Examples B in Parabolas

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Examples B

0. Find c, u, and v assuming that c, u, and v are constant, and that

$x = f(y) = 2(y - 1)^2 + 2$, $y = g(x) = (x - \frac{3}{2})^2 + 2$, and $g = T_2 \bullet T_1 \bullet f$.

T_1: $(x, y) \longrightarrow (cy, 2x)$.
T_2: $(x, y) \longrightarrow (x + u, y + v)$.

1. Specify a function g assuming that c, s, and t are constant, and that

$f(x) = 2x^2 - 2cx + c$ where $c \geq -1$.

The curve of $y = f(x)$ is a parabola, and the vertex is (s, t).

T: $c \longrightarrow (s, t)$, where $t = g(s)$.

2. Find a function g assuming that c, s, and t are constant, that $c \geq -1$, that the vertex of a parabola $x = y^2 - 2(c + 1)y + 2c^2$ is (s, t), and that

T: $c \longrightarrow (s, t)$, where $t = g(s)$.

Suggestions or Solutions
To the Problem in the Example 0

Find c, u, and v assuming that c, u, and v are constant, and that
$x = f(y) = 2(y-1)^2 + 2$, $y = g(x) = (x - \frac{3}{2})^2 + 2$, and $g = T_2 \bullet T_1 \bullet f$, and that
$T_1: (x, y) \longrightarrow (cy, 2x)$, and $T_2: (x, y) \longrightarrow (x + u, y + v)$.

Suppose first, $h = T_1 \bullet f$, and (s, t) is the arbitrary point of the function h. Then, we have
$T_1: (x, y) \longrightarrow (s, t) = (cy, 2x)$.

So we get $s = cy \Rightarrow y = \frac{s}{c}$, and $t = 2x \Rightarrow x = \frac{t}{2}$. Thus, we get

$$x = 2(y-1)^2 + 2 \Rightarrow \frac{t}{2} = 2(\frac{s}{c}-1)^2 + 2 \Rightarrow t = h(s) = 4(\frac{s}{c})^2 - 8 \cdot \frac{s}{c} + 8 = (\frac{2}{c})^2 s^2 - \frac{8}{c} s + 8.$$

Putting the function h in the x-y system, we get $y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8$.

Suppose now, (s, t) is the arbitrary point of g.
Then, we have $T_2: (x, y) \longrightarrow (s, t) = (x + u, y + v)$.
So $s = x + u \Rightarrow x = s - u$, and $t = y + v \Rightarrow y = t - v$.

If T_2 is applied to a parabola, the steepness (the coefficient of x^2) doesn't get changed.

We have $y = g(x) = (x - \frac{3}{2})^2 + 2$, $g = T_2 \bullet h$, and $h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8$.

So we get $(\frac{2}{c})^2 = 1 \Rightarrow \frac{2}{c} = \pm 1 \Rightarrow c = \pm 2$. Thus, we have two cases for c.

To begin with, we get $c = 2 \Rightarrow y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8 = x^2 - 4x + 8 = (x-2)^2 + 4$.
Thus, $y = h(x) = (x-2)^2 + 4 \Rightarrow t - v = (s - u - 2)^2 + 4 \Rightarrow t = g(s) = (s - u - 2)^2 + 4 + v$.
So we get $y = g(x) = (x - u - 2)^2 + 4 + v = \{x - (u + 2)\}^2 + 4 + v$.

Next, we get $c = -2 \Rightarrow y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8 = x^2 + 4x + 8 = (x+2)^2 + 4$.

So $y = h(x) = (x+2)^2 + 4 \Rightarrow t - v = (s - u + 2)^2 + 4 \Rightarrow t = g(s) = (s - u + 2)^2 + 4 + v$.

Putting g in the x-y system, we get $y = g(x) = \{x - (u - 2)\}^2 + 4 + v$.

Now, we have $y = g(x) = (x - \frac{3}{2})^2 + 2$, and

When $c = 2$, $y = g(x) = \{x - (u+2)\}^2 + 4 + v$.

When $c = -2$, $y = g(x) = \{x - (u-2)\}^2 + 4 + v$.

Thus,

when $c = 2$, we get $u + 2 = \frac{3}{2} \Rightarrow u = -\frac{1}{2}$, and $4 + v = 2 \Rightarrow v = -2$.

When $c = -2$, we get $u - 2 = \frac{3}{2} \Rightarrow u = \frac{7}{2}$, and $4 + v = 2 \Rightarrow v = -2$.

If not quite sure of the idea behind the processes above, follow the steps below.

In this example, we want to find a particular transformation where a parabola with the axis of symmetry parallel to the y-axis gets transformed to a parabola with the axis of symmetry parallel to the x-axis.

The transformation we want to find has a structure preset, is a composite transformation, and is $T_2 \bullet T_1$, where T_1: $(x, y) \longrightarrow (cy, 2x)$, and T_2: $(x, y) \longrightarrow (x + u, y + v)$.

So we want to find c, u, and v, for which $T_2 \bullet T_1$ does the work.

• Now, going over briefly the idea of transformation, we can put it the way below, too.

In this case, transforming a function called f, we get a function called g.

That is, putting f into a machine called a transformation, the transformation produces g.

So to speak, if we input f into a transformation, the transformation outputs g.

So f can be called a function input, and g can be called a function output.

So putting an input called f into a transformation called $T_2 \bullet T_1$, we get an output called g.

In other words, a transformation is no other than a function, and is a function itself.

In fact, transforming a data called an input, a function produces a data called an output.

So understanding the ideas of functions, we can understand transformations better.

Now, let's get the solution to this problem.

Suppose now, $h = T_1 \bullet f$, and (s, t) is the arbitrary point of h. Then, we have
$T_1 : (x, y) \longrightarrow (s, t) = (cy, 2x)$.

So we get $s = cy \Rightarrow y = \frac{s}{c}$, and $t = 2x \Rightarrow x = \frac{t}{2}$. Thus, we get

$$x = 2(y-1)^2 + 2 \Rightarrow \tfrac{t}{2} = 2(\tfrac{s}{c} - 1)^2 + 2 = 2(\tfrac{s}{c})^2 - 4 \cdot \tfrac{s}{c} + 4 \Rightarrow t = 4(\tfrac{s}{c})^2 - 8 \cdot \tfrac{s}{c} + 8$$

$\Rightarrow t = (\frac{2}{c})^2 s^2 - \frac{8}{c} s + 8$, which is not in the x-y coordinate system but the s-t system.

Putting h in the x-y system, we just replace s and t with x and y respectively.

Then, we get $y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8$.

Next, let's move on to $T_2 \bullet T_1 \bullet f$, which is g.
Since $h = T_1 \bullet f$, we have $g = T_2 \bullet h$.
The T_2 is a parallel transformation where a curve gets translated only.
So if the T_2 is applied to a parabola, the steepness doesn't get changed.

We have $y = g(x) = (x - \frac{3}{2})^2 + 2$, which is a parabola where the steepness is 1.
And we have $g = T_2 \bullet h$, too, so the steepness of h needs to be 1, also.

We have $y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8$. So we get $(\frac{2}{c})^2 = 1 \Rightarrow \frac{2}{c} = \pm 1 \Rightarrow c = \pm 2$.

Thus, we have two choices for c, so let's begin with the case where $c = 2$.

Then, first, we get $c = 2 \Rightarrow y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8 = x^2 - 4x + 8 = (x-2)^2 + 4$.

Next, we want to find u and v in the case where $c = 2$. How do we get u and v, though?

The transformation T_2 has the constants u and v. So let's apply the T_2 to the function h.

Then, we get g, so suppose now, (s, t) is the arbitrary point of g.

Then, we have $T_2 : (x, y) \longrightarrow (s, t) = (x + u, y + v)$.
So we get $(s, t) = (x + u, y + v) \Rightarrow s = x + u \Rightarrow x = s - u$, and $t = y + v \Rightarrow y = t - v$.

Therefore, we get $y = h(x) = (x-2)^2 + 4 \Rightarrow t - v = (s-u-2)^2 + 4$
$\Rightarrow t = g(s) = (s-u-2)^2 + 4 + v$.

Putting g is in the x-y system, we just replace s and t with x and y respectively.
Then, we get $y = g(x) = (x-u-2)^2 + 4 + v = \{x-(u+2)\}^2 + 4 + v$.
And we have $y = g(x) = (x - \frac{3}{2})^2 + 2$, too.
Therefore, when $c = 2$, we get $u + 2 = \frac{3}{2} \Rightarrow u = -\frac{1}{2}$, and $4 + v = 2 \Rightarrow v = -2$.

Let's next, move on to the case where $c = -2$.
Then, we get first $c = -2 \Rightarrow y = h(x) = (\frac{2}{c})^2 x^2 - \frac{8}{c} x + 8 = x^2 + 4x + 8 = (x+2)^2 + 4$.

Next, in the transformation T_2 stated above, we have $x = s - u$, and $y = t - v$.

Therefore, we get $y = h(x) = (x+2)^2 + 4 \Rightarrow t - v = (s-u+2)^2 + 4$
$\Rightarrow t = g(s) = (s-u+2)^2 + 4 + v$.

Putting g is in the x-y system, we replace s and t with x and y respectively.
Then, we get $y = g(x) = (x-u+2)^2 + 4 + v = \{x-(u-2)\}^2 + 4 + v$.

Also, we have $y = g(x) = (x - \frac{3}{2})^2 + 2$.

Thus, when $c = -2$, we get $u - 2 = \frac{3}{2} \Rightarrow u = \frac{7}{2}$, and $4 + v = 2 \Rightarrow v = -2$.

Note:

The two solution sets mean two paths where the function $f(y)$ can change to the function $g(x)$. One of the two is as follows.

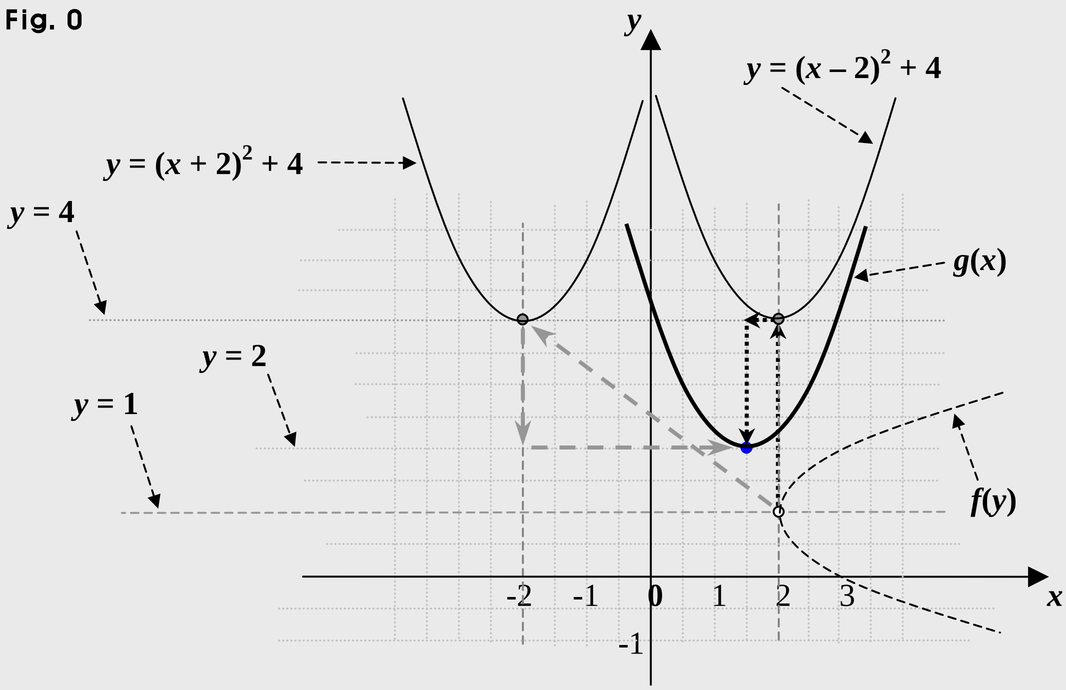
T_1: $(x, y) \longrightarrow (2y, 2x)$. T_2: $(x, y) \longrightarrow (x - \frac{1}{2}, y - 2)$.
In short, $T_2 \bullet T_1$: $(x, y) \longrightarrow (2y - \frac{1}{2}, 2x - 2)$.

And the other path is as follows.

T_1: $(x, y) \longrightarrow (-2y, 2x)$. T_2: $(x, y) \longrightarrow (x + \frac{7}{2}, y - 2)$.
In short, $T_2 \bullet T_1$: $(x, y) \longrightarrow (-2y + \frac{7}{2}, 2x - 2)$.

Both of the two paths lead to the same result, of course. Suppose ***A*** is the first of the two paths, and ***B*** is the second. Then, putting the curves in a graph, we can get

Fig. 0



The path ***A*** follows the arrows in black.

The path ***B*** follows the arrows in gray.

Suggestions or Solutions
To the Problem in the Example 1

Specify a function g assuming that c, s, and t are constant, and that
$f(x) = 2x^2 - 2cx + c$ where $c \geq -1$.
The curve of $y = f(x)$ is a parabola, and the vertex is (s, t).
T: $c \longrightarrow (s, t)$, where $t = g(s)$.

First, $\boldsymbol{y = 2x^2 - 2cx + c = 2(x^2 - cx + \frac{c^2}{4} - \frac{c^2}{4}) + c = 2(x - \frac{c}{2})^2 - \frac{c^2}{2} + c}$ for $\boldsymbol{c \geq -1}$.

Therefore, the vertex is $\boldsymbol{(\frac{c}{2}, -\frac{c^2}{2} + c)}$ where $\boldsymbol{c \geq -1}$.

Next, $\boldsymbol{(s, t)}$ is the vertex. So we get $\boldsymbol{T: c \longrightarrow (s, t) = (\frac{c}{2}, -\frac{c^2}{2} + c)}$ where $\boldsymbol{c \geq -1}$.

Thus, we get $\boldsymbol{s = \frac{c}{2}}$ and $\boldsymbol{t = -\frac{c^2}{2} + c}$ where $\boldsymbol{c \geq -1}$.

So first, $\boldsymbol{s = \frac{c}{2} \Rightarrow c = 2s}$. Thus, $\boldsymbol{c \geq -1 \Rightarrow 2s \geq -1 \Rightarrow s \geq -\frac{1}{2}}$.

Next, $\boldsymbol{c = 2s \Rightarrow t = -\frac{c^2}{2} + c = -\frac{4s^2}{2} + 2s = -2s^2 + 2s}$ for $\boldsymbol{s \geq -\frac{1}{2}}$.

Therefore, $\boldsymbol{t = g(s) = -2s^2 + 2s}$ for $\boldsymbol{s \geq -\frac{1}{2}}$.

Thus, we get $\boldsymbol{y = g(x) = -2x^2 + 2x}$ for $\boldsymbol{x \geq -\frac{1}{2}}$.

If not quite sure of the idea behind the processes above, follow the steps below.

This transformation ***T*** is quite different from the ones covered earlier.

The actual input to the ***T*** is not a function or equation but a number ray.

That is, each and every number in a ray is an input to the ***T***.

More specifically, every number ≥ **-1** is input into the ***T***. Then, for every number input, the ***T*** outputs a point and the point belongs to the curve of a function $\boldsymbol{y = g(x)}$.

Thus, although we don't input a function into the ***T***, the ***T*** still produces a function as the output. The input is a set of all numbers ≥ -1, and the output is a function called ***g***.

So in sum, if we input into the ***T*** a set of all numbers ≥ -1, the ***T*** outputs the function ***g***.

The ***T*** is called a parametric transformation, where the parameter is ***c***.

For details on parametric transformations, refer to the book, **GRAPH OPERATIONS**.

Let's now, start solving the problem beginning with the vertex of the curve of ***f***.

$\boldsymbol{y = 2x^2 - 2cx + c = 2(x^2 - cx + \frac{c^2}{4} - \frac{c^2}{4}) + c = 2(x - \frac{c}{2})^2 - \frac{c^2}{2} + c}$ for $\boldsymbol{c \geq -1}$.

Therefore, the vertex is $\boldsymbol{(\frac{c}{2}, -\frac{c^2}{2} + c)}$ where $\boldsymbol{c \geq -1}$.

Next, we have $\boldsymbol{T\!: c \longrightarrow (s, t)}$, where ***s*** and ***t*** are the coordinates at the vertex.

So ***s*** and ***t*** will get expressed in terms of the parameter ***c*** respectively.

Therefore, ***s*** and ***t*** will be functions of ***c*** respectively.

For instance, we can possibly get $\boldsymbol{s = f(c)}$, and $\boldsymbol{t = g(c)}$ for $\boldsymbol{c \geq -1}$.

So ***s*** and ***t*** will be connected via the parameter ***c***.

Next, $\boldsymbol{(s, t)}$ is the vertex of the parabola $\boldsymbol{y = 2x^2 - 2cx + c = 2(x - \frac{c}{2})^2 - \frac{c^2}{2} + c}$ for $\boldsymbol{c \geq -1}$.

So the vertex is $\boldsymbol{(\frac{c}{2}, -\frac{c^2}{2} + c)}$ where $\boldsymbol{c \geq -1}$.

Thus, we now, have $\boldsymbol{T\!: c \longrightarrow (s, t) = (\frac{c}{2}, -\frac{c^2}{2} + c)}$ where $\boldsymbol{c \geq -1}$.

So we get $\boldsymbol{s = \frac{c}{2}}$ and $\boldsymbol{t = -\frac{c^2}{2} + c}$ where $\boldsymbol{c \geq -1}$.

Now, ***c*** is the parameter, and thus, is just a catalyst if you will. So what we need to do is to remove ***c*** from the two equations above so that ***s*** and ***t*** can get connected. That is, we get the connective expression between ***s*** and ***t***, and the expression is for the function ***g***.

Then first, $\boldsymbol{s = \frac{c}{2} \Rightarrow c = 2s}$.

So we get $\boldsymbol{c \geq -1 \Rightarrow 2s \geq -1 \Rightarrow s \geq -\frac{1}{2}}$.

Next, we get $\boldsymbol{c = 2s \Rightarrow t = -\frac{c^2}{2} + c = -\frac{4s^2}{2} + 2s = -2s^2 + 2s}$ for $\boldsymbol{s \geq -\frac{1}{2}}$.

Therefore, we get $\boldsymbol{t = g(s) = -2s^2 + 2s}$ for $\boldsymbol{s \geq -\frac{1}{2}}$.

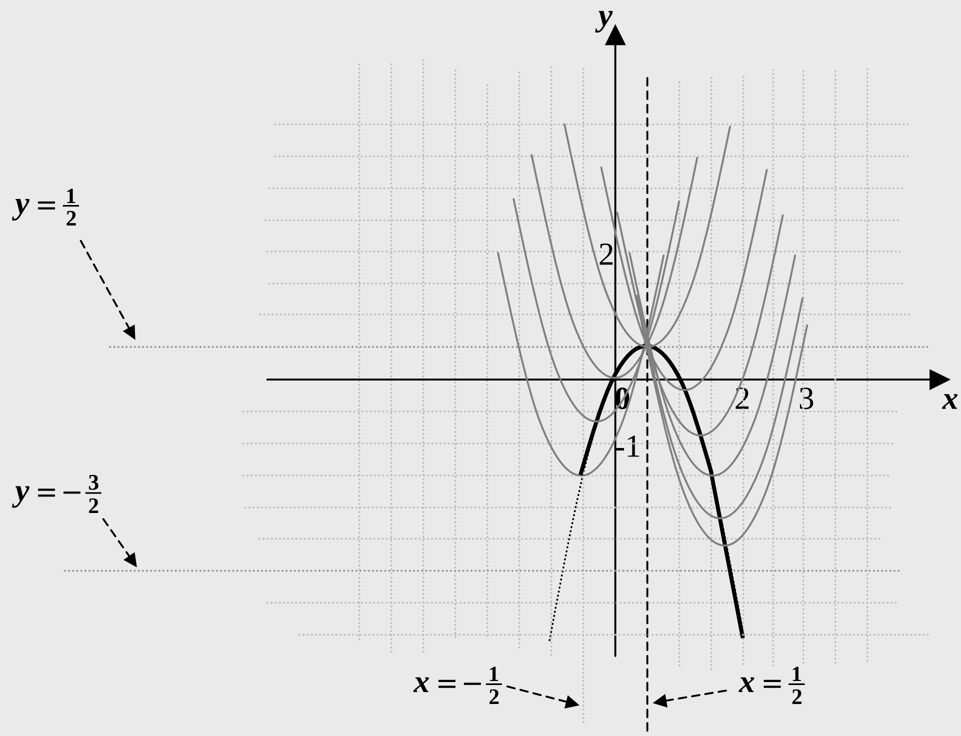
Putting ***g*** in the ***x-y*** system, we replace ***s*** and ***t*** with ***x*** and ***y*** respectively.

Then, we get $\boldsymbol{y = g(x) = -2x^2 + 2x}$ for $\boldsymbol{x \geq -\frac{1}{2}}$.

Fig. 0

$y = g(x) = -2x^2 + 2x = -2(x - \frac{1}{2})^2 + \frac{1}{2}$ for $x \geq -\frac{1}{2}$.

$y = f(x) = 2x^2 - 2cx + c$ for $c \geq -1$.



Now, as can be seen in the graph above, the curve of ***g*** can be said to indicate a trace that the vertex of ***f*** makes as ***c*** changes, and the trace is a parabolic ray.

By the way, as in the cases of equations for lines, we can put in one equation many parabolas or lines, or combinations of both.

Suppose for instance, that $k = 0, 1, 2, \ldots$ and a_k, b_k, etc. are constants.

Then, we can put in one equation,

- Two parabolas, $(a_0y + b_0x^2 + c_0x + d_0)(a_1y + b_1x^2 + c_1x + d_1) = 0$.

- Many parabolas,
$(a_0y + b_0x^2 + c_0x + d_0)(a_1y + b_1x^2 + c_1x + d_1)\ldots(a_ny + b_nx^2 + c_nx + d_n) = 0$.

• A parabola and a line, $(ay + bx^2 + cx + d)(ey + fx + g) = 0$.

• A parabola and two lines, $(ay + bx^2 + cx + d)(ey + fx + g)(hy + ix + j) = 0$.

• Many parabolas and lines, $(a_0y + b_0x^2 + c_0x + d_0)(a_1y + b_1x^2 + c_1x + d_1)\ldots$
$(a_ny + b_nx^2 + c_nx + d_n)(a_1x + b_1y + c_1)(a_2x + b_2y + c_2)\ldots(a_nx + b_ny + c_n) = 0$.

How?

For instance,

$(ay + bx^2 + cx + d)(ey + fx + g) = 0 \Rightarrow ay + bx^2 + cx + d = 0$ or $ey + fx + g = 0$.

$ay + bx^2 + cx + d = 0$ indicates a parabola.

$ey + fx + g = 0$ indicates a line.

In short:

First, $y = 2x^2 - 2cx + c = 2(x^2 - cx + \frac{c^2}{4} - \frac{c^2}{4}) + c = 2(x - \frac{c}{2})^2 - \frac{c^2}{2} + c$ for $c \geq -1$.

Therefore, the vertex is $(\frac{c}{2}, -\frac{c^2}{2} + c)$ where $c \geq -1$.

Next, (s, t) is the vertex. So we get $T: c \longrightarrow (s, t) = (\frac{c}{2}, -\frac{c^2}{2} + c)$ where $c \geq -1$.

Thus, we get $s = \frac{c}{2}$ and $t = -\frac{c^2}{2} + c$ where $c \geq -1$.

So first, $s = \frac{c}{2} \Rightarrow c = 2s$.   Thus, $c \geq -1 \Rightarrow 2s \geq -1 \Rightarrow s \geq -\frac{1}{2}$.

Next, $c = 2s \Rightarrow t = -\frac{c^2}{2} + c = -\frac{4s^2}{2} + 2s = -2s^2 + 2s$ for $s \geq -\frac{1}{2}$.

Therefore, $t = g(s) = -2s^2 + 2s$ for $s \geq -\frac{1}{2}$.

Thus, we get $y = g(x) = -2x^2 + 2x$ for $x \geq -\frac{1}{2}$.

Suggestions or Solutions
To the Problem in the Example 2

Find a function g assuming that c, s, and t are constant, and $c \geq -1$, and that the vertex of a parabola $x = y^2 - 2(c+1)y + 2c^2$ is (s, t), and
$T: c \longrightarrow (s, t)$, where $t = g(s)$.

First, $x = y^2 - 2(c+1)y + 2c^2 = y^2 - 2(c+1)y + (c+1)^2 - (c+1)^2 + 2c^2$
$= \{y - (c+1)\}^2 - (c+1)^2 + 2c^2 = \{y - (c+1)\}^2 - c^2 - 2c - 1 + 2c^2$
$= \{y - (c+1)\}^2 + c^2 - 2c - 1 = \{y - (c+1)\}^2 + (c-1)^2 - 2$ where $c \geq -1$.

Therefore, the vertex is $\{(c-1)^2 - 2, c+1\}$ where $c \geq -1$.

Next, (s, t) is the vertex.
So $T: c \longrightarrow (s, t) = \{(c-1)^2 - 2, c+1\}$ where $c \geq -1$.
Thus, we get $s = (c-1)^2 - 2$ and $t = c + 1$ where $c \geq -1$.
So $s = (c-1)^2 - 2 \Rightarrow (c-1)^2 = s + 2 \Rightarrow c = \pm\sqrt{s+2} + 1$.

So next, we have $c = \sqrt{s+2} + 1 \geq 1$, yet we have $c = -\sqrt{s+2} + 1 \leq 1$, too.
Thus, we need to find the extent of s that can make c always ≥ -1.
$c \geq -1 \Rightarrow c = -\sqrt{s+2} + 1 \geq -1 \Rightarrow \sqrt{s+2} \leq 2 \Rightarrow 0 \leq s + 2 \leq 4 \Rightarrow -2 \leq s \leq 2$.
Meanwhile, we get $t = c + 1 \Rightarrow c = t - 1$.
So we get $t - 1 = \pm\sqrt{s+2} + 1 \Rightarrow t = \pm\sqrt{s+2} + 2$.

Therefore, we get
$t = g(s) = \sqrt{s+2} + 2$ for $s \geq -2$. $t = g(s) = -\sqrt{s+2} + 2$ for $-2 \leq s \leq 2$.

Putting the function g in the x-y system, we get

$y = g(x) = \sqrt{x+2} + 2$ for $x \geq -2$.

$y = g(x) = -\sqrt{x+2} + 2$ for $-2 \leq x \leq 2$.

If not quite sure of the idea behind the processes above, follow the steps below.

This time, too, the $\boldsymbol{T}$ is a parametric transformation where the parameter is $\boldsymbol{c}$.

By means of a parameter, we can connect two functions, each of which is a function of the parameter. The same is true for equations, too. Usually, a parametric transformation is input a number line or a part of it, and outputs a function (or equation). For details on parametric transformations, refer to the book **Function Transformations**.

Let's start now, solving this problem beginning with the vertex. Then, we get first,

$$\boldsymbol{x = y^2 - 2(c+1)y + 2c^2 = y^2 - 2(c+1)y + (c+1)^2 - (c+1)^2 + 2c^2}$$
$$\boldsymbol{= \{y-(c+1)\}^2 - (c+1)^2 + 2c^2 = \{y-(c+1)\}^2 - c^2 - 2c - 1 + 2c^2}$$
$$\boldsymbol{= \{y-(c+1)\}^2 + c^2 - 2c - 1 = \{y-(c+1)\}^2 + (c-1)^2 - 2} \text{ where } \boldsymbol{c \geq -1}.$$

Therefore, the vertex is $\boldsymbol{\{(c-1)^2 - 2,\ c+1\}}$ where $\boldsymbol{c \geq -1}$.

Next, we have $\boldsymbol{T: c \longrightarrow (s, t)}$, where $\boldsymbol{s}$ and $\boldsymbol{t}$ are the coordinates at the vertex.

So $\boldsymbol{s}$ and $\boldsymbol{t}$ will get expressed in terms of the parameter $\boldsymbol{c}$ respectively.

Therefore, $\boldsymbol{s}$ and $\boldsymbol{t}$ will be functions of $\boldsymbol{c}$ respectively.

For instance, we can possibly get $\boldsymbol{s = f(c)}$ and $\boldsymbol{t = g(c)}$ for $\boldsymbol{c \geq -1}$.
Thus, $\boldsymbol{s}$ and $\boldsymbol{t}$ can be connected via the parameter $\boldsymbol{c}$. So let's get them connected now.

To begin with, $\boldsymbol{(s, t)}$ is the vertex, which is $\boldsymbol{\{(c-1)^2 - 2,\ c+1\}}$ where $\boldsymbol{c \geq -1}$.

Thus, we now, have $\boldsymbol{T: c \longrightarrow (s, t) = \{(c-1)^2 - 2,\ c+1\}}$ where $\boldsymbol{c \geq -1}$.

So we get $\boldsymbol{s = (c-1)^2 - 2}$, and $\boldsymbol{t = c + 1}$ where $\boldsymbol{c \geq -1}$.

Thus, we get $\boldsymbol{s = (c-1)^2 - 2 \Rightarrow (c-1)^2 = s + 2 \Rightarrow c = \pm\sqrt{s+2} + 1}$.

And we have $\boldsymbol{t = c + 1}$, too. So we get $\boldsymbol{c = t - 1 = \pm\sqrt{s+2} + 1 \Rightarrow t = \pm\sqrt{s+2} + 2}$.

Next, let's move on to this: $\boldsymbol{c \geq -1}$. Then first of all, $\boldsymbol{c = \pm\sqrt{s+2} + 1}$ has to hold anyway.

Then, what's inside the square root sign has to be $\geq \boldsymbol{0}$. Thus, we get $\boldsymbol{s + 2 \geq 0 \Rightarrow s \geq -2}$.

So next, we get $\boldsymbol{c = \sqrt{s+2} + 1 \geq 1}$, yet we get $\boldsymbol{c = -\sqrt{s+2} + 1 \leq 1}$, too. Why?

That's because we have $s \geq -2$, so we get $\sqrt{s+2} \geq 0$, and $-\sqrt{s+2} \leq 0$.

So since $c = -\sqrt{s+2}+1 \leq 1$ as well as $c = \sqrt{s+2}+1 \geq 1$, we can't always have $c \geq -1$.

Thus, we need to find the extent of s that can make c always ≥ -1.

Finding it, we get $c \geq -1 \Rightarrow c = -\sqrt{s+2}+1 \geq -1 \Rightarrow \sqrt{s+2} \leq 2 \Rightarrow 0 \leq s+2 \leq 4$.

Why not just $s + 2 \leq 4$, though?

That is because what's inside the square root is ≥ 0. Consequently, we get $-2 \leq s \leq 2$.
Therefore, we get

$t = g(s) = \sqrt{s+2}+2$ for $s \geq -2$.

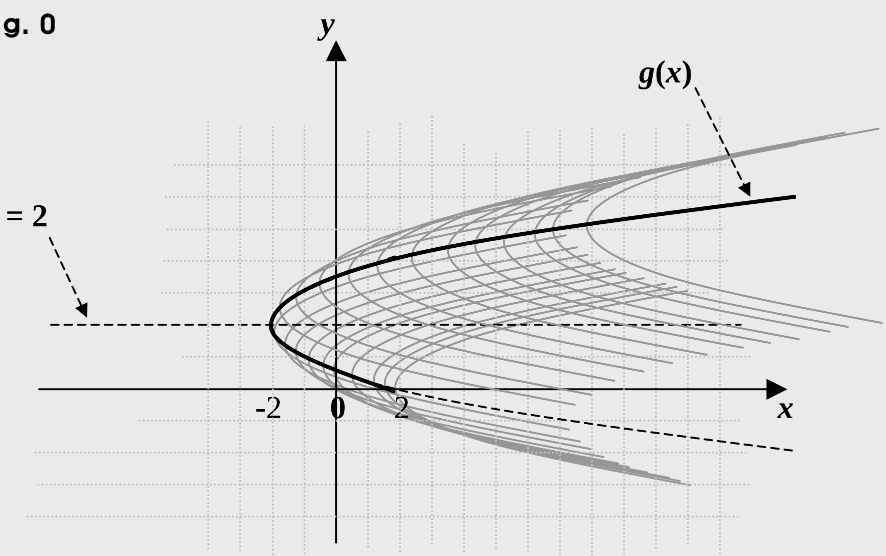
$t = g(s) = -\sqrt{s+2}+2$ for $-2 \leq s \leq 2$.

Replacing s and t with x and y respectively, we can put the function g in the x-y system.
Then, we get

$y = g(x) = \sqrt{x+2}+2$ for $x \geq -2$. Note that this curve is a half parabola (upper-half).

$y = g(x) = -\sqrt{x+2}+2$ for $-2 \leq x \leq 2$. Note that this curve is a parabolic segment.

Fig. 0



In the graph above, each parabola in gray can be indicated by $\boldsymbol{x = y^2 - 2(c + 1)y + 2c^2}$ for each value of $\boldsymbol{c}$, which is $\geq$ **-1**. The curve of $\boldsymbol{g(x)}$ is a parabolic ray in black, and is made of the vertices of all the parabolas in gray. That is, the curve of $\boldsymbol{g}$ is a trace that the vertex of $\boldsymbol{x = y^2 - 2(c + 1)y + 2c^2}$ makes as $\boldsymbol{c}$ changes, and the trace is the parabolic ray.

In short:

First, $x = y^2 - 2(c + 1)y + 2c^2 = y^2 - 2(c + 1)y + (c + 1)^2 - (c + 1)^2 + 2c^2$
$= \{y - (c + 1)\}^2 - (c + 1)^2 + 2c^2 = \{y - (c + 1)\}^2 - c^2 - 2c - 1 + 2c^2$
$= \{y - (c + 1)\}^2 + c^2 - 2c - 1 = \{y - (c + 1)\}^2 + (c - 1)^2 - 2$ where $c \geq -1.$

Therefore, the vertex is $\{(c - 1)^2 - 2, c + 1\}$ where $c \geq -1$.

Next, (s, t) is the vertex.

So $T: c \longrightarrow (s, t) = \{(c - 1)^2 - 2, c + 1\}$ where $c \geq -1$.

Thus, we get $s = (c - 1)^2 - 2$ and $t = c + 1$ where $c \geq -1$.

So $s = (c - 1)^2 - 2 \Rightarrow (c - 1)^2 = s + 2 \Rightarrow c = \pm\sqrt{s+2} + 1$.

So next, we have $c = \sqrt{s+2} + 1 \geq 1$, yet we have $c = -\sqrt{s+2} + 1 \leq 1$, too.

Thus, we need to find the extent of s that can make c always ≥ -1.

$c \geq -1 \Rightarrow c = -\sqrt{s+2} + 1 \geq -1 \Rightarrow \sqrt{s+2} \leq 2 \Rightarrow 0 \leq s + 2 \leq 4 \Rightarrow -2 \leq s \leq 2$.

Meanwhile, we get $t = c + 1 \Rightarrow c = t - 1$.

So we get $t - 1 = \pm\sqrt{s+2} + 1 \Rightarrow t = \pm\sqrt{s+2} + 2$.

Therefore, we get

$t = g(s) = \sqrt{s+2} + 2$ for $s \geq -2$. $t = g(s) = -\sqrt{s+2} + 2$ for $-2 \leq s \leq 2$.

Putting the function g in the x-y system, we get

$y = g(x) = \sqrt{x+2} + 2$ for $x \geq -2$.

$y = g(x) = -\sqrt{x+2} + 2$ for $-2 \leq x \leq 2$.