{5}{4}$
-2
y = -3
$y = 2(x - \frac{1}{8})^2 - \frac{121}{32}$
$y = -\frac{121}{32}$

Suggestions or Solutions
To the Problem in the Example 3

Assuming a and c are constant, and a parabola $y = ax^2 - \frac{1}{2}x + c$ includes two points at which another parabola $y = -2x^2 + \frac{3}{2}x + \frac{9}{4}$ meets a line $y = \frac{1}{2}x - \frac{3}{4}$, find a and c.

Suppose that $\boldsymbol{B}$ is the parabola $y = ax^2 - \frac{1}{2}x + c$.
Then, the parabola $\boldsymbol{B}$ passes through two points, at which the other parabola meets the line. So putting into $\boldsymbol{B}$ each of the two points, we can get one equation for the two constants $\boldsymbol{a}$ and $\boldsymbol{c}$. Thus, we can get a system of two equations for the two constants.

And notice that, the other parabola and the line are that same as those in the example 2.
So the two points are $(\frac{3}{2}, 0)$ and $(-1, -\frac{5}{4})$.
And if $\boldsymbol{A}$ is the parabola $y = 2x^2 - \frac{1}{2}x - \frac{15}{4}$, the coefficients of the $\boldsymbol{x}$-terms in $\boldsymbol{A}$ and $\boldsymbol{B}$ are the same, and are $-\frac{1}{2}$. So is the parabola $\boldsymbol{A}$ the same as the parabola $\boldsymbol{B}$?
In other words, is the value of $\boldsymbol{a}$ equal to 2, and is the value of $\boldsymbol{c}$ equal to $-\frac{15}{4}$?

Probably the same. Let's check to see if it really is the case.

The parabola $\boldsymbol{B}$ is $y = ax^2 - \frac{1}{2}x + c$, and includes the two points $(\frac{3}{2}, 0)$ and $(-1, -\frac{5}{4})$.

So we get $0 = a(\frac{3}{2})^2 - \frac{1}{2}\cdot\frac{3}{2} + c = \frac{9a}{4} - \frac{3}{4} + c \Rightarrow 9a - 3 + 4c = 0 \Rightarrow 9a + 4c = 3$.

And next, we get $-\frac{5}{4} = a(-1)^2 - \frac{1}{2}(-1) + c = a + \frac{1}{2} + c \Rightarrow a + c = -\frac{7}{4}$.

So next, solving the system, we get $(9a + 4c) - 4(a + c) = 3 - (-7) = 10$

$\Rightarrow 5a = 10 \Rightarrow a = 2 \Rightarrow a + c = 2 + c = -\frac{7}{4} \Rightarrow c = -\frac{7}{4} - 2 = -\frac{15}{4} \Rightarrow c = -\frac{15}{4}$.

So two parabolas $\boldsymbol{A}$ and $\boldsymbol{B}$ are the same. What if $\boldsymbol{B}$ is $y = ax^2 + bx - \frac{15}{4}$, though?

It will be the same as the parabola $\boldsymbol{A}$, too.

That's because there is only one particular equation for one particular parabola.

Same equations can look different, though.
For instance, $y = 2x^2 + 3x + \frac{5}{2}$ is the same as $4x^2 + 6x - 2y + 5 = 0$.

Suggestions or Solutions
To the Problem in the Example 4

Assuming *a* and *c* are constant, and a parabola $x = ay^2 - \frac{1}{2}y + c$ includes two points at which another parabola $y = 2x^2 + x - 3$ meets a line $y = 3x + 1$, find *a* and *c*.

This example is just about the same as the previous two examples.
So finding first, the ***x***-coordinates of the two points, we get

$2x^2 + x - 3 = 3x + 1 \Rightarrow 2x^2 - 2x - 4 = 0 \Rightarrow 2(x + 1)(x - 2) = 0 \Rightarrow x = -1$ or 2.

So next, $x = -1 \Rightarrow y = 3x + 1 = -3 + 1 = -2$, and $x = 2 \Rightarrow y = 3x + 1 = 6 + 1 = 7$.

Therefore, the other parabola and the line meet at **(-1, -2)** and **(2, 7)**.

Next, the parabola $x = ay^2 - \frac{1}{2}y + c$ includes the two points above.   So we get

$-1 = a(-2)^2 - \frac{1}{2}(-2) + c = 4a + 1 + c \Rightarrow 4a + c = -2$.

$2 = 7^2 a - \frac{7}{2} + c = 49a - \frac{7}{2} + c \Rightarrow 49a + c = \frac{11}{2}$.

And next, solving the system of the two equations above, we get

$(49a + c) - (4a + c) = \frac{11}{2} - (-2) = \frac{15}{2} \Rightarrow 45a = \frac{15}{2} \Rightarrow 3a = \frac{1}{2} \Rightarrow a = \frac{1}{6}$.

$a = \frac{1}{6} \Rightarrow 4a + c = 4 \cdot \frac{1}{6} + c = -2 \Rightarrow c = -2 - \frac{2}{3} = -\frac{8}{3} \Rightarrow c = -\frac{8}{3}$.

Let's now, put the two parabolas and the line in one graph.

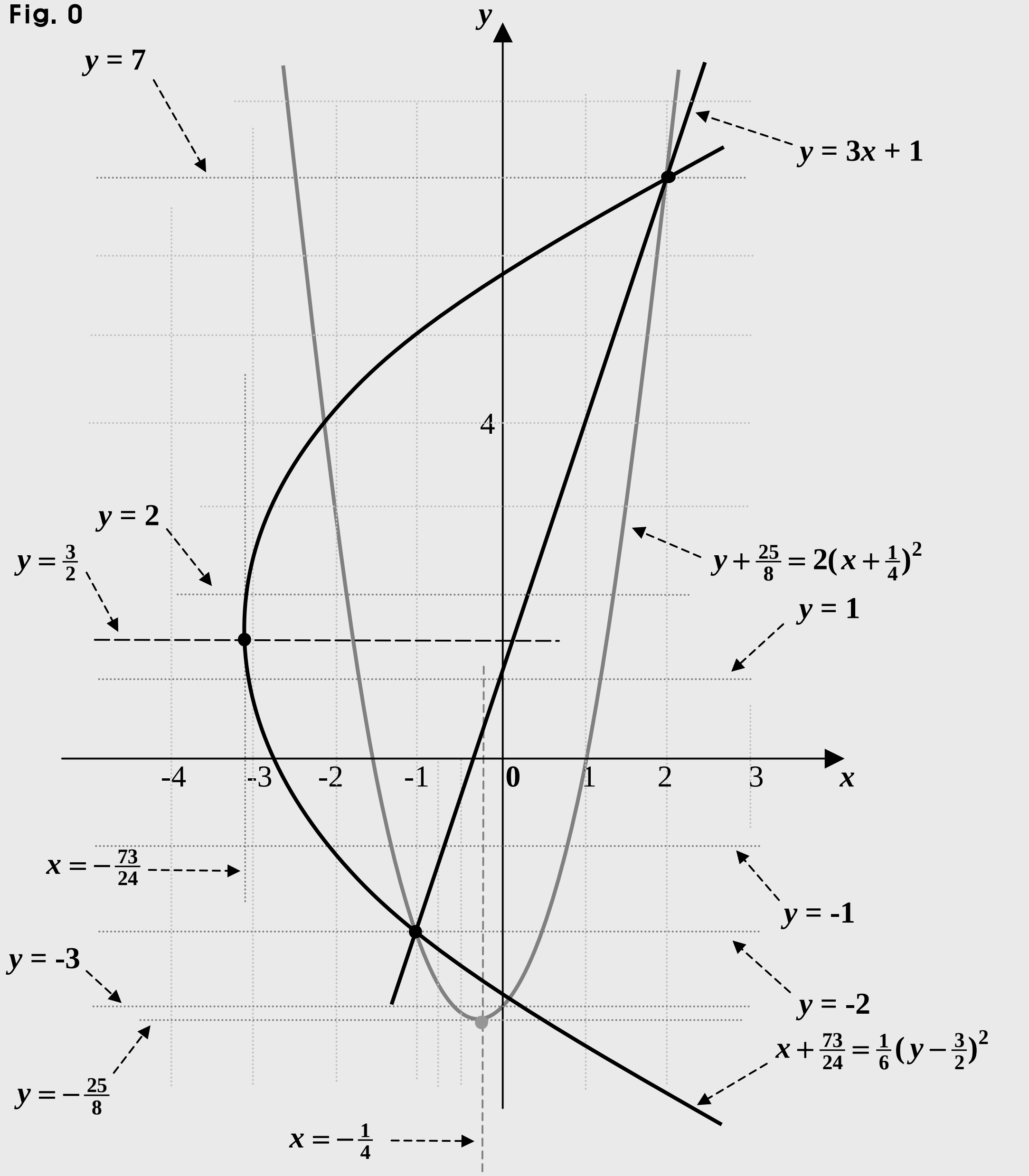
$x = \frac{1}{6}y^2 - \frac{1}{2}y - \frac{8}{3} \Rightarrow 6x = y^2 - 3y - 16 = y^2 - 3y + (\frac{3}{2})^2 - (\frac{3}{2})^2 - 16 = (y - \frac{3}{2})^2 - \frac{9}{4} - 16$

$\Rightarrow 6x + \frac{73}{4} = (y - \frac{3}{2})^2 \Rightarrow x + \frac{73}{24} = \frac{1}{6}(y - \frac{3}{2})^2$. So the vertex is $(-\frac{73}{24}, \frac{3}{2}) \approx (-3.04, 1.5)$.

$y = 2x^2 + x - 3 = 2\{x^2 + \frac{1}{2}x + (\frac{1}{4})^2 - (\frac{1}{4})^2\} - 3 = 2(x + \frac{1}{4})^2 - \frac{1}{8} - 3 \Rightarrow y + \frac{25}{8} = 2(x + \frac{1}{4})^2$.

So the vertex is $(-\frac{1}{4}, -\frac{25}{8}) = (-0.25, -3.125)$.

Fig. 0


y
y = 7
y = 3x + 1
4
y = 2
$y = \frac{3}{2}$
$y + \frac{25}{8} = 2(x + \frac{1}{4})^2$
y = 1
-4 -3 -2 -1 0 1 2 3 x
$x = -\frac{73}{24}$
y = -1
y = -3
y = -2
$x + \frac{73}{24} = \frac{1}{6}(y - \frac{3}{2})^2$
$y = -\frac{25}{8}$
$x = -\frac{1}{4}$