Examples A in Parabolas

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Examples A

This book is in fact, for preparations for calculus, And the same is true, too, for most books the author writes. Doing calculus, we often need to do some operations on graphs. Doing such operations, we manipulate (move or modify) curves, and the curves are from functions or equations. Thus, what we actually manipulate is functions or equations. And such operations are often called function (or equation) transformations.

For details on such transformations, refer to the book **Function Transformations**. And briefly, visiting the idea of such transformations, we can get some ideas as follows.

Transforming a function, we change the function. Changing a function, we move or modify the function. Doing it, we use the curve the function has. And doing it, we say that we do a function transformation or apply a transformation to a function. Doing a function transformation, we make a new function, and get a new curve, which is the curve of the new function.

Let's now call a function to be transformed, the original function.
Transforming a function then, we change the curve the original function has, and make a new curve. So we get a new function. Changing the curve, that is, transforming the curve, we transform each and every point in the curve the original function has. However, we cannot actually work with each of all the points at a time.   What then, can we do?

Transforming a curve, we can't actually work with each and every point in the curve, but can transform all the points at once. And in fact, we have to do it that way. How though?

We use a special point, which is random, but represents all the points in the curve.
What special point then, is it?

It is called an arbitrary point in the curve. So transforming the arbitrary point, we make the entire curve transformed And thus, we apply a transformation to the arbitrary point in the curve. An arbitrary point is no other than a variable we use for an equation or function, and is in fact, actually made of variables used in an equation or function.

For instance, (x, y) is an arbitrary point in the curve of a function $y = f(x) = x^2 + x$.

And (x, y) is an arbitrary point in the curve of an equation $x^2 + xy - y = 0$.

So doing problems in this set of examples, we will do some practices on such operations covering or going over some basics on such, and the curves are parabolas, of course.

0. Find u and v assuming that u and v are constant, and that

$y = f(x) = 2(x + 1)^2$, and $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$.
T_1: $(x, y) \longrightarrow (3 - x, -2 - y)$.
T_2: $(x, y) \longrightarrow (-x, -y)$.
T_3: $(x, y) \longrightarrow (x + u, y + v)$.

1. Find the function g assuming that

$y = f(x) = 2x^2 - 2x + 3$, and $g = T_2 \bullet T_1 \bullet f$.
T_1: $(x, y) \rightarrow (x - 3, y - 2)$.
T_2: $(x, y) \rightarrow (y, x)$.

2. Find u and v assuming that u and v are constant, and that

$y = f(x) = -2x^2 + 8x - 5$, $y = g(x) = 2x^2 - 6x + \frac{5}{2}$, and $g = T \bullet f$.
T: $(x, y) \longrightarrow (u - x, v - y)$.

Suggestions or Solutions
To the Problem in the Example 0

Find u and v assuming that u and v are constant, and that

$y = f(x) = 2(x + 1)^2$, and $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$.

T_1: $(x, y) \longrightarrow (3 - x, -2 - y)$.

T_2: $(x, y) \longrightarrow (-x, -y)$.

T_3: $(x, y) \longrightarrow (x + u, y + v)$.

$T_2 \bullet T_1$: $(x, y) \longrightarrow (-x, -y) \longrightarrow (-x + 3, -y + 2) \longrightarrow \{-(-x + 3), -(-y + 2)\} = (x - 3, y - 2)$.

So $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3, y - 2) \longrightarrow \{(x - 3) + u, (y - 2) + v\}$.

Thus, $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3 + u, y - 2 + v)$.

And we have $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$, so we need to have $(x - 3 + u, y - 2 + v) = (x, y)$.

Thus, $x - 3 + u = x \Rightarrow u = 3$, and $y - 2 + v = y \Rightarrow v = 2$.

If not quite sure of the idea behind the processes above, follow the steps below.

In this example, we get to apply to the given function f the transformations called T_1, T_2, and T_3. The transformations get applied in the order of T_1, T_2, and T_3.

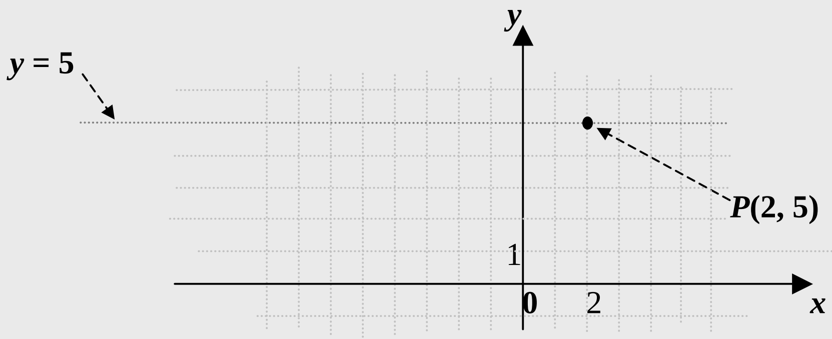
We can do such transformations analytically or algebraically. In this case though, we can try a graph method. Such a method is quite useful when the transformation is not too complicated and we want to quickly get the final solution only.

Suppose now, a point P in the curve of f is at **(2, 5)**, and we move P by a transformation.

Then, the rest of the points in the curve, too, will move in the same manner as P moves.

Putting the point P in the x-y plane, we can get a graph as follows.

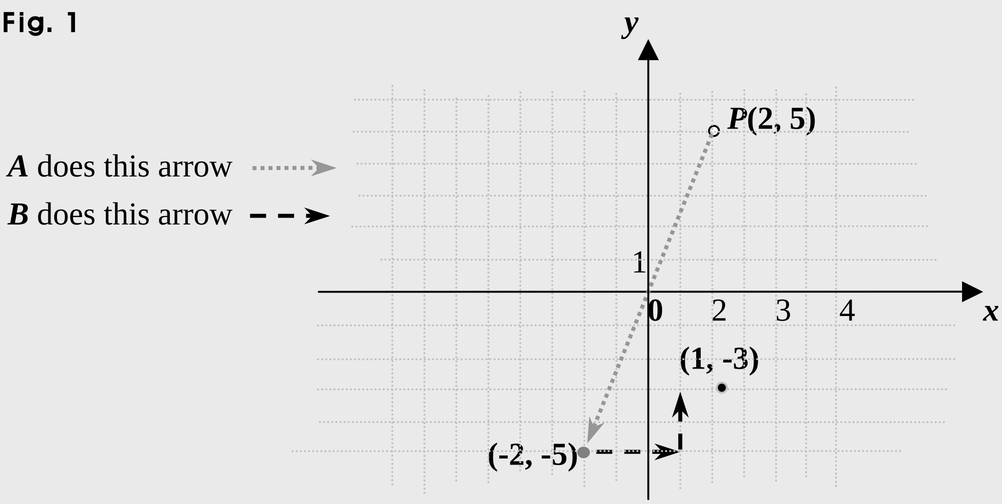
Fig. 0



Let's now, begin with the application of the transformation $\boldsymbol{T_1}$ to the point $\boldsymbol{P}$.
The transformation is $\boldsymbol{T_1}$**: $(x, y) \longrightarrow (3 - x, 2 - y)$.**

$\boldsymbol{T_1}$ is in fact, a composite transformation of two transformations ***A*** and ***B*** as follows.
A: $(x, y) \longrightarrow (-x, -y)$. (***A*** is a transformation symmetric about the origin)
B: $(x, y) \longrightarrow (x + 3, y + 2)$. (***B*** is a parallel transformation, that is, a translation)

Fig. 1



In the composite transformation $\boldsymbol{T_1 = B \bullet A}$, the transformation ***A*** gets applied first, and then, the transformation ***B*** gets applied.

That is, $\boldsymbol{T_1 = B \bullet A}$: $(x, y) \longrightarrow (-x, -y) \longrightarrow (-x + 3, -y + 2) = (3 - x, 2 - y)$.

Therefore, $\boldsymbol{T_1}$: $(x, y) \longrightarrow (3 - x, 2 - y)$.

Let' see now, what happens to the point ***P*** at **(2, 5)** when the T_1 gets applied to ***P***.

The point ***P*** is now, at **(2, 5)**.

In the first place, the transformation ***A*** gets applied to ***P***.

Then, the point ***P*** moves to a position symmetric to the position **(2, 5)** about the origin, and thus, gets positioned at **(-2, -5)**.
In turn, the transformation ***B*** acts on the point ***P*** at **(-2, -5)**.

Then, ***P*** gets translated by 3 in the direction of the ***x***-axis, and by 2 in the direction of the ***y***-axis, and thus, gets positioned at **(-2 + 3, -5 + 2) = (1, -3)**.

Next, the T_2 is as follows. T_2**:** $(x, y) \longrightarrow (-x, -y)$.

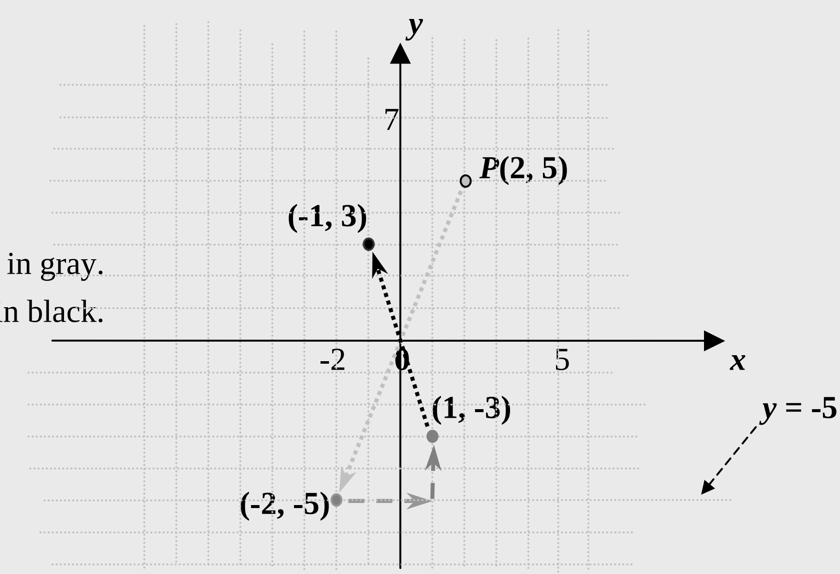
The T_2 is actually the same as the transformation ***A*** above.

If the T_2 is applied to ***P*** at **(1, -3)**, the point ***P*** gets moved symmetrically about the origin, and thus, gets positioned at **(-1, 3)**. So putting the T_1 and the T_2 in a graph, we get

Fig. 2

T_1 does the arrows in gray.

T_2 does the arrow in black.



Putting threads together, we get

$T_2 \bullet T_1$**:** $(x, y) \longrightarrow (-x, -y) \longrightarrow (-x + 3, -y + 2) \longrightarrow \{-(-x + 3), -(-y + 2)\} = (x - 3, y - 2)$.

In short, we have $T_2 \bullet T_1$**:** $(x, y) \longrightarrow (x - 3, y - 2)$.

Next, the T_3 is as follows. T_3: $(x, y) \longrightarrow (x + u, y + v)$ where u and v are constant.

If the T_3 is applied to P at $(-1, 3)$, the point P gets translated by u in the direction of the x-axis, and by v in the direction of the y-axis, and thus, gets positioned at $(-1 + u, 3 + v)$.

Let's now, get back to the beginning part of the problem.

We have $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$.
So after all the transformations, the new function is the same as the original.
We know that the point P was at $(2, 5)$ at the beginning of all the transformations.
Thus, P has to be at $(2, 5)$ at the end of all the transformations, too.
So we need to have $(-1 + u, 3 + v) = (2, 5)$.
Thus, we get $-1 + u = 2 \Rightarrow u = 3$, and also, $3 + v = 5 \Rightarrow v = 2$.

Let's put together all the three transformations.

We have
$T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3, y - 2)$.
T_3: $(x, y) \longrightarrow (x + u, y + v)$.

So we get $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3, y - 2) \longrightarrow \{(x - 3) + u, (y - 2) + v\}$

In short, we have $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3 + u, y - 2 + v)$.

Now, since we have $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$, we need to have $(x - 3 + u, y - 2 + v) = (x, y)$.
So we get $x - 3 + u = x \Rightarrow u = 3$, and $y - 2 + v = y \Rightarrow v = 2$.

In short:

$T_2 \bullet T_1$: $(x, y) \longrightarrow (-x, -y) \longrightarrow (-x + 3, -y + 2) \longrightarrow \{-(-x + 3), -(-y + 2)\} = (x - 3, y - 2)$.

So $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3, y - 2) \longrightarrow \{(x - 3) + u, (y - 2) + v\}$.

Thus, $T_3 \bullet T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3 + u, y - 2 + v)$.

Now, since $f(x) = T_3 \bullet T_2 \bullet T_1 \bullet f$, we need to have $(x - 3 + u, y - 2 + v) = (x, y)$.

Thus, $x - 3 + u = x \Rightarrow u = 3$, and $y - 2 + v = y \Rightarrow v = 2$.

Suggestions or Solutions
To the Problem in the Example 1

Find a function g assuming that $y = f(x) = 2x^2 - 2x + 3$, and $g = T_2 \bullet T_1 \bullet f$, and that T_1: $(x, y) \longrightarrow (x - 3, y - 2)$. T_2: $(x, y) \longrightarrow (y, x)$.

Suppose (x, y) is an arbitrary point of f, and (u, v) is that of g.

Then, $T_2 \bullet T_1$: $(x, y) \longrightarrow (x - 3, y - 2) \longrightarrow (y - 2, x - 3) = (u, v)$.

In short, $T_2 \bullet T_1$: $(x, y) \longrightarrow (u, v) = (y - 2, x - 3)$.

Thus, $u = y - 2 \Rightarrow y = u + 2$, and $v = x - 3 \Rightarrow x = v + 3$.

So next, $y = 2x^2 - 2x + 3 = 2(x^2 - x) + 3 = 2(x - \frac{1}{2})^2 - \frac{1}{2} + 3 = 2(x - \frac{1}{2})^2 + \frac{5}{2}$

$\Rightarrow y = 2(x - \frac{1}{2})^2 + \frac{5}{2} \Rightarrow u + 2 = 2\{(v + 3) - \frac{1}{2}\}^2 + \frac{5}{2} = 2(v + \frac{5}{2})^2 + \frac{5}{2} \Rightarrow u = 2(v + \frac{5}{2})^2 + \frac{1}{2}$.

So we get $u = g(v) = 2(v + \frac{5}{2})^2 + \frac{1}{2}$.

Since g is in the x-y system, we get $x = g(y) = 2(y + \frac{5}{2})^2 + \frac{1}{2}$.

If not quite sure of the idea behind the processes above, follow the steps below.

We can do the transformations above analytically, that is, we can do them by algebra. This time, too, we can do them using the graph method.

Like the one in the example 0, $T_2 \bullet T_1$ is a transformation, too, and specifically, is called a composite transformation, which begins with the transformation T_1, and end up with the transformation T_2.

So to begin with, in the T_1, a curve moves by -3 in the direction of the x-axis, and by -2 in the direction of the y-axis.

Therefore, if the T_1 is applied to the function f, every point in the curve of f moves by -3 in the direction of the x-axis, and by -2 in the direction of the y-axis.

Then, the collection of all the points moved will form a new curve, and a new function will have the new curve, of course. What then, is the new function?

It is $\boldsymbol{T_1 \bullet f}$, which will be transformed by the $\boldsymbol{T_2}$, but is kind of bulky to carry around with.

So giving a name to it, we can set $\boldsymbol{h = T_1 \bullet f}$. Then, we move on to $\boldsymbol{T_2 \bullet h}$.

In the $\boldsymbol{T_2}$, an original curve gets transformed and a new curve is made so that the original is symmetric to the new curve about the line $\boldsymbol{y = x}$.

So if the $\boldsymbol{T_2}$ is applied to the function $\boldsymbol{h}$, the curve of $\boldsymbol{h}$ gets transformed so that the new curve is symmetric to the curve of $\boldsymbol{h}$, about the line $\boldsymbol{y = x}$. What then, has the new curve?

It's a new function, of course, which is $\boldsymbol{T_2 \bullet h}$, which is $\boldsymbol{T_2 \bullet T_1 \bullet f}$, which is the function $\boldsymbol{g}$.

• Now, let's get the solution using the graph method.

Then, we are going to apply the $\boldsymbol{T_2 \bullet T_1}$ to one particular point in the curve of $\boldsymbol{f}$.

Let's choose the vertex for the particular point.

That's because locating or finding vertices, we can readily find or place the parabolas.

Then, the vertex will move to a new position after the composite transformation $\boldsymbol{T_2 \bullet T_1}$.

Then, all the other points in the curve move in the same way as the vertex moves.

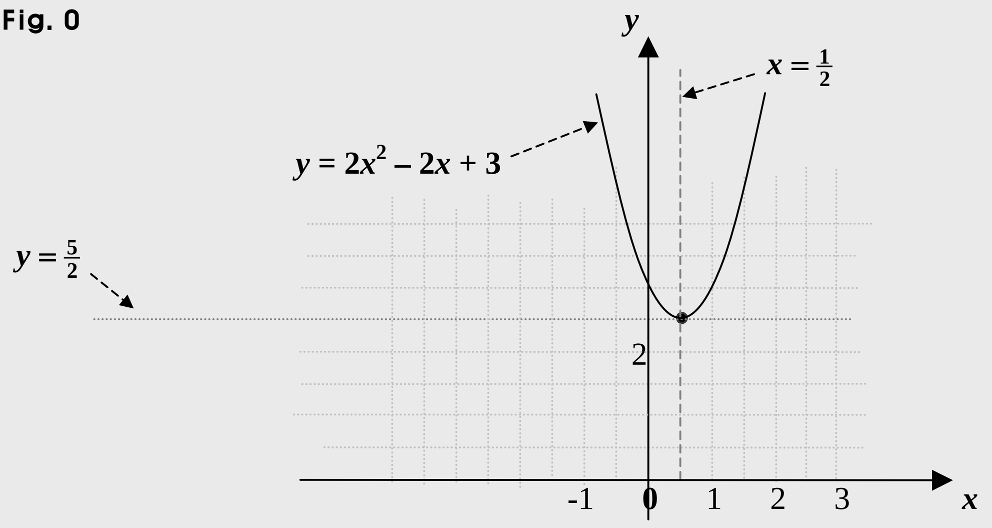
Let's see first, the particulars of the curve of $\boldsymbol{f}$.

Putting it in the vertex form, we get
$\boldsymbol{y = 2x^2 - 2x + 3 = 2(x^2 - x) + 3 = 2(x - \frac{1}{2})^2 - \frac{1}{2} + 3 = 2(x - \frac{1}{2})^2 + \frac{5}{2}}$.

So the steepness, that is, the coefficient of $\boldsymbol{x^2}$ is 2, and thus, it is concave-up, the vertex is at $\boldsymbol{(\frac{1}{2}, \frac{5}{2})}$, and the axis of symmetry is $\boldsymbol{x = -\frac{1}{2}}$.

Now, let's apply the transformations to the vertex.

Fig. 0

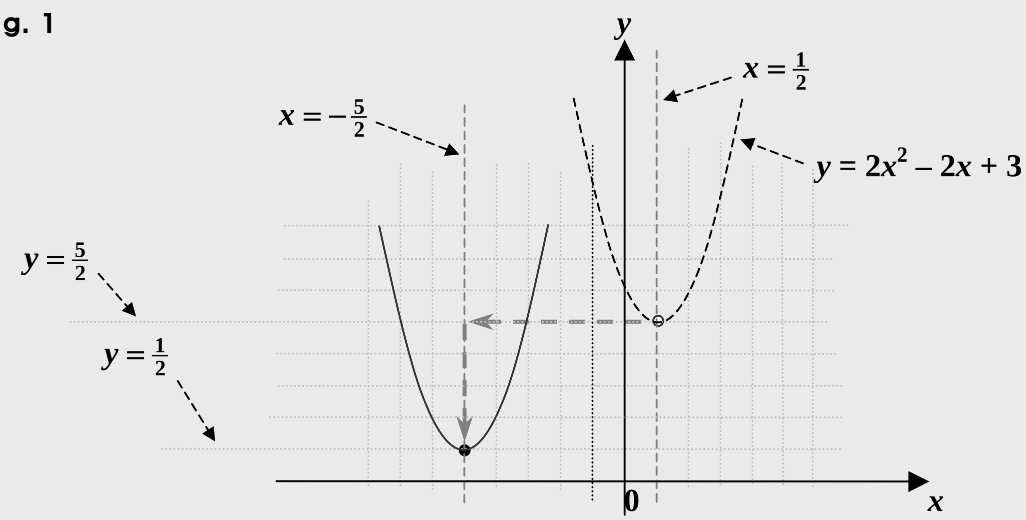


To begin with, we have $\boldsymbol{T_1}$**:** $\boldsymbol{(x, y) \longrightarrow (x-3, y-2)}$.

If the $\boldsymbol{T_1}$ is applied to the vertex, the vertex moves by -3 in the direction of the $\boldsymbol{x}$-axis, and by -2 in the direction of the $\boldsymbol{y}$-axis. So the vertex will be at $\boldsymbol{(\frac{1}{2}-3, \frac{5}{2}-2) = (-\frac{5}{2}, \frac{1}{2})}$.

The entire curve moves the way the vertex moves.
The concavity and steepness remain the same though, since the curve translates only.

Fig. 1



Next, we have $\boldsymbol{T_2}$**:** $\boldsymbol{(x, y) \longrightarrow (y, x)}$.

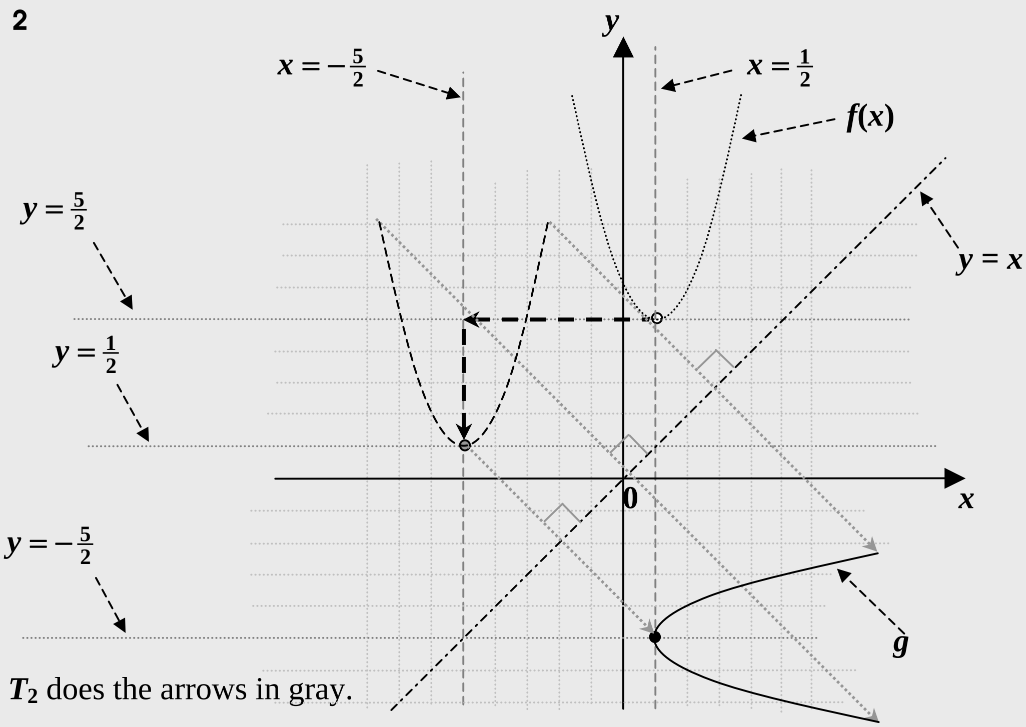
The $\boldsymbol{T_2}$ makes the vertex move to a position symmetric to $\boldsymbol{(-\frac{5}{2}, \frac{1}{2})}$ about the line $\boldsymbol{y = x}$.

So the vertex will be at $\boldsymbol{(\frac{1}{2}, -\frac{5}{2})}$ after the $\boldsymbol{T_2}$ gets applied to it.

This time, the concavity of the parabola gets changed through the transformation.

The original parabola is concave-up, but the new one is concave-right. In other words, the new parabola is open to the right.
However, the symmetric transformation does not affect the magnitude of the steepness.
Therefore, the magnitude of the steepness remains 2.

Fig. 2



The $\boldsymbol{T_2}$ does the arrows in gray.

So the parabola concave-right is the result of the composite transformation, $\boldsymbol{T_1 \bullet T_2}$.

Now, let’s find the equation of the curve of $\boldsymbol{g}$, which is the parabola concave-right.

We may want to use the vertex form, since we know the vertex.

The vertex form is $\boldsymbol{x - m = a(y - n)^2}$, where $\boldsymbol{(m, n)}$ is the vertex, and $\boldsymbol{a}$ is the steepness. The steepness is 2 since the curve is concave-*right*. Therefore, the equation is as follows.

$$x - \tfrac{1}{2} = 2\{y - (-\tfrac{5}{2})\}^2 = 2(y + \tfrac{5}{2})^2 \Rightarrow x - \tfrac{1}{2} = 2(y + \tfrac{5}{2})^2 \Rightarrow x = 2(y + \tfrac{5}{2})^2 + \tfrac{1}{2}.$$

What if we want to do the transformation analytically?

Then, let's get back to the beginning of the problem.

We have $\boldsymbol{y = f(x) = 2x^2 - 2x + 3}$, and $\boldsymbol{g = T_2 \bullet T_1 \bullet f}$.

$\boldsymbol{T_1: (x, y) \longrightarrow (x - 3, y - 2)}$.

$\boldsymbol{T_2: (x, y) \longrightarrow (y, x)}$.

Suppose now, $\boldsymbol{(x, y)}$ is the arbitrary point of $\boldsymbol{f}$, and $\boldsymbol{(u, v)}$ is that of $\boldsymbol{g}$.

Then, $\boldsymbol{T_2 \bullet T_1: (x, y) \longrightarrow (x - 3, y - 2) \longrightarrow (y - 2, x - 3) = (u, v)}$.

In short, $\boldsymbol{T_2 \bullet T_1: (x, y) \longrightarrow (u, v) = (y - 2, x - 3)}$.

So we get $\boldsymbol{u = y - 2 \Rightarrow y = u + 2}$, and also, $\boldsymbol{v = x - 3 \Rightarrow x = v + 3}$.

Putting in the vertex form the original parabola, which is the curve of $\boldsymbol{f}$, we get

$$y = 2x^2 - 2x + 3 = 2(x^2 - x) + 3 = 2(x - \tfrac{1}{2})^2 - \tfrac{1}{2} + 3 = 2(x - \tfrac{1}{2})^2 + \tfrac{5}{2}$$

$$\Rightarrow y = 2(x - \tfrac{1}{2})^2 + \tfrac{5}{2}.$$

Then, since $\boldsymbol{y = u + 2}$, and $\boldsymbol{x = v + 3}$, we get

$$y = 2(x - \tfrac{1}{2})^2 + \tfrac{5}{2} \Rightarrow u + 2 = 2\{(v + 3) - \tfrac{1}{2}\}^2 + \tfrac{5}{2} = 2(v + \tfrac{5}{2})^2 + \tfrac{5}{2} \Rightarrow u = 2(v + \tfrac{5}{2})^2 + \tfrac{1}{2}.$$

So we get $\boldsymbol{u = g(v) = 2(v + \tfrac{5}{2})^2 + \tfrac{1}{2}}$.

Now, the curve of $\boldsymbol{g}$ has to be in the $\boldsymbol{x}$-$\boldsymbol{y}$ plane, and $\boldsymbol{(u, v)}$ is the arbitrary point of $\boldsymbol{g}$.

So replacing $\boldsymbol{u}$ and $\boldsymbol{v}$ with $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively, we can put the function $\boldsymbol{g}$ in the $\boldsymbol{x}$-$\boldsymbol{y}$ plane.

Thus, we get $\boldsymbol{x = g(y) = 2(y + \tfrac{5}{2})^2 + \tfrac{1}{2}}$.

Suggestions or Solutions
To the Problem in the Example 2

Find u and v assuming that u and v are constant, and that

$y = f(x) = -2x^2 + 8x - 5$, $y = g(x) = 2x^2 - 6x + \frac{5}{2}$, and $g = T \bullet f$.

$T: (x, y) \longrightarrow (u - x, v - y)$

$y = f(x) = -2x^2 + 8x - 5 = -2(x^2 - 4x + 4 - 4) - 5 = -2(x - 2)^2 + 8 - 5 = -2(x - 2)^2 + 3$

$\Rightarrow y = f(x) = -2(x - 2)^2 + 3$.

$y = g(x) = 2x^2 - 6x + \frac{5}{2} = 2(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) + \frac{5}{2} = 2(x - \frac{3}{2})^2 - \frac{9}{2} + \frac{5}{2} = 2(x - \frac{3}{2})^2 - 2$.

$\Rightarrow y = g(x) = 2(x - \frac{3}{2})^2 - 2$.

Suppose now, (x, y) is the arbitrary point of f, and (s, t) is that of g.

Then, we get $T: (x, y) \longrightarrow (s, t) = (u - x, v - y)$.

Thus, we get $s = u - x \Rightarrow x = u - s$, and $t = v - y \Rightarrow y = v - t$.

So next, we get $y = f(x) = -2(x - 2)^2 + 3 \Rightarrow v - t = -2(u - s - 2)^2 + 3$

$\Rightarrow t = 2(s - u + 2)^2 + v - 3 = 2\{s - (u - 2)\}^2 + v - 3$.

Since the function g is in the x-y system, we get $y = g(x) = 2\{x - (u - 2)\}^2 + v - 3$.

And we have $y = g(x) = 2(x - \frac{3}{2})^2 - 2$, too.

Thus, $u - 2 = \frac{3}{2} \Rightarrow u = \frac{7}{2}$, and $v - 3 = -2 \Rightarrow v = 1$.

If not quite sure of the idea behind the processes above, follow the steps below.

This time, too, we get to do a composite transformation.
That is, the transformation T can be broken apart into two transformations. How?

We can reset $u - x = -x + u$, and $v - y = -y + v$.

Thus, the transformation $\boldsymbol{T}$ can be taken for a composite transformation of the two below.
$\boldsymbol{A}$: $(x, y) \longrightarrow (-x, -y)$, and $\boldsymbol{B}$: $(x, y) \longrightarrow (x + u, y + v)$

In the composite transformation $\boldsymbol{T} = \boldsymbol{B} \bullet \boldsymbol{A}$, the transformation $\boldsymbol{A}$ gets applied first, and then, the $\boldsymbol{B}$ gets applied.

If the $\boldsymbol{A}$ is applied to $\boldsymbol{f}$, every point in the curve of $\boldsymbol{f}$ gets moved symmetrically about the origin, and becomes a point in a new curve. A new function has the new curve, of course.

Suppose $\boldsymbol{h}$ is the new function. Then, if the $\boldsymbol{B}$ is applied to $\boldsymbol{h}$, every point in the curve of $\boldsymbol{h}$ gets moved by $\boldsymbol{u}$ in the direction of the $\boldsymbol{x}$-axis, and by $\boldsymbol{v}$ in the direction of the $\boldsymbol{y}$-axis, and belongs to the curve of another new function. What is the other new function?

It is the very function $\boldsymbol{g(x)}$. This time also, we can get the solution by the graph method. So to begin with, let's put in a graph the curves of the two functions $\boldsymbol{f}$ and $\boldsymbol{g}$.

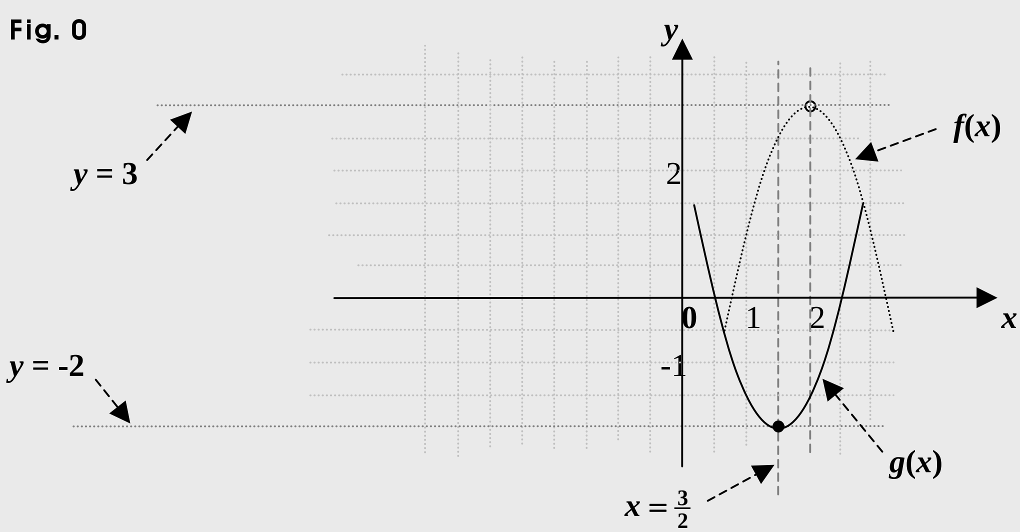
Then, we may want to put $\boldsymbol{f}$ and $\boldsymbol{g}$ in the vertex form. Then, we get

$$y = f(x) = -2x^2 + 8x - 5 = -2(x^2 - 4x + 4 - 4) - 5 = -2(x - 2)^2 + 8 - 5 = -2(x - 2)^2 + 3.$$

$$y = g(x) = 2x^2 - 6x + \tfrac{5}{2} = 2(x^2 - 3x + \tfrac{9}{4} - \tfrac{9}{4}) + \tfrac{5}{2} = 2(x - \tfrac{3}{2})^2 - \tfrac{9}{2} + \tfrac{5}{2} = 2(x - \tfrac{3}{2})^2 - 2.$$

Now, we can see that
the curve of $\boldsymbol{f}$ is of steepness **-2**, and thus, is concave-down. And the vertex is at **(2, 3)**.
The curve of $\boldsymbol{g}$ is of steepness **2**, so it is concave-up. And the vertex is at $(\tfrac{3}{2}, -2)$.

Fig. 0



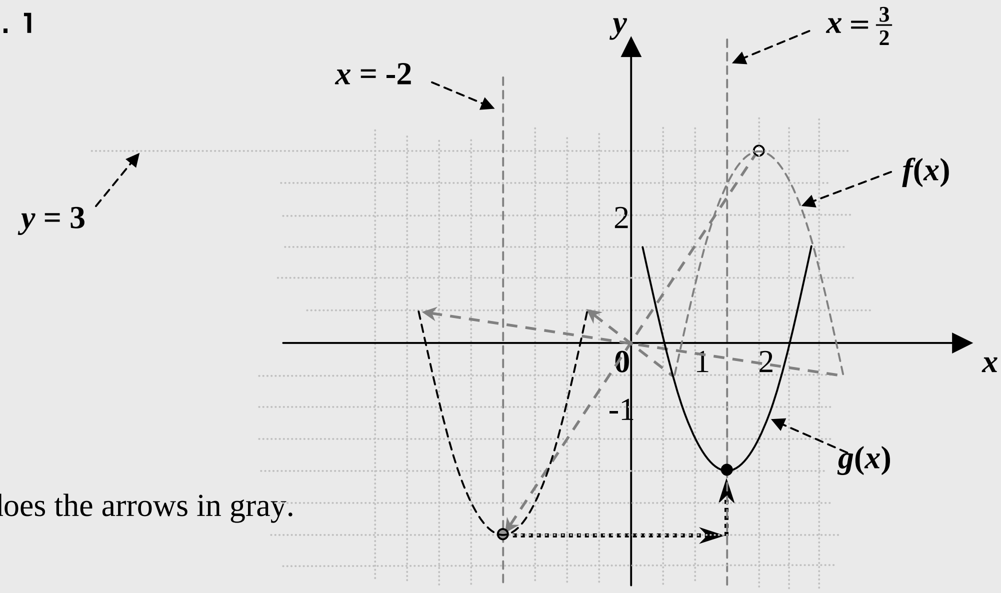
In the transformation $T = B \bullet A$, two transformations happen in turn as follows.

A: (x, y) $\longrightarrow$ **(-x, -y)**, and then, **B: (x, y)** $\longrightarrow$ **(x + u, y + v)**.

First, if the ***A*** is applied to the vertex of the parabola ***f***, the vertex moves symmetrically about the origin, and then, gets positioned at **(-2, -3)**.

So the entire parabola of ***f*** gets transformed to the parabola dashed in black in the figure below.

Fig. 1



A does the arrows in gray.

Next, it the ***B*** is applied to the vertex at **(-2, -3)**, the vertex moves in the amount of ***u*** along the ***x***-axis, and in the amount of ***v*** along the ***y***-axis.

So the entire parabola dashed in black gets transformed to the parabola solid in black in the figure above. The parabola in black is the curve of the function ***g***.

So the vertex of the curve of ***g*** is $(\frac{3}{2}, -2)$. Now, we know **B: (x, y)** $\longrightarrow$ **(x + u, y + v)**.

Thus, **(-2, -3)** $\longrightarrow$ **(-2 + u, -3 + v)** = $(\frac{3}{2}, -2)$.

So we get $-2 + u = \frac{3}{2}$, and $-3 + v = -2$. Thus, we get $u = \frac{7}{2}$, and $v = 1$.

What if we want to do the transformation analytically?

Then, getting back to the beginning of the problem, we have

$\boldsymbol{y = f(x) = -2x^2 + 8x - 5}$, $\boldsymbol{y = g(x) = 2x^2 - 6x + \frac{5}{2}}$, and $\boldsymbol{g = T \bullet f}$.

$\boldsymbol{T: (x, y) \longrightarrow (u - x, v - y)}$.

We may want to put the curves of ***f*** and ***g*** in the vertex form. Then, we can avoid the quadratic terms, so life can get easier, that is, calculations get simpler. Then, we get

$$\boldsymbol{y = f(x) = -2x^2 + 8x - 5 = -2(x^2 - 4x + 4 - 4) - 5 = -2(x - 2)^2 + 8 - 5 = -2(x - 2)^2 + 3}$$

$$\boldsymbol{\Rightarrow y = f(x) = -2(x - 2)^2 + 3}.$$

$$\boldsymbol{y = g(x) = 2x^2 - 6x + \tfrac{5}{2} = 2(x^2 - 3x + \tfrac{9}{4} - \tfrac{9}{4}) + \tfrac{5}{2} = 2(x - \tfrac{3}{2})^2 - \tfrac{9}{2} + \tfrac{5}{2} = 2(x - \tfrac{3}{2})^2 - 2}.$$

$$\boldsymbol{\Rightarrow y = g(x) = 2(x - \tfrac{3}{2})^2 - 2}.$$

Suppose now, $\boldsymbol{(x, y)}$ is the arbitrary point in the curve of ***f***, and $\boldsymbol{(s, t)}$ is that of ***g***.

Then, we get $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (u - x, v - y)}$.

So we get $\boldsymbol{s = u - x \Rightarrow x = u - s}$, and $\boldsymbol{t = v - y \Rightarrow y = v - t}$.

Then, $\boldsymbol{y = f(x) = -2(x - 2)^2 + 3 \Rightarrow v - t = -2(u - s - 2)^2 + 3 \Rightarrow t = 2(u - s - 2)^2 + v - 3}$.

Meanwhile, $\boldsymbol{(u - s - 2)^2 = (-s + u - 2)^2 = \{-(s - u + 2)\}^2 = (s - u + 2)^2}$.

So we get $\boldsymbol{t = 2(s - u + 2)^2 + v - 3 = 2\{s - (u - 2)\}^2 + v - 3}$.

Now, the curve of the function ***g*** is in the ***x-y*** plane, and $\boldsymbol{(s, t)}$ is the arbitrary point of ***g***. Replacing ***s*** and ***t*** with ***x*** and ***y*** respectively, we can put the function ***g*** in the ***x-y*** plane.

Then, we get $\boldsymbol{y = g(x) = 2\{x - (u - 2)\}^2 + v - 3}$.

We have $\boldsymbol{y = g(x) = 2(x - \tfrac{3}{2})^2 - 2}$, too.

Thus, we get $\boldsymbol{u - 2 = \tfrac{3}{2} \Rightarrow u = \tfrac{7}{2}}$, and $\boldsymbol{v - 3 = -2 \Rightarrow v = 1}$.