Examples 6 in Parabolas

Click this to start.

Examples 6

Construct the graphs of the equations as follows.

0. $y=\sqrt{x}$, $y=-\sqrt{x}$, $y=\sqrt{-x}$, and $y=-\sqrt{-x}$

1. $y=\sqrt{2x}$, $y=\sqrt{2x-1}$, $y=\sqrt{\frac{x}{2}}$, and $y=\sqrt{\frac{x}{2}-1}$

2. $2y=\sqrt{x}$, $2y-1=\sqrt{x}$, $\frac{y}{2}=\sqrt{x}$, and $\frac{y}{2}-1=\sqrt{x}$

3. $2y-1=\sqrt{2x-1}$, and $\frac{y}{2}-1=-\sqrt{\frac{x}{2}-1}$

4. $y=\sqrt{|x|}$, and $y=\sqrt{|x-1|}$

5. $x=-\sqrt{|y|}$, and $x=\sqrt{|y-1|}$

Suggestions or Solutions
To the Problem in the Example 0

Graph the equations as follows. $y=\sqrt{x}$, $y=-\sqrt{x}$, $y=\sqrt{-x}$, and $y=-\sqrt{-x}$.

Suppose first, A is $y=\sqrt{x}$, B is $y=-\sqrt{x}$, C is $y=\sqrt{-x}$, and D is $y=-\sqrt{-x}$.

Then, since what's inside the square root sign is $\geq \mathbf{0}$, we can say that

in the equations A and B, we get $x \geq 0$.
In the equations C and D, we get $-x \geq 0 \Rightarrow x \leq 0$.

Next, a square root of a number is $\geq \mathbf{0}$, and thus, we get

A: $y \geq 0$, B: $y \leq 0$, C: $y \geq 0$, and D: $y \leq 0$.

Next, squaring both sides of each equation, we can remove the square root sign. Then,

A: $y=\sqrt{x} \Rightarrow x=y^2$.
B: $y=-\sqrt{x} \Rightarrow x=y^2$.
C: $y=\sqrt{-x} \Rightarrow y^2=-x \Rightarrow x=-y^2$.
D: $y=-\sqrt{-x} \Rightarrow y^2=-x \Rightarrow x=-y^2$.

Thus, we get

A: $x = y^2$ for $x \geq 0$ and $y \geq 0$. B: $x = y^2$ for $x \geq 0$ and $y \leq 0$.
C: $x = -y^2$ for $x \leq 0$ and $y \geq 0$. D: $x = -y^2$ for $x \leq 0$ and $y \leq 0$.

Each of the equations above is a half of a parabola, where the axis of symmetry is perpendicular to the y-axis, and the vertex is at the origin.

Each of A and B is a half of a parabola concave-right.
Each of C and D is a half of a parabola concave-left.

Each of A and C is an upper half of a parabola.
Each of B and D is a lower half of a parabola.

Fig. 0

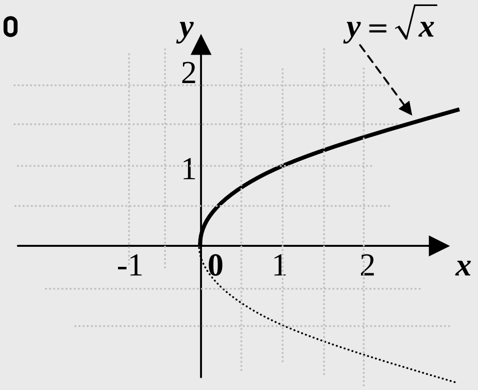


Fig. 1

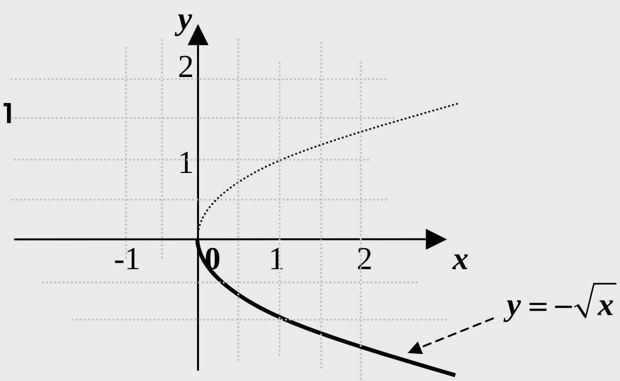


Fig. 2

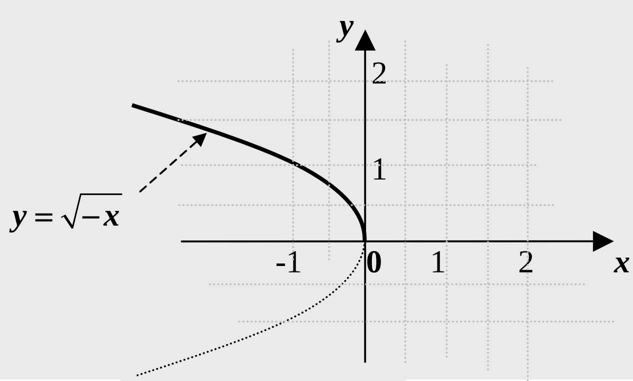
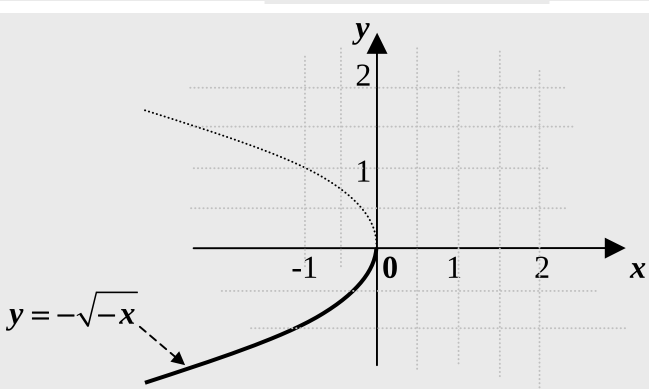


Fig. 3



Suggestions or Solutions
To the Problem in the Example 1

Graph the equations as follows. $\boldsymbol{y=\sqrt{2x}}$, $\boldsymbol{y=\sqrt{2x-1}}$, $\boldsymbol{y=\sqrt{\frac{x}{2}}}$, and $\boldsymbol{y=\sqrt{\frac{x}{2}-1}}$.

Suppose first, $\boldsymbol{A}$ is $\boldsymbol{y=\sqrt{2x}}$, $\boldsymbol{B}$ is $\boldsymbol{y=\sqrt{2x-1}}$, $\boldsymbol{C}$ is $\boldsymbol{y=\sqrt{\frac{x}{2}}}$, and $\boldsymbol{D}$ is $\boldsymbol{y=\sqrt{\frac{x}{2}-1}}$.

Then, what's inside the square root sign is $\geq \mathbf{0}$, so we get

$\boldsymbol{A}$: $\boldsymbol{2x\geq 0 \Rightarrow x\geq 0}$, $\boldsymbol{B}$: $\boldsymbol{2x-1\geq 0 \Rightarrow x\geq \frac{1}{2}}$.

$\boldsymbol{C}$: $\boldsymbol{\frac{x}{2}\geq 0 \Rightarrow x\geq 0}$, $\boldsymbol{D}$: $\boldsymbol{\frac{x}{2}-1\geq 0 \Rightarrow x\geq 2}$.

Next, squaring both sides of each equation, we can remove the square root sign. Then,

$\boldsymbol{A}$: $\boldsymbol{y=\sqrt{2x} \Rightarrow y^2=2x \Rightarrow x=\frac{1}{2}y^2}$.

$\boldsymbol{B}$: $\boldsymbol{y=\sqrt{2x-1} \Rightarrow y^2=2x-1 \Rightarrow 2x=y^2+1 \Rightarrow x=\frac{1}{2}y^2+\frac{1}{2}}$.

$\boldsymbol{C}$: $\boldsymbol{y=\sqrt{\frac{x}{2}} \Rightarrow y^2=\frac{x}{2} \Rightarrow x=2y^2}$.

$\boldsymbol{D}$: $\boldsymbol{y=\sqrt{\frac{x}{2}-1} \Rightarrow y^2=\frac{x}{2}-1 \Rightarrow \frac{x}{2}=y^2+1 \Rightarrow x=2y^2+2}$.

And also, a square root of a number is $\geq \mathbf{0}$. So we get

$\boldsymbol{A}$: $\boldsymbol{x=\frac{1}{2}y^2}$ for $\boldsymbol{x\geq 0}$ and $\boldsymbol{y\geq 0}$.

$\boldsymbol{B}$: $\boldsymbol{x=\frac{1}{2}y^2+\frac{1}{2}}$ for $\boldsymbol{x\geq \frac{1}{2}}$ and $\boldsymbol{y\geq 0}$.

$\boldsymbol{C}$: $\boldsymbol{x=2y^2}$ for $\boldsymbol{x\geq 0}$ and $\boldsymbol{y\geq 0}$.

$\boldsymbol{D}$: $\boldsymbol{x=2y^2+2}$ for $\boldsymbol{x\geq 2}$ and $\boldsymbol{y\geq 0}$.

Each of the equations above is a half of a parabola that has an axis of symmetry perpendicular to the $\boldsymbol{y}$-axis, and is concave-right.

Each of $\boldsymbol{A}$ and $\boldsymbol{C}$ is a half of a parabola where the vertex is at the origin.

$\boldsymbol{B}$ is a half of a parabola where the vertex is at $\boldsymbol{(\frac{1}{2}, 0)}$.

$\boldsymbol{D}$ is a half of a parabola where the vertex is at $\boldsymbol{(2, 0)}$.

Fig. 0

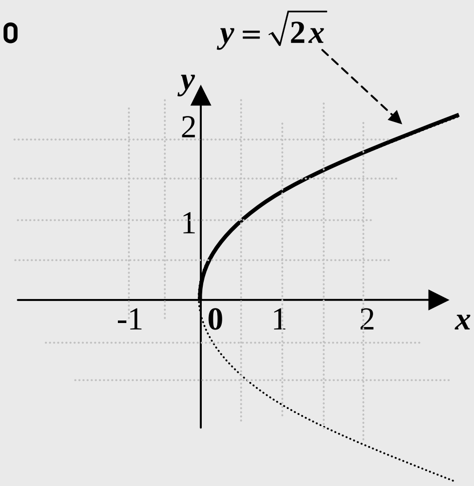


Fig. 1

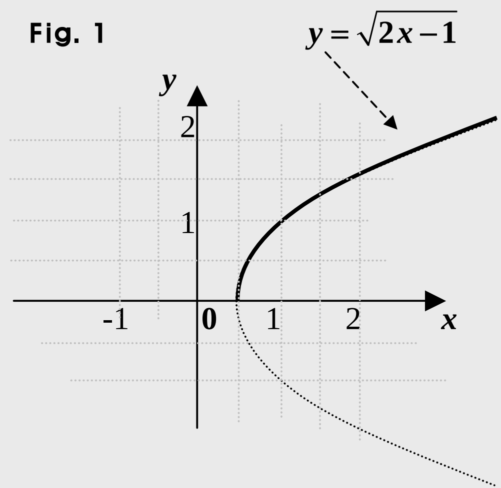


Fig. 2

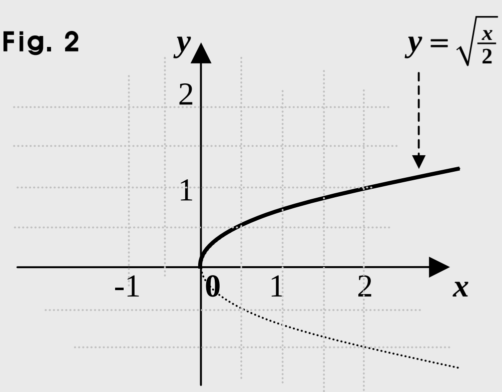
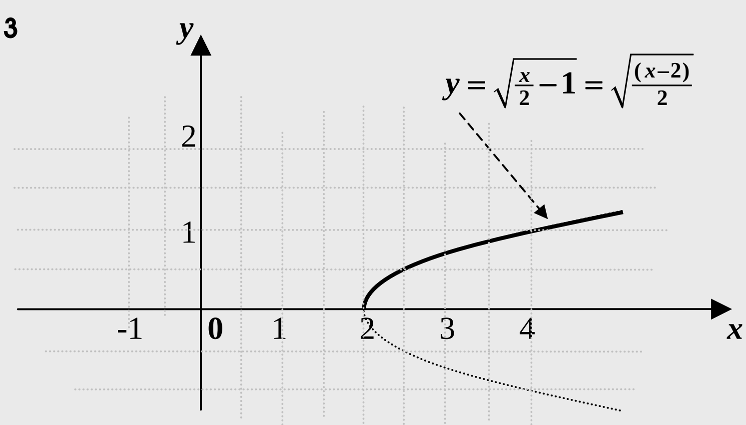


Fig. 3



Suggestions or Solutions
To the Problem in the Example 2

Graph the equations as follows. $2y = \sqrt{x}$, $2y - 1 = \sqrt{x}$, $\frac{y}{2} = \sqrt{x}$, and $\frac{y}{2} - 1 = \sqrt{x}$.

Suppose first, A is $2y = \sqrt{x}$, B is $2y - 1 = \sqrt{x}$, C is $\frac{y}{2} = \sqrt{x}$, and D is $\frac{y}{2} - 1 = \sqrt{x}$.

Then, what's inside the square root sign is ≥ 0, so we get $x \geq 0$.

Besides, a square root of a number is ≥ 0, so we get

A: $2y \geq 0 \Rightarrow y \geq 0$.　　B: $2y - 1 \geq 0 \Rightarrow y \geq \frac{1}{2}$.

C: $\frac{y}{2} \geq 0 \Rightarrow y \geq 0$.　　D: $\frac{y}{2} - 1 \geq 0 \Rightarrow y \geq 2$.

Next, squaring both sides of each equation, we can remove the square root sign.

A: $2y = \sqrt{x} \Rightarrow 4y^2 = x$.

B: $2y - 1 = \sqrt{x} \Rightarrow (2y - 1)^2 = x$.

C: $\frac{y}{2} = \sqrt{x} \Rightarrow \frac{y^2}{4} = x$.

D: $\frac{y}{2} - 1 = \sqrt{x} \Rightarrow (\frac{y}{2} - 1)^2 = x$.

Thus, we get

A: $x = 4y^2$ for $x \geq 0$ and $y \geq 0$.

B: $x = (2y - 1)^2 = 4(y - \frac{1}{2})^2$ for $x \geq 0$ and $y \geq \frac{1}{2}$.

C: $x = \frac{y^2}{4}$ for $x \geq 0$ and $y \geq 0$.

D: $x = (\frac{y}{2} - 1)^2 = \frac{1}{4}(y - 2)^2$ for $x \geq 0$ and $y \geq 2$.

Each of the above is a half of a parabola that has an axis of symmetry perpendicular to the y-axis, and is concave-right.

Each of A and C is a half of a parabola where the vertex is at the origin.

B is a half of a parabola where the vertex is at $(0, \frac{1}{2})$.

D is a half of a parabola where the vertex is at $(0, 2)$.

Fig. 0

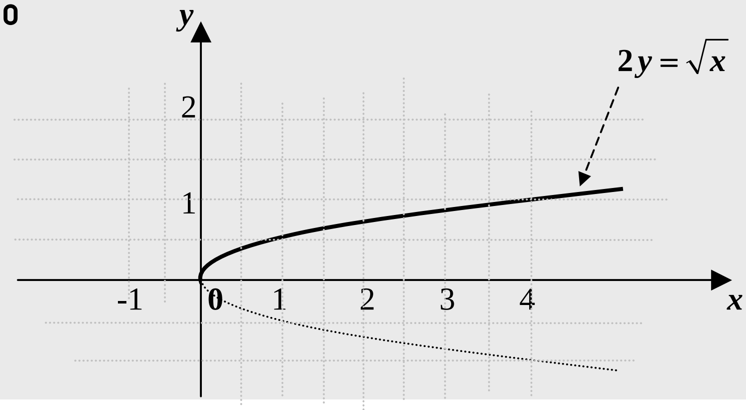


Fig. 1

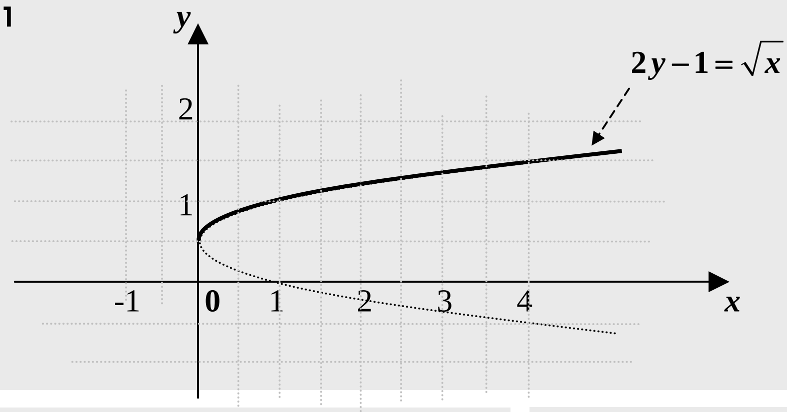


Fig. 2

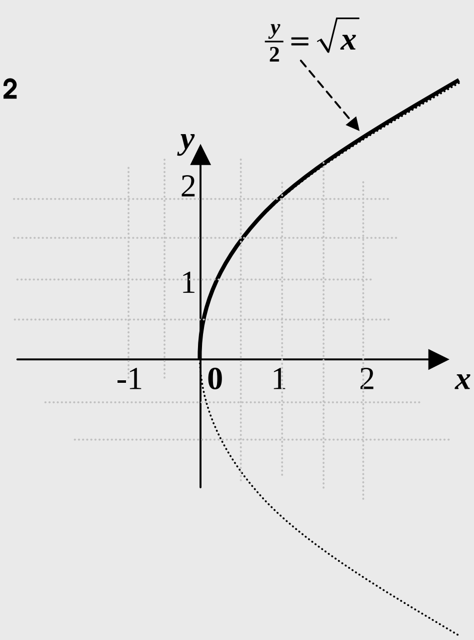
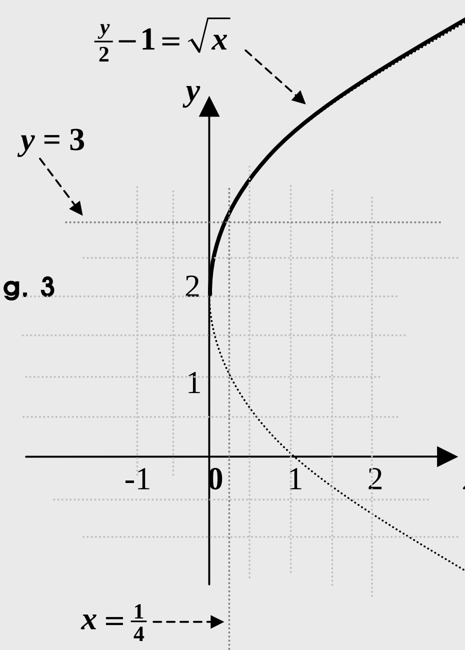


Fig. 3



Suggestions or Solutions
To the Problem in the Example 3

Graph the equations as follows. $2y-1=\sqrt{2x-1}$, and $\frac{y}{2}-1=-\sqrt{\frac{x}{2}-1}$.

To begin with, suppose A is $2y-1=\sqrt{2x-1}$, and B is $\frac{y}{2}-1=-\sqrt{\frac{x}{2}-1}$.

Then, what's inside the square root sign is ≥ 0, and thus, we get

A: $2x-1\geq 0 \Rightarrow x\geq \frac{1}{2}$, and B: $\frac{x}{2}-1\geq 0 \Rightarrow x\geq 2$.

Besides, a square root of a number is ≥ 0, so we get:

A: $2y-1\geq 0 \Rightarrow y\geq \frac{1}{2}$.

B: $\frac{y}{2}-1\leq 0 \Rightarrow y\leq 2$ due to the minus sign in front of the root sign.

Next, squaring both sides of each equation, we can remove the square root sign.

A: $2y-1=\sqrt{2x-1} \Rightarrow (2y-1)^2=2x-1 \Rightarrow x=\frac{1}{2}(2y-1)^2+\frac{1}{2}=2(y-\frac{1}{2})^2+\frac{1}{2}$
$\Rightarrow x=2(y-\frac{1}{2})^2+\frac{1}{2}$.

B: $\frac{y}{2}-1=-\sqrt{\frac{x}{2}-1} \Rightarrow (\frac{y}{2}-1)^2=\frac{x}{2}-1 \Rightarrow x=2(\frac{y}{2}-1)^2+2=\frac{1}{2}(y-2)^2+2$
$\Rightarrow x=\frac{1}{2}(y-2)^2+2$.

Thus, we have

A: $x=2(y-\frac{1}{2})^2+\frac{1}{2}$ for $x\geq \frac{1}{2}$ and $y\geq \frac{1}{2}$.

B: $x=\frac{1}{2}(y-2)^2+2$ for $x\geq 2$ and $y\leq 2$.

Each of the above is a half of a parabola that has an axis of symmetry perpendicular to the y-axis, and is concave-right.

A is an upper half of a parabola where the vertex is at $(\frac{1}{2}, \frac{1}{2})$.

B is a lower half of a parabola where the vertex is at $(2, 2)$.

Fig. 0

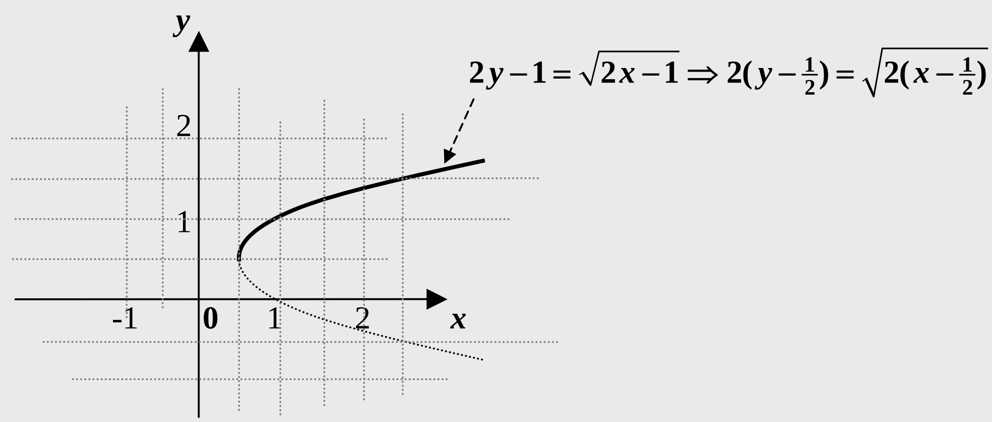
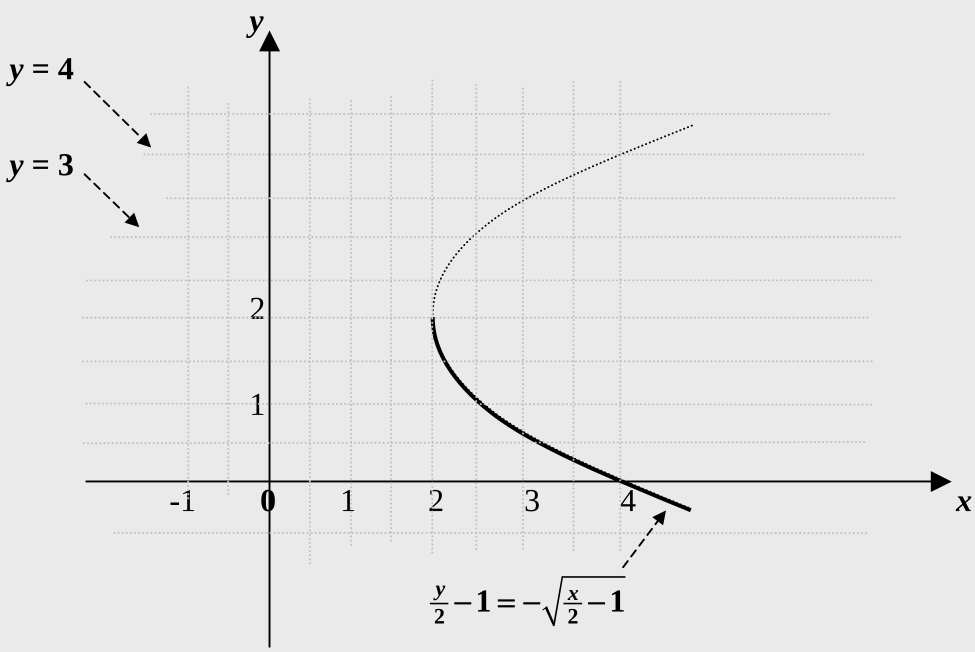

y
$2y-1=\sqrt{2x-1} \Rightarrow 2(y-\frac{1}{2})=\sqrt{2(x-\frac{1}{2})}$
2
1
-1
0
1
2
x

Fig. 1


y
y = 4
y = 3
2
1
-1
0
1
2
3
4
x
$\frac{y}{2}-1=-\sqrt{\frac{x}{2}-1}$

Suggestions or Solutions
To the Problem in the Example 4

Graph the equations as follows. $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}\,|}$, and $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}-\mathbf{1}\,|}$.

To begin with, what's inside the square root sign is an absolute value, and thus, is $\geq \mathbf{0}$.
Thus, $\boldsymbol{x}$ can be any real number in both $\boldsymbol{A}$ and $\boldsymbol{B}$.

Next, suppose $\boldsymbol{A}$ is $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}\,|}$, and $\boldsymbol{B}$ is $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}-\mathbf{1}\,|}$.

Then, since a square root of a number is $\geq \mathbf{0}$, we get $\boldsymbol{y} \geq \mathbf{0}$ in both $\boldsymbol{A}$ and $\boldsymbol{B}$.

Next, squaring both sides of each equation, we can remove the square root sign.

$\boldsymbol{A}$: $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}\,|} \Rightarrow \boldsymbol{y}^2 = |\boldsymbol{x}|$.

$\boldsymbol{B}$: $\boldsymbol{y} = \sqrt{|\,\boldsymbol{x}-\mathbf{1}\,|} \Rightarrow \boldsymbol{y}^2 = |\boldsymbol{x} - \mathbf{1}|$.

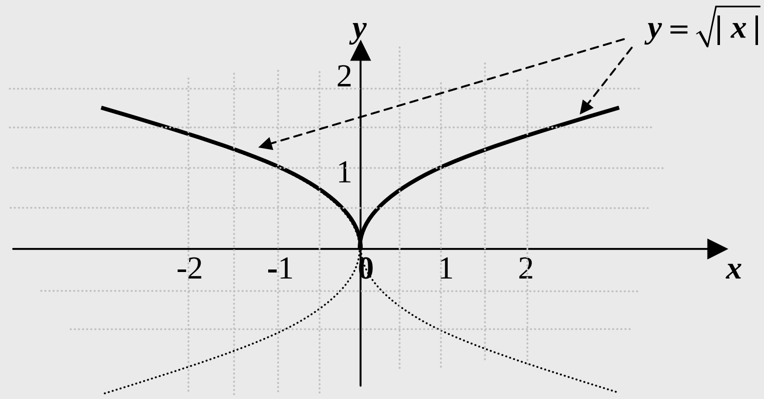
Let's now, begin with $\boldsymbol{A}$. We know $\boldsymbol{x}$ can be any real number, but $\boldsymbol{y} \geq \mathbf{0}$ only. So we get

$\boldsymbol{x} = \boldsymbol{y}^2$ for $\boldsymbol{x} \geq \mathbf{0}$ and $\boldsymbol{y} \geq \mathbf{0}$.

$\boldsymbol{x} = -\boldsymbol{y}^2$ for $\boldsymbol{x} < \mathbf{0}$ and $\boldsymbol{y} \geq \mathbf{0}$.

Each of the above is an upper half of a parabola where the vertex is at the origin.
The first of the above is concave-right, but the second is concave-left.

Fig. 0



Next, let's move on to ***B***.

We have $\boldsymbol{y = \sqrt{|x-1|} \Rightarrow y^2 = |x-1|}$.

We know that $\boldsymbol{x}$ can be any real number, but $\boldsymbol{y \geq 0}$ only.

Besides, removing the absolute sign, we wan to consider two cases as follows.

$\boldsymbol{x - 1 \geq 0}$, and $\boldsymbol{x - 1 < 0}$.

So we get

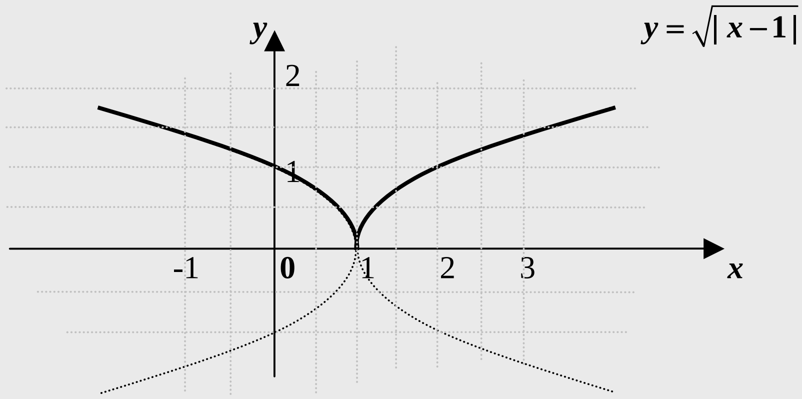
$\boldsymbol{x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow x - 1 = y^2}$ for $\boldsymbol{x \geq 1}$ and $\boldsymbol{y \geq 0}$.

$\boldsymbol{x - 1 < 0 \Rightarrow x < 1 \Rightarrow -(x - 1) = y^2 \Rightarrow x - 1 = -y^2}$ for $\boldsymbol{x < 1}$ and $\boldsymbol{y \geq 0}$.

Each of the above is an upper half of a parabola where the vertex is at **(1, 0)**.

The first of the above is concave-right, but the second is concave-left.

Fig. 1



Suggestions or Solutions
To the Problem in the Example 5

Graph the equations as follows. $x = -\sqrt{|y|}$, **and** $x = \sqrt{|y-1|}$.

To begin with, what's inside the square root sign is an absolute value, and thus, is $\geq \mathbf{0}$. Thus, y can be any real number in both A and B.

Next, suppose A is $x = -\sqrt{|y|}$, and B is $x = \sqrt{|y-1|}$.

Then, since a square root of a number is $\geq \mathbf{0}$, we get $x \geq 0$ in B, but we get $x \leq 0$ in A due to the minus sign in front of the root sign.

Next, squaring both sides of each equation, we can remove the square root sign.

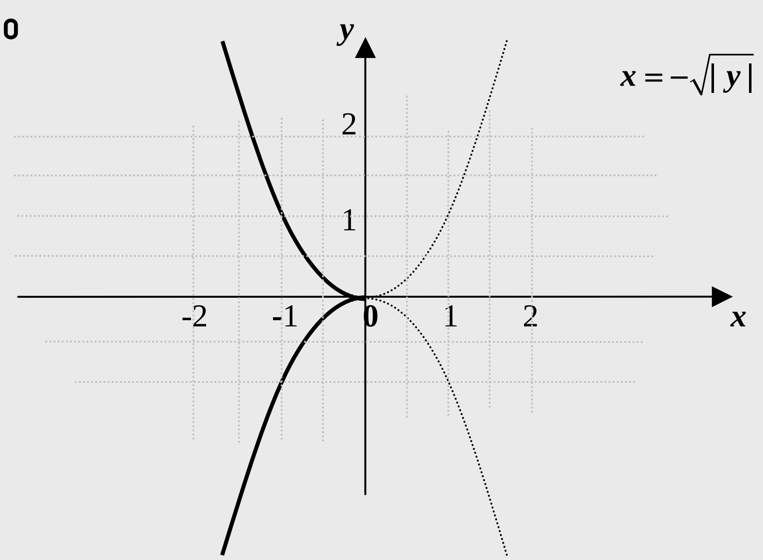
A: $x = -\sqrt{|y|} \Rightarrow x^2 = |y|$, and B: $x = \sqrt{|y-1|} \Rightarrow x^2 = |y-1|$.

Let's now, begin with A. We know y can be any real number, but $x \leq 0$ only. So we get

$y = x^2$ for $x \leq 0$ and $y \geq 0$. $y = -x^2$ for $x \leq 0$ and $y < 0$.

Each of the equations above is a left half of a parabola where the vertex is at the origin. The first of the above is concave-up, but the second is concave-down.

Fig. 0



Next, let's move on to ***B***.

We have ***B***: $x = \sqrt{|y-1|} \Rightarrow x^2 = |y-1|$.

We know that ***y*** can be any real number, but $x \geq 0$ only.
Besides, removing the absolute sign, we wan to consider two cases as follows.

$y - 1 \geq 0$, and $y - 1 < 0$.

So we get

$y - 1 \geq 0 \Rightarrow y \geq 1 \Rightarrow y - 1 = x^2$ for $y \geq 1$ and $x \geq 0$.
$y - 1 < 0 \Rightarrow y < 1 \Rightarrow -(y - 1) = x^2 \Rightarrow y - 1 = -x^2$ for $y < 1$ and $x \geq 0$.

Each of the above is a right half of a parabola where the vertex is at **(0, 1)**.
The first of the above is concave-up, but the second is concave-down.

Fig. 1