Examples 5 in Parabolas

Click this to start.

Examples 5

Graph each equation below.

0. $y = \sqrt{x^2 - 2x + 1}$

1. $y = \sqrt{(x^2 - 4x + 3)^2}$

2. $y = \sqrt{(x^2 - 2x + 2)^2}$

3. $y = \sqrt{(4x - x^2 - 3)^2}$

4. $y = -\sqrt{(x^2 - 2x)^2}$

5. $x = \sqrt{(y^2 - 4y + 3)^2}$

6. $x = \sqrt{(4y - y^2 - 3)^2}$

7. $x = -\sqrt{(2y - y^2)^2}$

Suggestions or Solutions
To the Problem in the Example 0

Put in a graph the curve of the equation as follows: $y = \sqrt{x^2 - 2x + 1}$**.**

First, we get $y = \sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = |x - 1|$. So next, we get

$x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow y = x - 1$ for $x \geq 1$.

$x - 1 < 0 \Rightarrow x < 1 \Rightarrow y = -(x - 1) = -x + 1$ for $x < 1$.

So putting in one graph, the curves of the two equations above, we get the curve of equation given.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, taking a square root of a number, we get a number.
What then is the sign of the number we get? Is it positive or negative?

It is positive or 0. So a square root of a number is greater than or equal to zero.

Putting a number into x in $\sqrt{x^2 - 2x + 1}$, we get a number, which is positive or 0.
So in equation $y = \sqrt{x^2 - 2x + 1}$, the y-value is suppose to be ≥ 0.

It is quite obvious or self-evident, yet we often make errors on it.
So when removing a square root sign, we want to make sure that the result is greater than or equal to zero. How then can we make sure?

We can do so using the absolute sign, that is, applying the absolute sign to the result.

Now, getting back to the problem in this example, we have $y = \sqrt{x^2 - 2x + 1}$.

Then, first, factorizing what's inside the root, we can get $y = \sqrt{(x-1)^2}$.

Next, we know a square root of a number is positive or 0, so $\sqrt{x^2 - 2x + 1}$ is positive or 0.

In other words, we have $y \geq 0$, so we need to set $y = |x - 1|$. What then is the next?

We want to remove the absolute sign. And we know $|x - 1| \geq 0$.

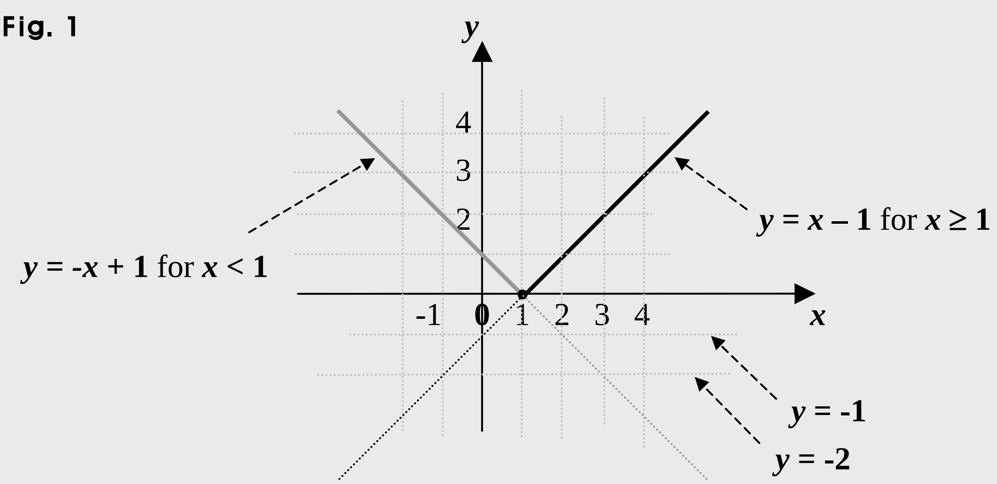
So we need to consider two cases as follows: $x - 1 \geq 0$, and $x - 1 < 0$. So we get

$x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow y = x - 1$ for $x \geq 1$.

$x - 1 < 0 \Rightarrow x < 1 \Rightarrow y = -(x - 1) = -x + 1$ for $x < 1$.

Thus, we can put the equation $y = \sqrt{x^2 - 2x + 1}$ in a graph the way as follows.

Fig. 1



Of course, the curve of the equation given is made of the two rays in gray and black.

And we can put the equation given the way below, too.

$$y = \sqrt{x^2 - 2x + 1} = \begin{cases} x - 1 \text{ for } x \geq 1 \\ \\ -x + 1 \text{ for } x < 1 \end{cases}$$

Suggestions or Solutions
To the Problem in the Example 1

Graph the equation as follows: $y = \sqrt{(x^2 - 4x + 3)^2}$**.**

First, we can get $y = \sqrt{(x^2 - 4x + 3)^2} = |x^2 - 4x + 3| = |(x - 1)(x - 3)|$. So next, we get

$(x - 1)(x - 3) \geq 0 \Rightarrow x \geq 3$ or $x \leq 1 \Rightarrow y = (x - 1)(x - 3)$ for $x \geq 3$ or $x \leq 1$.

$(x - 1)(x - 3) < 0 \Rightarrow 1 < x < 3 \Rightarrow y = -(x - 1)(x - 3)$ for $1 < x < 3$.

Therefore, the curve of $y = \sqrt{(x^2 - 4x + 3)^2}$ is composed of two curves.

One is the curve of $y = (x - 1)(x - 3)$ for $x \geq 3$ or $x \leq 1$.

And the other is the curve of $y = -(x - 1)(x - 3)$ for $1 < x < 3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, what's inside the square sign is a complete square.
So we can easily remove the root sign.
Removing the sign though, we need to note that a square root of a number is ≥ 0.

That is to say that we have $\sqrt{(x^2 - 4x + 3)^2} \geq 0$, and thus, we have $y \geq 0$.
So we want to make sure that $y \geq 0$ for all values of x.

How then can we make sure that?

We can use an absolute sign. So using it, we can set, for now

$y = \sqrt{(x^2 - 4x + 3)^2} = |x^2 - 4x + 3|$. So we get $y = |x^2 - 4x + 3|$.

And next, if putting its curve in a graph, we want to remove the absolute sign.
We don't just remove such a sign, of course.
Removing it, we need to get the expression equivalent, without the sign, of course.

How then do we remove the sign?

It may **not** be the case where $x^2 - 4x + 3 \geq 0$ for some values of x.
In other words, it can be the case we get $x^2 - 4x + 3 < 0$ for some values of x.

So we cannot just set $y = x^2 - 4x + 3$, because we have to ge: $y \geq 0$ for all values of x.

How then do we know if $(x^2 - 4x + 3)$ can be negative?

We know that the curve of $y = x^2 - 4x + 3$ is a parabola.
So we want to check to see if the parabola can go below the x-axis.
If it can, $(x^2 - 4x + 3)$ is negative for some values of x.

How then can we check to see if it can go below the x-axis?

We can do so taking the discriminant of $(x^2 - 4x + 3)$.

If discriminant is positive, a part of the parabola is below the x-axis, since the coefficient of x^2 is positive, so $(x^2 - 4x + 3)$ has two roots, and thus, is negative for some values of x. Then, we need to take care of the case.

So taking the discriminant of $(x^2 - 4x + 3)$ now, we get $(-4)^2 - 4 \cdot 1 \cdot 3 = 16 - 12 = 4$, which is positive, of course, so the parabola can go below the x-axis.
What then can we do?

We want to see for what values of x, the parabola is below the x-axis. That is, we want to find the extend of x for which $(x^2 - 4x + 3)$ is negative.   How then can we find it?

We want to get the roots of it first, so we want to solve $x^2 - 4x + 3 = 0$.

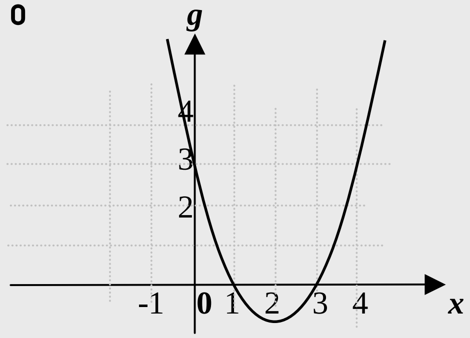
So solving it, we get $x^2 - 4x + 3 = (x - 1)(x - 3) = 0 \Rightarrow x = 1$ or 3.

Let's this time, put the parabola in a graph, and see how it behaves as x-value changes.

So suppose now, that $g(x) = x^2 - 4x + 3$.

Then, since $x^2 - 4x + 3 = (x - 1)(x - 3)$, we can put the curve of g the way as follows.

Fig. 0



Then, we can see that when $1 < x < 3$, we get $g(x) < 0$.

Thus, it is not true that $g(x) \geq 0$ for all x, and thus, we cannot just set $y = x^2 - 4x + 3$.

So we want to set $y = \sqrt{(x^2 - 4x + 3)^2} = |x^2 - 4x + 3| = |(x - 1)(x - 3)|$.

Then, removing the absolute sign, we need to consider two cases.

One is $(x - 1)(x - 3) \geq 0$, and the other is $(x - 1)(x - 3) < 0$.

Then, using the graph above, we can take care of each of the two cases above.

In one case, $(x - 1)(x - 3) \geq 0 \Rightarrow x \geq 3$ or $x \leq 1 \Rightarrow y = (x - 1)(x - 3)$ for $x \geq 3$ or $x \leq 1$.

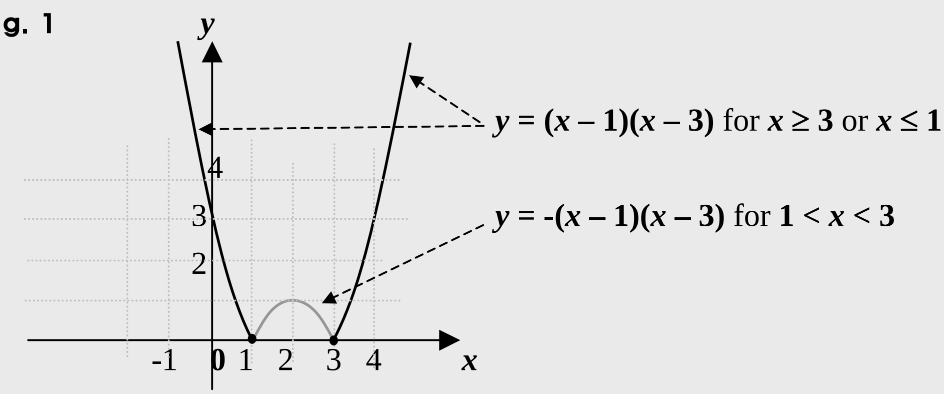
And in the other case, $(x - 1)(x - 3) < 0 \Rightarrow 1 < x < 3 \Rightarrow y = -(x - 1)(x - 3)$ for $1 < x < 3$.

Therefore, the curve of $y = \sqrt{(x^2 - 4x + 3)^2}$ is composed of two curves.

One is the curve of $(x - 1)(x - 3)$ for $x \geq 3$ or $x \leq 1$, which is made of two parabolic rays.

And the other is the curve of $-(x - 1)(x - 3)$ for $1 < x < 3$, which is a parabolic segment as shown in the graph below.

Fig. 1



Suggestions or Solutions
To the Problem in the Example 2

Graph the equation as follows: $y = \sqrt{(x^2 - 2x + 2)^2}$.

First, we can set $y = \sqrt{(x^2 - 2x + 2)^2} = |x^2 - 2x + 2|$.

Next, we can get $x^2 - 2x + 3 = (x - 1)^2 + 1 \geq 1$.

So the curve of $y = \sqrt{(x^2 - 2x + 2)^2}$ is the same as the curve of $y = x^2 - 2x + 3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, what's inside the square root sign is a complete square.
So we can easily remove the square root sign. Keep in mind though, the fact below.

A square root of a number is ≥ 0.

So we need to make sure that $y \geq 0$.
So we want to apply the absolute sign to what is squared inside the root sign.
That is, we need to set $y = |x^2 - 2x + 2|$. What then is the next?

We need to remove the absolute sign.
Removing the sign, we need to consider two cases.

One is $x^2 - 2x + 2 \geq 0$, and the other is $x^2 - 2x + 2 < 0$. What then is the next?

We want to find the extent of x, in which, we get $x^2 - 2x + 2 \geq 0$, and another extent of x, in which, we get $x^2 - 2x + 2 < 0$. How then can we get those extents?

We can get the extents solving this equation: $x^2 - 2x + 2 = 0$.
And factorizing it, we can get the roots, and in turn, we can get the extents.
It doesn't seem though, we can readily factorize it. What then?

We can use the quadratic formula to get the solution, which doesn't seem to be however, the best idea. We can quickly put $\boldsymbol{x^2 - 2x + 2}$ in the vertex form. Then, we can see if the parabola can go below the $\boldsymbol{x}$-axis, that is, we can see if $\boldsymbol{x^2 - 2x + 2}$ can be negative.

So putting it in the vertex form, we get $\boldsymbol{x^2 - 2x + 3 = (x - 1)^2 + 1 \geq 1}$.

So we can see that it cannot be negative, that is, what's inside the absolute sign cannot be negative, and thus, we don't need to get the extents of $\boldsymbol{x}$.

So in this particular case, we can just set $\boldsymbol{y = x^2 - 2x + 2}$.

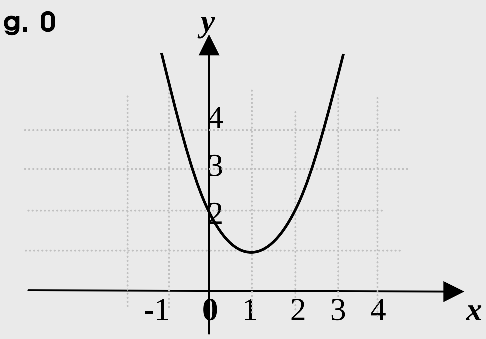
Also, we can do the same, too, checking to see if the discriminant of $\boldsymbol{x^2 - 2x + 2}$ is negative.

If it is negative, the entire curve of $\boldsymbol{y = x^2 - 2x + 2}$, that is, the whole parabola is above the $\boldsymbol{x}$-axis. That's because the coefficient of $\boldsymbol{x^2}$ is positive.

The discriminant is $\boldsymbol{(-2) - 4 \cdot 1 \cdot 2 = -4}$. So there is no $\boldsymbol{x}$-value for which $\boldsymbol{y < 0}$.

Thus, the curve of the given equation is the same as the curve of $\boldsymbol{y = x^2 - 2x + 2}$, which is the parabola below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 3

Graph the equation as follows: $y = \sqrt{(4x - x^2 - 3)^2}$.

First, we can set $y = \sqrt{(4x - x^2 - 3)^2} = |4x - x^2 - 3| = |(1 - x)(x - 3)|$. So next, we get

$(1 - x)(x - 3) \geq 0 \Rightarrow (x - 1)(x - 3) \leq 0 \Rightarrow 1 \leq x \leq 3 \Rightarrow y = (1 - x)(x - 3)$ for $1 \leq x \leq 3$.

$(1 - x)(x - 3) < 0 \Rightarrow (x - 1)(x - 3) > 0 \Rightarrow x < 1$ or $x > 3$
$\Rightarrow y = -(1 - x)(x - 3) = (x - 1)(x - 3)$ for $x < 1$ or $x > 3$.

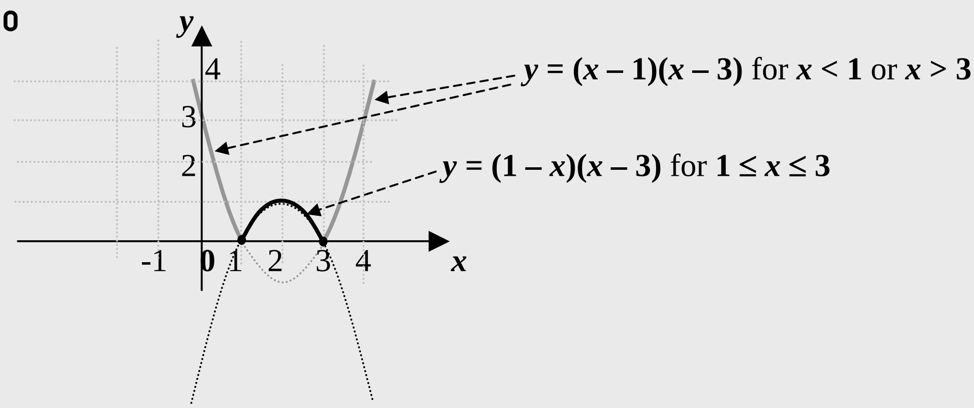
Therefore, the curve of $y = \sqrt{(x^2 - 4x + 3)^2}$ is composed of two curves.

One is the curve of $y = (1 - x)(x - 3)$ for $1 \leq x \leq 3$.

And the other is the curve of $y = (x - 1)(x - 3)$ for $x < 1$ or $x > 3$.

So we can graph the equation given the way below.

Fig. 0



We can notice that the graph above is the same as the one in the example 1.
Structureally though, it's a bit different.

The two points at **(1, 0)** and **(3, 0)** belong to the curve of $y = (1 - x)(x - 3)$ in the graph above. However, they belong to the curve of $y = (x - 1)(x - 3)$ in the example 1.

Suggestions or Solutions
To the Problem in the Example 4

Graph the equation as follows: $y = -\sqrt{(x^2 - 2x)^2}$.

First, we can set $y = -\sqrt{(x^2 - 2x)^2}$ = **-|x^2 – 2*x*| = -|*x*(*x* – 2)|**. So next, we get

***x*(*x* – 2) ≥ 0 ⇒ *x* ≥ 2** or ***x* ≤ 0 ⇒ *y* = -*x*(*x* – 2)** for ***x* ≥ 2** or ***x* ≤ 0**.

***x*(*x* – 2) < 0 ⇒ 0 < *x* < 2 ⇒ *y* = -{-*x*(*x* – 2)} = *x*(*x* – 2)** for **0 < *x* < 2**.

Therefore, the curve of $y = -\sqrt{(x^2 - 2x)^2}$ is composed of two curves.

One is the curve of ***y* = -*x*(*x* – 2)** for ***x* ≥ 2** or ***x* ≤ 0**.

And the other is the curve of ***y* = *x*(*x* – 2)** for **0 < *x* < 2**.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we need to remove the square root sign.
A square root of a number is ≥ **0**, yet in this case, ***y*** is not ≥ **0** but ≤ **0**, because a negative sign is in front of the root sign.

Thus, we want to first, set ***y* = -|x^2 – 2*x*| = -|*x*(*x* – 2)|**.
How then can we get rid of the absolute sign?

The expression **|*x*(*x* – 2)|** itself has to be still positive or 0.
So removing the absolute sign, we need to consider two cases.

One is ***x*(*x* – 2) ≥ 0**, and the other is ***x*(*x* – 2) < 0**. So we get

***x*(*x* – 2) ≥ 0 ⇒ *x* ≥ 2** or ***x* ≤ 0 ⇒ *y* = -*x*(*x* – 2)** for ***x* ≥ 2** or ***x* ≤ 0**.

***x*(*x* – 2) < 0 ⇒ 0 < *x* < 2 ⇒ *y* = -{-*x*(*x* – 2)} = *x*(*x* – 2)** for **0 < *x* < 2**.

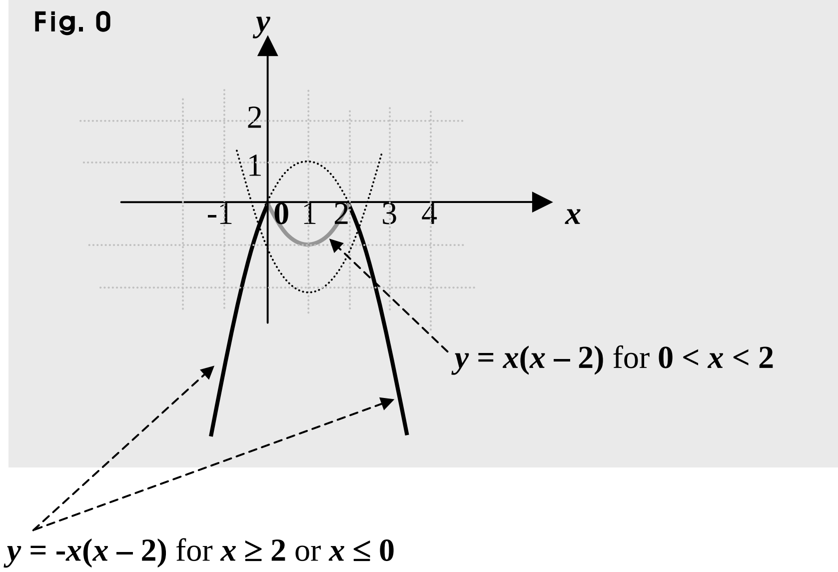
Therefore, the curve of $\boldsymbol{y = -\sqrt{(x^2 - 2x)^2}}$ is composed of two curves.

One is the curve of $\boldsymbol{y = -x(x - 2)}$ for $\boldsymbol{x \geq 2}$ or $\boldsymbol{x \leq 0}$.

And the other is the curve of $\boldsymbol{y = x(x - 2)}$ for $\boldsymbol{0 < x < 2}$.

So we can graph the equation given the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 5

Graph the equation as follows: $x = \sqrt{(y^2 - 4y + 3)^2}$ **.**

First, we can set $\boldsymbol{x} = \sqrt{(y^2 - 4y + 3)^2} = |y^2 - 4y + 3| = |(y - 1)(y - 3)|$. So next, we get

$(y - 1)(y - 3) \geq 0 \Rightarrow y \geq 3$ or $y \leq 1 \Rightarrow x = (y - 1)(y - 3)$ for $y \geq 3$ or $y \leq 1$.

$(y - 1)(y - 3) < 0 \Rightarrow 1 < y < 3 \Rightarrow x = -(y - 1)(y - 3)$ for $1 < y < 3$.

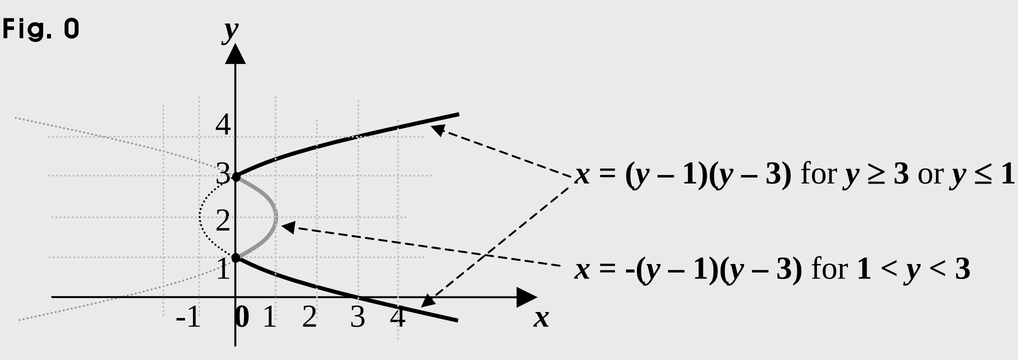
Therefore, the curve of $x = \sqrt{(y^2 - 4y + 3)^2}$ is composed of two curves.

One is the curve of $x = (y - 1)(y - 3)$ for $y \geq 3$ or $y \leq 1$.

And the other is the curve of $x = -(y - 1)(y - 3)$ for $1 < y < 3$.

So we can graph the equation given the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 6

Graph the equation as follows: $x = \sqrt{(4y - y^2 - 3)^2}$.

First, we can set $x = \sqrt{(4y - y^2 - 3)^2} = |4y - y^2 - 3| = |(1 - y)(y - 3)|$. So next, we get

$(1 - y)(y - 3) \geq 0 \Rightarrow (y - 1)(y - 3) \leq 0 \Rightarrow 1 \leq y \leq 3 \Rightarrow x = (1 - y)(y - 3)$ for $1 \leq y \leq 3$.

$(1 - y)(y - 3) < 0 \Rightarrow (y - 1)(y - 3) > 0 \Rightarrow y < 1$ or $y > 3$

$\Rightarrow x = -(1 - y)(y - 3) = (y - 1)(y - 3)$ for $y < 1$ or $y > 3$.

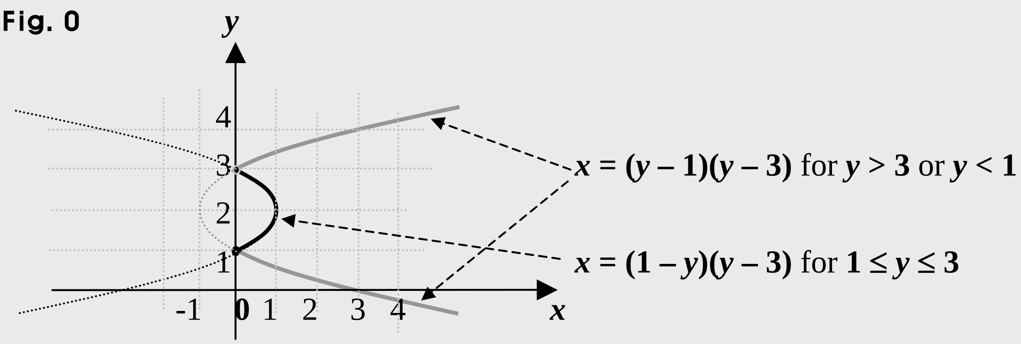
Therefore, the curve of $x = \sqrt{(y^2 - 4y + 3)^2}$ is composed of two curves.

One is the curve of $x = (1 - y)(y - 3)$ for $1 \leq y \leq 3$..

And the other is the curve of $x = (y - 1)(y - 3)$ for $y < 1$ or $y > 3$.

So we can graph the equation given the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 7

Graph the curve of the equation as follows: $x = -\sqrt{(2y - y^2)^2}$ **.**

First, we can set $\boldsymbol{x} = -\sqrt{(2y - y^2)^2} = -|2y - y^2| = -|y(2 - y)|$. So next, we get

$\boldsymbol{y(2 - y) \geq 0 \Rightarrow y(y - 2) \leq 0 \Rightarrow 0 \leq y \leq 2 \Rightarrow x = -y(2 - y) = y(y - 2)}$ for $\boldsymbol{0 \leq y \leq 2}$.

$\boldsymbol{y(2 - y) < 0 \Rightarrow y(y - 2) > 0 \Rightarrow y < 0}$ or $\boldsymbol{y > 2}$
$\boldsymbol{\Rightarrow x = -\{-y(2 - y)\} = y(2 - y)}$ for $\boldsymbol{y < 0}$ or $\boldsymbol{y > 2}$.

Therefore, the curve of $\boldsymbol{y} = -\sqrt{(x^2 - 2x)^2}$ is composed of two curves.

One is the curve of $\boldsymbol{x = y(y - 2)}$ for $\boldsymbol{0 \leq y \leq 2}$..

And the other is the curve of $\boldsymbol{x = y(2 - y)}$ for $\boldsymbol{y < 0}$ or $\boldsymbol{y > 2}$.

So we can graph the equation given the way below.

Fig. 0