Examples 4 in Parabolas

Click this to start.

Examples 4

Using a pair of vertical bars around an expression as in |-3|, $|a|$, or $|2a - 1|$, we mean the absolute value of the expression. An absolute value is positive or 0, and cannot be negative. So for instance, we get: $|a| \geq 0$, and $|2a - 1| \geq 0$, and we get: |-3| = 3, and |0| = 0. And we call the pair of vertical bars, the absolute sign.

When graphing the curve of an equation with the absolute sign, we need to remove the sign, first. We don't just remove the sign, of course. By definition, we have: $|x| \geq 0$ for all values of x. So when removing the absolute sign, we need to consider two cases.

- One is the case where what's inside the absolute sign is greater than or equal to 0.
- The other is the case where what's inside the sign is less than 0.

In the case where it is less than 0, we need to add a minus sign (-) to it after the removal of the absolute sign. Assuming for instance: $t = |s|$, we need to consider two cases below.

If $s \geq 0$, we get $t = s$; that is, $|s| = s$.
And if $s < 0$, we get $t = -s$; that is, $|s| = -s$, because $|s|$ is positive.

Let's now, construct the graphs of the equations as follows.

0. $y = 2(x - 1)^2 - 2|x - 2| + 1$
1. $y = (|2x| - 1)(x + 1)$
2. $y = |x^2 + 3x - 2|$
3. $y = |x|^2 + 2|x| - 3$
4. $|y| = x^2 + 2x - 3$
5. $|y| = |x|^2 + 2|x| - 3$
6. $|y| = x^2 + |2x - 3|$

Suggestions or Solutions
To the Problem in the Example 0

Put in a graph, the curve of $y = 2(x-1)^2 - 2|x-2| + 1$.

First, $x - 2 \geq 0 \Rightarrow x \geq 2 \Rightarrow y = 2(x-1)^2 - 2(x-2) + 1 = 2x^2 - 4x + 2 - 2x + 4 + 1$

$= 2x^2 - 6x + 7 = 2(x^2 - 3x) + 7 = 2(x - \frac{3}{2})^2 - 2(\frac{3}{2})^2 + 7 = 2(x - \frac{3}{2})^2 + \frac{5}{2}$.

So we get an equation $y = 2(x - \frac{3}{2})^2 + \frac{5}{2}$ for $x \geq 2$.

Next, $x - 2 < 0 \Rightarrow x < 2 \Rightarrow y = 2(x-1)^2 - \{-2(x-2)\} + 1 = 2x^2 - 4x + 2 + 2x - 4 + 1$

$= 2x^2 - 2x - 1 = 2(x^2 - x) - 1 = 2(x - \frac{1}{2})^2 - 2(\frac{1}{2})^2 - 1 = 2(x - \frac{1}{2})^2 - \frac{3}{2}$.

So we get another equation $y = 2(x - \frac{1}{2})^2 - \frac{3}{2}$ for $x < 2$.

So putting in a graph, the curves of the two equations above, we get the graph of the given equation $y = 2(x-1)^2 - 2|x-2| + 1$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, as stated earlier, we need to remove the absolute sign first.

And when removing the sign, we need to consider the fact that we have: $|x - 2| \geq 0$ by the definition for the absolute sign.

So we cannot just put it this way: $y = 2(x-1)^2 - 2(x-2) + 1 = 2(x-1)^2 - 2x + 5$.

When removing the absolute sign, we need to consider two cases.

- One is the case where what’s inside the absolute sign is greater than or equal to 0.
- The other is the case where what’s inside the sign is less than 0.

And in this problem, what's inside the absolute sign is $\boldsymbol{x - 2}$.

So we need to consider two cases, one is $\boldsymbol{x - 2 \geq 0}$, and the other is $\boldsymbol{x - 2 < 0}$.

First, if $\boldsymbol{x - 2 \geq 0}$, we get $\boldsymbol{|x - 2| = x - 2}$. That is to say that $\boldsymbol{|x - 2| = x - 2}$ for $\boldsymbol{x \geq 2}$.

Next, if $\boldsymbol{x - 2 < 0}$, we get $\boldsymbol{|x - 2| = -(x - 2) = -x + 2}$. That is, $\boldsymbol{|x - 2| = -x + 2}$ for $\boldsymbol{x < 2}$.

So we get these:

First, $\boldsymbol{x - 2 \geq 0 \Rightarrow x \geq 2 \Rightarrow y = 2(x - 1)^2 - 2(x - 2) + 1 = 2x^2 - 4x + 2 - 2x + 4 + 1}$

$\boldsymbol{= 2x^2 - 6x + 7 = 2(x^2 - 3x) + 7 = 2(x - \frac{3}{2})^2 - 2(\frac{3}{2})^2 + 7 = 2(x - \frac{3}{2})^2 + \frac{5}{2}}$ for $\boldsymbol{x \geq 2}$.

So we get an equation $\boldsymbol{y = 2(x - \frac{3}{2})^2 + \frac{5}{2}}$ for $\boldsymbol{x \geq 2}$.

Next, $\boldsymbol{x - 2 < 0 \Rightarrow x < 2 \Rightarrow y = 2(x - 1)^2 - \{-2(x - 2)\} + 1 = 2x^2 - 4x + 2 + 2x - 4 + 1}$

$\boldsymbol{= 2x^2 - 2x - 1 = 2(x^2 - x) - 1 = 2(x - \frac{1}{2})^2 - 2(\frac{1}{2})^2 - 1 = 2(x - \frac{1}{2})^2 - \frac{3}{2}}$ for $\boldsymbol{x < 2}$.

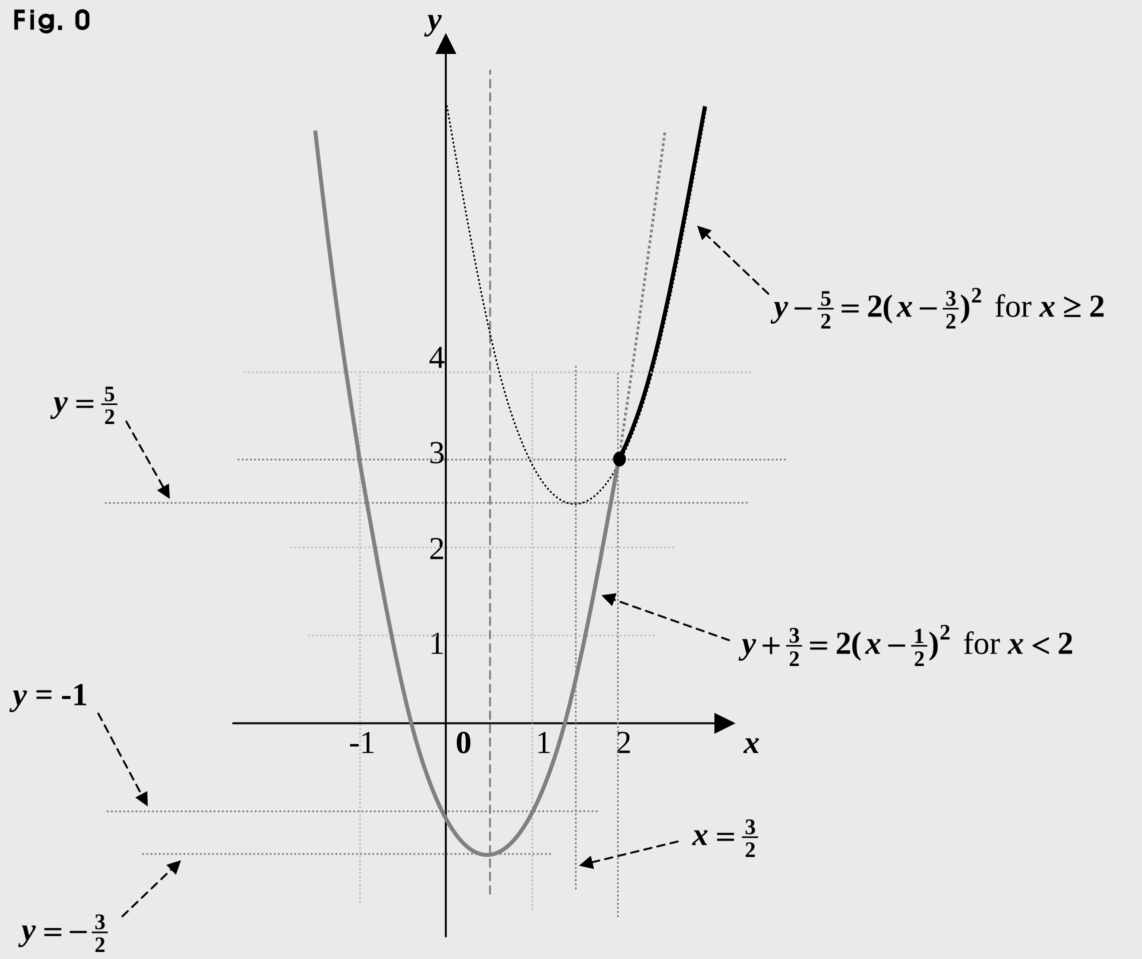
So we get another equation $\boldsymbol{y = 2(x - \frac{1}{2})^2 - \frac{3}{2}}$ for $\boldsymbol{x < 2}$.

Thus, graphing the given equation $\boldsymbol{y = 2(x - 1)^2 - 2|x - 2| + 1}$, we need to put in one graph, the two curves of two equations.

One is $\boldsymbol{y = 2(x - \frac{3}{2})^2 + \frac{5}{2}}$ for $\boldsymbol{x \geq 2}$. And the other is $\boldsymbol{y = 2(x - \frac{1}{2})^2 - \frac{3}{2}}$ for $\boldsymbol{x < 2}$.

And putting in one graph, the curves of the two equations above, we can put them the way below.

Fig. 0



So we can say that the curve of the equation given is made of two parabola rays. One is in gray, and the other is in black in the graph above.

And note that the point (2, 3) belongs to the parabola ray in black and not the ray in gray.

Suggestions or Solutions
To the Problem in the Example 1

Put in a graph, the curve of $y = (|2x| - 1)(x + 1)$.

First, $x \geq 0 \Rightarrow y = (2x - 1)(x + 1)$. So we get $y = (2x - 1)(x + 1)$ for $x \geq 0$.

And next, $x < 0 \Rightarrow y = -(2x + 1)(x + 1)$. So we get $y = -(2x + 1)(x + 1)$ for $x < 0$.

And thus, graphing the equation $y = (|2x| - 1)(x + 1)$, we need to put in a graph, the two curves of the two equations above.

If not quite sure of the idea behind the processes above, follow the steps below.

First, we have $|2x| \geq 0$ by the definition for the absolute sign.
So we have to take care of two cases, one is $x \geq 0$, and the other is $x < 0$.
Where is 2, though?

We don't need to use it. That's because 2 is positive, so it does not affect the sign of $2x$.

If $x \geq 0$, we simply get $|2x| = 2x$, and if $x < 0$, we just get $|2x| = -2x$.

And in fact, $2x \geq 0 \Rightarrow x \geq 0$, and, and $2x < 0 \Rightarrow x < 0$.

So we need to consider two cases. One is $x \geq 0$, and, the other is $x < 0$.

And considering each of the two cases above, we get an equation for each case.
That is, we will get to have two equations, which altogether, indicate the equation given.

In other words, putting together the curves of the two equations, we get the curve of the equation given in this problem. And we can get the two equations the way below.

First, $x \geq 0 \Rightarrow y = (|2x| - 1)(x + 1) = (2x - 1)(x + 1)$ for $x \geq 0$.

So we get $y = (2x - 1)(x + 1)$ for $x \geq 0$.

And next, $x < 0 \Rightarrow y = (|2x| - 1)(x + 1) = (-2x - 1)(x + 1) = -(2x + 1)(x + 1)$ for $x < 0$.

So we get $y = -(2x + 1)(x + 1)$ for $x < 0$.

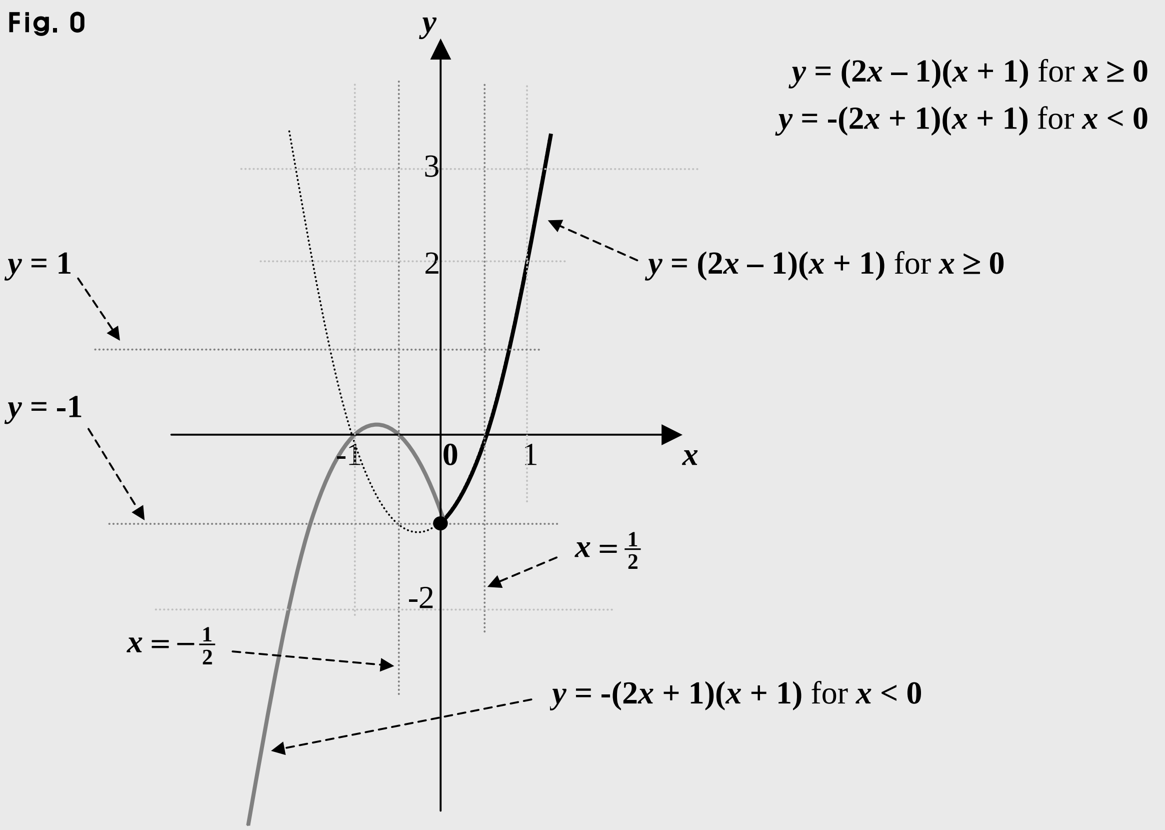
Thus, we can express the given equation $y = (|2x| - 1)(x + 1)$ using two equations.

One is $y = (2x - 1)(x + 1)$ for $x \geq 0$. And the other is $y = -(2x + 1)(x + 1)$ for $x < 0$.

So assuming A is the equation $y = (|2x| - 1)(x + 1)$, and graphing the equation A, we need to put in one graph, the two curves of the two equations above.

Thus, we can graph the equation $y = (|2x| - 1)(x + 1)$ the way below.

Fig. 0



Notice the curve of A has three x-intercepts, that is, three roots. Two of the three belongs to the second of the two equations above, and the two are $-\frac{1}{2}$ and -1. And the other root belongs to the first of the two equations, and the root is $\frac{1}{2}$, because its domain is $x \geq 0$.

Suggestions or Solutions
To the Problem in the Example 2

Graph the equation as follows: $y = |x^2 + 3x - 2|$.

First, $x^2 + 3x - 2 = (x + 1)(x + 2) \geq 0 \Rightarrow x \geq -1$ or $x \leq -2$

So we get an equation: $y = (x + 1)(x + 2)$ for $x \geq -1$ or $x \leq -2$.

And next, $x^2 + 3x + 2 = (x + 1)(x + 2) < 0 \Rightarrow -2 < x < -1$

So we get another equation $y = -(x + 1)(x + 2)$ for $-2 < x < -1$.

So graphing $y = |x^2 + 3x - 2|$, we put in a graph, the curves of the two equations above:

If not quite sure of the idea behind the processes above, follow the steps below.

We have $|x^2 + 3x - 2| \geq 0$ by the definition for the absolute sign.

So first, we get: $x^2 + 3x - 2 \geq 0 \Rightarrow (x + 1)(x + 2) \geq 0 \Rightarrow x \geq -1$ or $x \leq -2$

$\Rightarrow y = |x^2 + 3x + 2| = x^2 + 3x + 2 = (x + 1)(x + 2)$ for $x \geq -1$ or $x \leq -2$.

So we get $y = (x + 1)(x + 2)$ for $x \geq -1$ or $x \leq -2$.

And its x-intercepts are -1 and -2.

And next, we get $x^2 + 3x + 2 = (x + 1)(x + 2) < 0 \Rightarrow -2 < x < -1$

$\Rightarrow y = |x^2 + 3x + 2| = -(x^2 + 3x + 2) = -(x + 1)(x + 2)$ for $-2 < x < -1$.

So we get $y = -(x + 1)(x + 2)$ for $-2 < x < -1$.

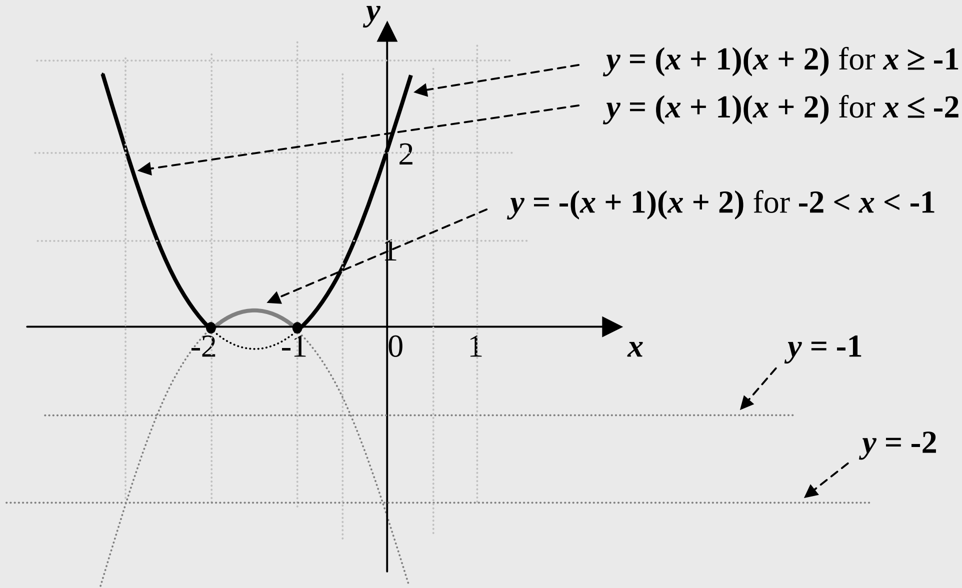
And it has no x-intercept, that is, no root, because its domain does not include -1 and -2.

So the curve of $y = |x^2 + 3x - 2|$ has two roots, and we can graph it the way below.

Fig. 0

$y = (x + 1)(x + 2)$ for $x \geq -1$ or $x \leq -2$

$y = -(x + 1)(x + 2)$ for $-2 < x < -1$



And we can express the equation given the way below, too.

$$y = |x^2 + 3x - 2| = \begin{cases} (x+1)(x+2) \text{ for } x \geq -1 \text{ or } x \leq -2 \\ -(x+1)(x+2) \text{ for } -2 < x < -1 \end{cases}$$

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph the curve of the equation as follows: $y = |x|^2 + 2|x| - 3$.

First, $x \geq 0 \Rightarrow y = |x|^2 + 2|x| - 3 = x^2 + 2x - 3 = (x - 1)(x + 3)$ for $x \geq 0$.

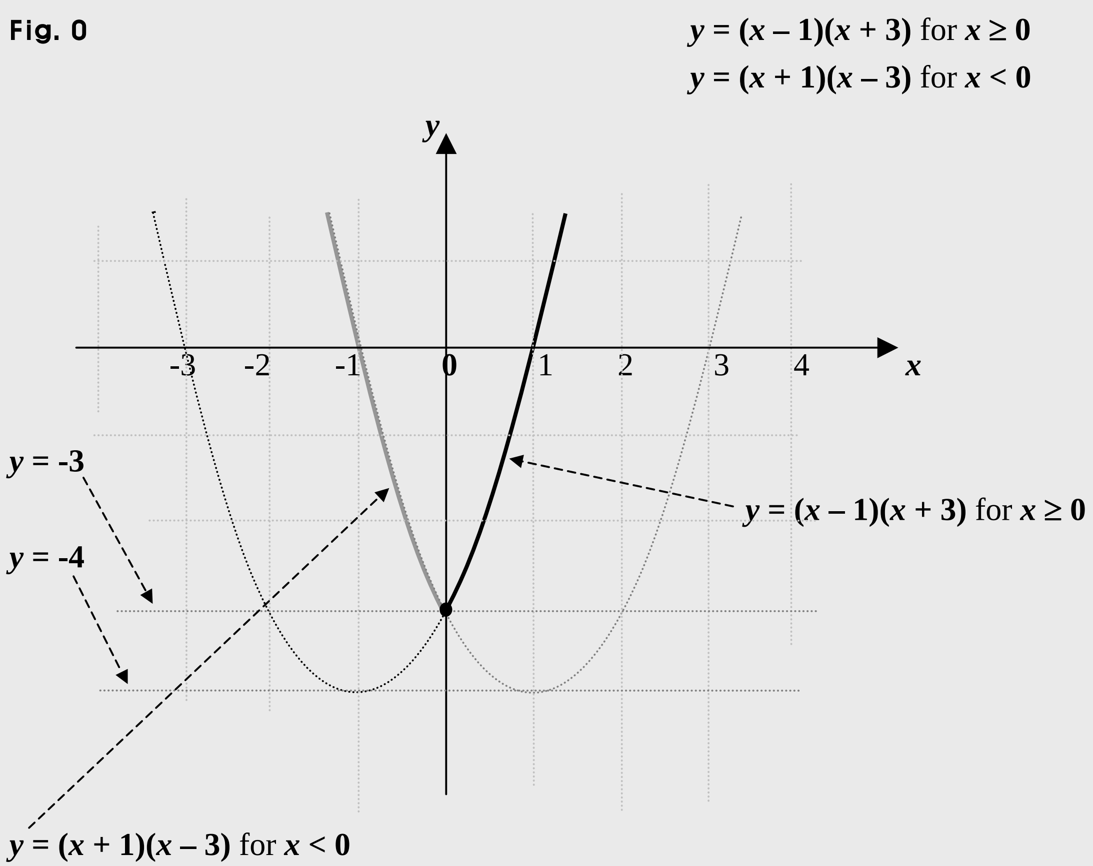
So we get $y = (x - 1)(x + 3)$ for $x \geq 0$. And it has one x-intercept, which is 1.

And next, $x < 0 \Rightarrow y = |x|^2 + 2|x| - 3 = x^2 - 2x - 3 = (x + 1)(x - 3)$ for $x < 0$.

So we get $y = (x + 1)(x - 3)$ for $x < 0$. And it one x-intercept, which is -1.

And thus, the equation $y = |x|^2 + 2|x| - 3$ has two x-intercepts, which are –1 and 1, and we can graph the equation the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 4

Graph the equation as follows: $|y| = x^2 + 2x - 3$.

First, we have: $|y| \geq 0$ by the definition of the absolute sign.
So we need to consider two cases. One is: $y \geq 0$, and the other is: $y < 0$.

If $y \geq 0$, we get: $|y| = y$, and if $y < 0$, we get: $|y| = -y$.

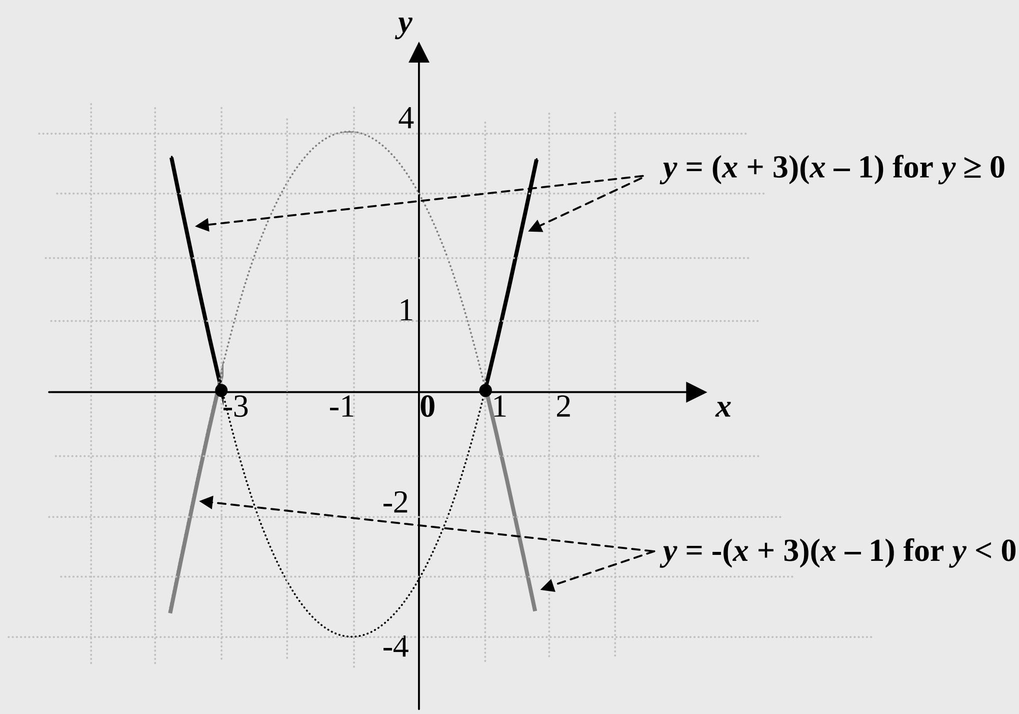
So first: $y \geq 0 \Rightarrow y = x^2 + 2x - 3$.
And we can put it this way, too: $y = (x + 3)(x - 1)$ for $y \geq 0$. And it has two x-intercepts, which are 1 and –3.

And next, $y < 0 \Rightarrow -y = x^2 + 2x - 3 \Rightarrow y = -(x^2 + 2x - 3) = -(x + 3)(x - 1)$.
So we get $y = -(x + 3)(x - 1)$ for $y < 0$. And it has no x-intercept, since y cannot be 0.

Fig. 0

$y = (x + 3)(x - 1)$ for $y \geq 0$

$y = -(x + 3)(x - 1)$ for $y < 0$



Suggestions or Solutions
To the Problem in the Example 5

Graph the equation as follows: $|y| = |x|^2 + 2|x| - 3$.

• $x \geq 0$ and $y \geq 0 \Rightarrow y = x^2 + 2x - 3 = (x - 1)(x + 3)$.

So we get $y = (x - 1)(x + 3)$ for $x \geq 0$ and $y \geq 0$.

• $x \geq 0$ and $y < 0 \Rightarrow -y = x^2 + 2x - 3 \Rightarrow y = -(x^2 + 2x - 3) = -(x - 1)(x + 3)$.

So we get $y = -(x - 1)(x + 3)$ for $x \geq 0$ and $y < 0$.

• $x < 0$ and $y \geq 0 \Rightarrow y = x^2 - 2x - 3 = (x + 1)(x - 3)$.

So we get $y = (x + 1)(x - 3)$ for $x < 0$ and $y \geq 0$.

• $x < 0$ and $y < 0 \Rightarrow -y = x^2 - 2x - 3 \Rightarrow y = -(x^2 - 2x - 3) = -(x + 1)(x - 3)$.

So we get $y = -(x + 1)(x - 3)$ for $x < 0$ and $y < 0$.

Thus, graphing the equation $|y| = |x|^2 + 2|x| - 3$, we need to put in a graph all the curves of the four equations above.

If not quite sure of the idea behind the processes above, follow the steps below.

In this example, x and y both are covered by the absolute signs.

So this time, we need to consider 4 different cases as follows.

$x \geq 0$ and $y \geq 0$, $x \geq 0$ and $y < 0$, $x < 0$ and $y \geq 0$, $x < 0$ and $y < 0$.

That is because, by definition, we have $|x| \geq 0$, and $|y| \geq 0$, and we have two cases for each of x and y, so we have four cases in total.

And in each case, we can get an equation the way as follows.

• $x \geq 0$ and $y \geq 0 \Rightarrow y = x^2 + 2x - 3 = (x - 1)(x + 3)$.
So we get $y = (x - 1)(x + 3)$ for $x \geq 0$ and $y \geq 0$.

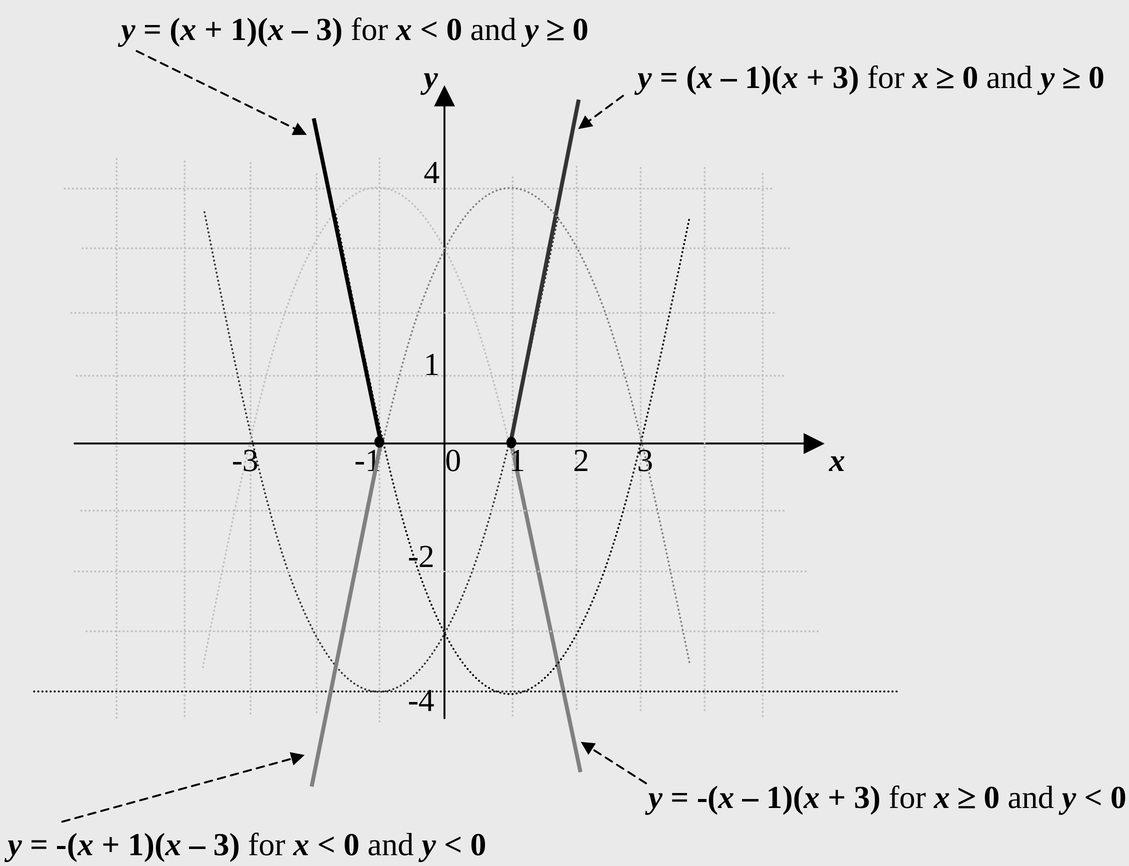
• $x \geq 0$ and $y < 0 \Rightarrow -y = x^2 + 2x - 3 \Rightarrow y = -(x^2 + 2x - 3) = -(x - 1)(x + 3)$.
So we get $y = -(x - 1)(x + 3)$ for $x \geq 0$ and $y < 0$.

• $x < 0$ and $y \geq 0 \Rightarrow y = x^2 - 2x - 3 = (x + 1)(x - 3)$.
So we get $y = (x + 1)(x - 3)$ for $x < 0$ and $y \geq 0$.

• $x < 0$ and $y < 0 \Rightarrow -y = x^2 - 2x - 3 \Rightarrow y = -(x^2 - 2x - 3) = -(x + 1)(x - 3)$.
So we get $y = -(x + 1)(x - 3)$ for $x < 0$ and $y < 0$.

And putting in one graph, the curves of the four equations above, we get the curve of the equation $|y| = |x|^2 + 2|x| - 3$, given in this problem. And we can put all the four curves in one graph the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 6

Graph the curve of the equation as follows: $|y| = x^2 + |2x - 3|$.

In the expression above, y and $(2x - 3)$ are covered by the absolute signs, so we need to consider two cases each for y and $(2x - 3)$, that is, 4 cases in total.

• $2x - 3 \geq 0$ and $y \geq 0 \Rightarrow x \geq \frac{3}{2}$ and $y \geq 0 \Rightarrow y = x^2 + 2x - 3 = (x - 1)(x + 3)$.

So we get $y = (x - 1)(x + 3)$ for $x \geq \frac{3}{2}$ and $y \geq 0$.

• $2x - 3 \geq 0$ and $y < 0 \Rightarrow x \geq \frac{3}{2}$ and $y < 0$

$\Rightarrow -y = x^2 + 2x - 3 \Rightarrow y = -(x^2 + 2x - 3) = -(x - 1)(x + 3)$.

So we get $y = -(x - 1)(x + 3)$ for $x \geq \frac{3}{2}$ and $y < 0$.

• $2x - 3 < 0$ and $y \geq 0 \Rightarrow x < \frac{3}{2}$ and $y \geq 0$

$\Rightarrow y = x^2 - (2x - 3) = x^2 - 2x + 3 = (x - 1)^2 - 1 + 3 = (x - 1)^2 + 2$.

So we get $y = (x - 1)^2 + 2$ for $x < \frac{3}{2}$ and $y \geq 0$.

• $2x - 3 < 0$ and $y < 0 \Rightarrow x < \frac{3}{2}$ and $y < 0 \Rightarrow -y = x^2 - (2x - 3) = x^2 - 2x + 3$

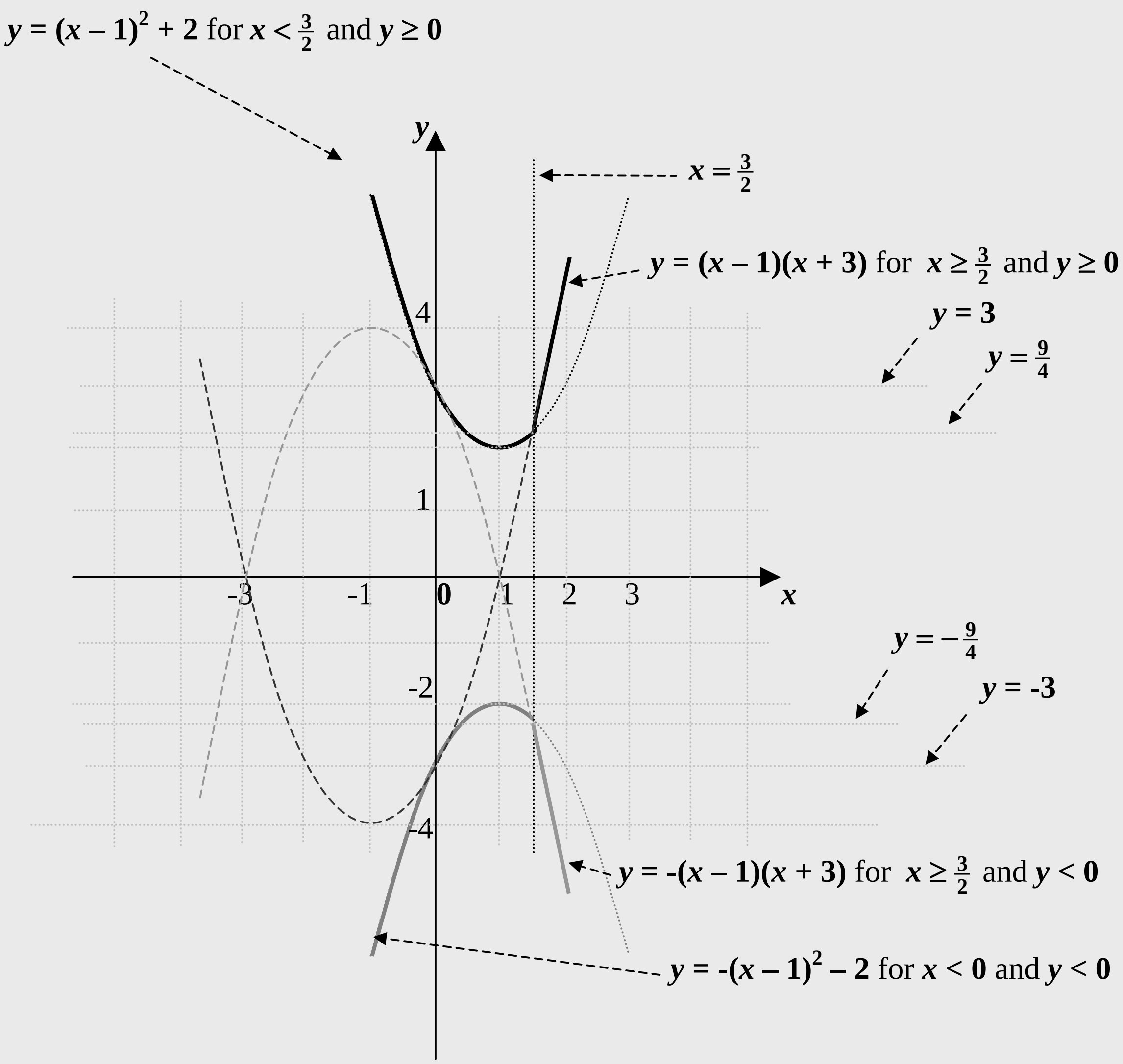
$\Rightarrow y = -(x^2 - 2x + 3) = -\{(x - 1)^2 + 2\} = -(x - 1)^2 - 2$.

So we get $y = -(x - 1)^2 - 2$ for $x < 0$ and $y < 0$.

So putting together the curves of the four equations above, we get the curve of the equation $|y| = x^2 + |2x - 3|$.

Thus, graphing the equation $|y| = x^2 + |2x - 3|$, we need to put in a graph all the curves of the four equations above, and we can put them in one graph the way below.

Fig. 0


$y = (x-1)^2 + 2$ for $x < \frac{3}{2}$ and $y \geq 0$
$x = \frac{3}{2}$
$y = (x-1)(x+3)$ for $x \geq \frac{3}{2}$ and $y \geq 0$
$y = 3$
$y = \frac{9}{4}$
$y = -\frac{9}{4}$
$y = -3$
$y = -(x-1)(x+3)$ for $x \geq \frac{3}{2}$ and $y < 0$
$y = -(x-1)^2 - 2$ for $x < 0$ and $y < 0$
y
x
4
1
-2
-4
-3
-1
0
1
2
3