Examples 1 in the Vertex Form

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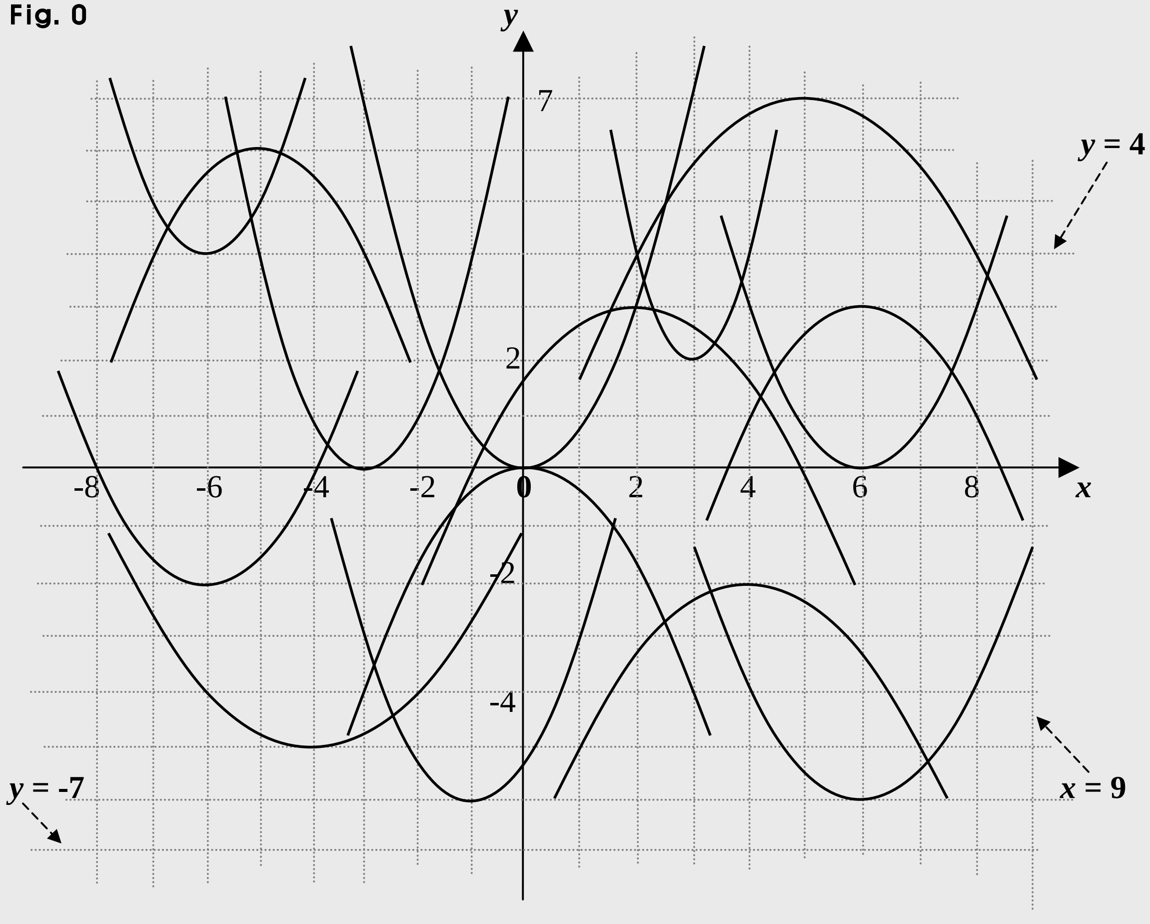
Examples 1 in the Vertex Form

Label each parabola below, and then find the equation of each of the parabolas.

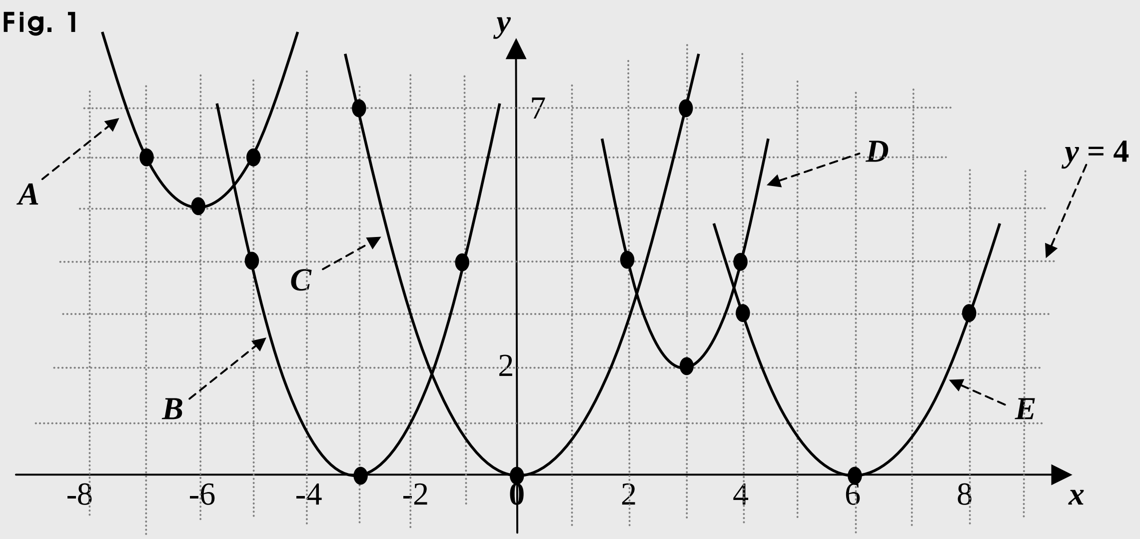
Fig. 0



Suggestions or Solutions
To the Problems in the Examples

Beginning with the parabolas below, we can see first, that the parabola ***A*** has the vertex at (-6, 5), and has a point at (-7, 6) or (-5, 6).

Fig. 1



And we can specify the vertex and a point each of all the other parabolas has.

Next, if a parabola is vertical, and we know the vertex and a point in the parabola, we can get the equation of the parabola using the vertex form as follows.

$\boldsymbol{y - t = a(x - s)^2}$, or $\boldsymbol{y = a(x - s)^2 + t}$, where $\boldsymbol{(s, t)}$ is the vertex.

And thus, beginning with the parabola ***A***, since the vertex is at (-6, 5), we can get first,

$\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x + 6)^2 + 5}$.

Next, since the parabola ***A*** has a point at (-5, 6), we can get

$\boldsymbol{y = a(x + 6)^2 + 5 \Rightarrow 6 = a(-5 + 6)^2 + 5 = a + 5 \Rightarrow a = 6 - 5 = 1}$.

So we get $\boldsymbol{y = (x + 6)^2 + 5}$, which is the equation of the parabola ***A***.

What if we use the point (-7, 6) instead of the point (-5, 6)?

The point (-7, 6) is in the parabola ***A***, too, so we will get the same equation, and we can get it the way as follows.
$\boldsymbol{y = a(x + 6)^2 + 5 \Rightarrow 6 = a(-7 + 6)^2 + 5 = a + 5 \Rightarrow a = 6 - 5 = 1 \Rightarrow y = (x + 6)^2 + 5}$.

Next, moving on to the parabola ***B***, since the vertex is at (-3, 0), we can get first,
$\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x + 3)^2 + 0 \Rightarrow y = a(x + 3)^2}$.

And next, since the parabola ***B*** has a point at (-5, 4), we can get
$\boldsymbol{y = a(x + 3)^2 \Rightarrow 4 = a(-5 + 3)^2 = 4a \Rightarrow a = 4/4 = 1}$.

So we get $\boldsymbol{y = (x + 3)^2}$, which is the equation of the parabola ***B***.

Next, looking at the parabola ***C***, we can see the vertex is at the origin (0, 0), so we can get first, $\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x + 0)^2 + 0 \Rightarrow y = ax^2}$.

And next, since the parabola ***C*** has a point at (3, 7), we can get
$\boldsymbol{y = ax^2 \Rightarrow 7 = 9a \Rightarrow a = 7/9}$.

So we get $\boldsymbol{y = \frac{7}{9}x^2}$, or $\boldsymbol{y = 7x^2/9}$, which is the equation of the parabola ***C***.

Next, examining the parabola ***D***, we can notice that the vertex is (3, 2), so we can get first, $\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x - 3)^2 + 2}$.

And next, the parabola ***D*** has a point at (2, 4), so we can get
$\boldsymbol{y = a(x - 3)^2 + 2 \Rightarrow 4 = a(2 - 3)^2 + 2 = a + 2 \Rightarrow a = 4 - 2 = 2}$.

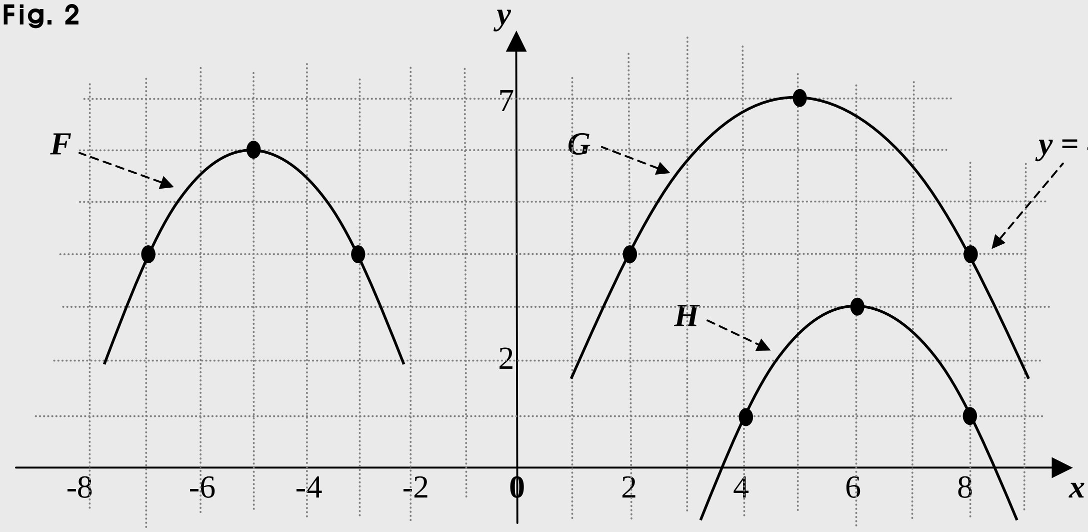
So we get $\boldsymbol{y = 2(x - 3)^2 + 2}$, which is the equation of the parabola ***D***.

Next, looking at the parabola ***E***, we can see that ***E*** has the vertex at (6, 0) and a point at (8, 3), so we can get $\boldsymbol{y = a(x - s)^2 + t \Rightarrow 3 = a(8 - 6)^2 + 0 = 4a \Rightarrow a = 3/4}$.

So we get $\boldsymbol{y = \frac{3}{4}(x - 6)^2}$, or $\boldsymbol{y = 3(x - 6)^2/4}$, which is the equation of the parabola ***E***.

Next, moving on to the parabolas below, we can see that the parabola ***F*** has the vertex at (-5, 6), and a point at (-7, 4) or (-3, 4).

Fig. 2



And we can specify the vertex and a point each of all the other parabolas has.
Next, if a parabola is vertical, and we know the vertex and a point in the parabola, we can get the equation of the parabola using the vertex form as follows.

$\boldsymbol{y - t = a(x - s)^2}$, or $\boldsymbol{y = a(x - s)^2 + t}$, where $\boldsymbol{(s, t)}$ is the vertex.

Beginning thus, with the parabola ***F***, since the vertex is at (-5, 6), we can get first,
$\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x + 5)^2 + 6}$.

And next, since the parabola ***F*** has a point at (-7, 4), we can get
$\boldsymbol{y = a(x + 5)^2 + 6 \Rightarrow 4 = a(-7 + 5)^2 + 6 = 4a + 6 \Rightarrow 4a = 4 - 6 = -2 \Rightarrow a = -1/2}$.

So we get $\boldsymbol{y = (-1/2)(x + 5)^2 + 6}$, which is the equation of the parabola ***F***, and we can put it the way as follows, too, of course. $\boldsymbol{y = -\frac{(x+5)^2}{2} + 6}$, or $\boldsymbol{y = 6 - (x + 5)^2/2}$.

Next, moving on to the parabola ***G***, since the vertex is at (5, 7), we can get first,
$\boldsymbol{y = a(x - s)^2 + t \Rightarrow y = a(x - 5)^2 + 7}$.

And next, since the parabola G has a point at (2, 4), we can get
$y = a(x - 5)^2 + 7 \Rightarrow 4 = a(2 - 5)^2 + 7 = 4a + 7 \Rightarrow 4a = 4 - 7 = -3 \Rightarrow a = -3/4$.

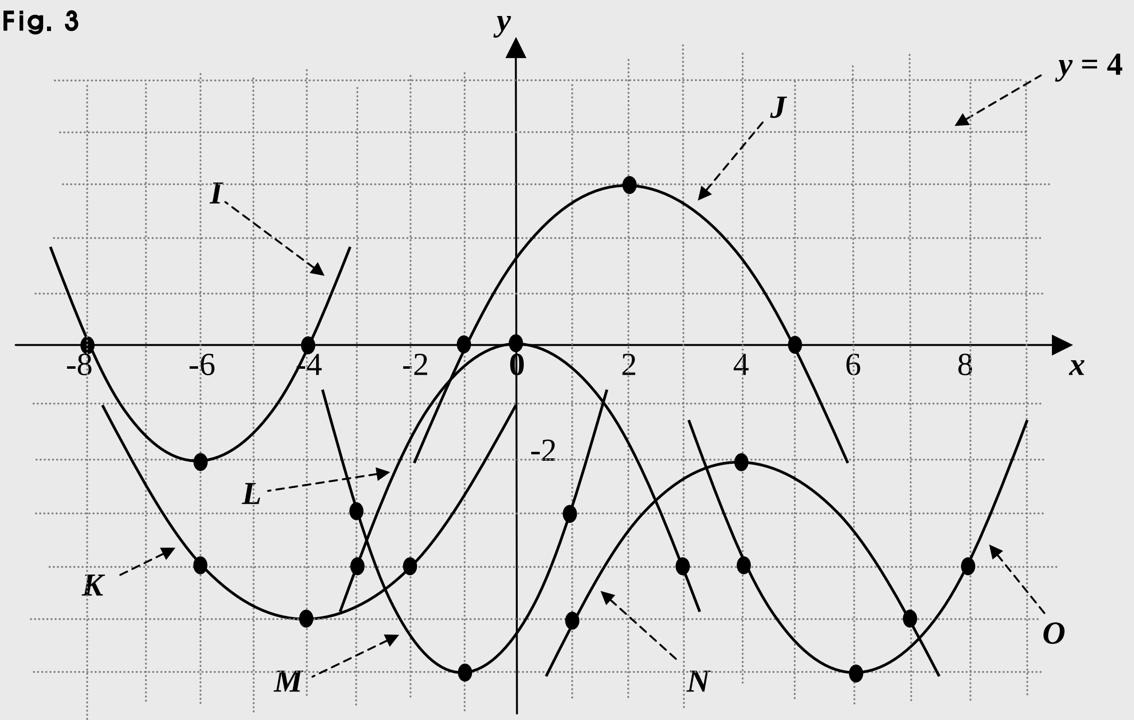
So we get $y = (-3/4)(x - 5)^2 + 7$, which is the equation of the parabola G, and we can put it this way, too, of course. $y = \frac{-3(x-5)^2}{4} + 7$, or $y = 7 - 3(x - 5)^2/4$.

Next, looking at the parabola H, we can see that H has the vertex at (6, 3) and a point at (8, 1), so we can get $y = a(x - s)^2 + t \Rightarrow 1 = a(8 - 6)^2 + 3 = 4a + 3 \Rightarrow a = -1/2$.

So we get $y = (-1/2)(x - 6)^2 + 3$, which is the equation of the parabola H, and, of course, we can put it this way, too: $y = -(x - 6)^2/2 + 3$, or $y = 3 - (x - 6)^2/2$.

Next, moving on to the other parabolas, we can put them the way below, and can see first, that the parabola I has the vertex at (-6, -2), and a point at (-8, 0) or (-4, 0).

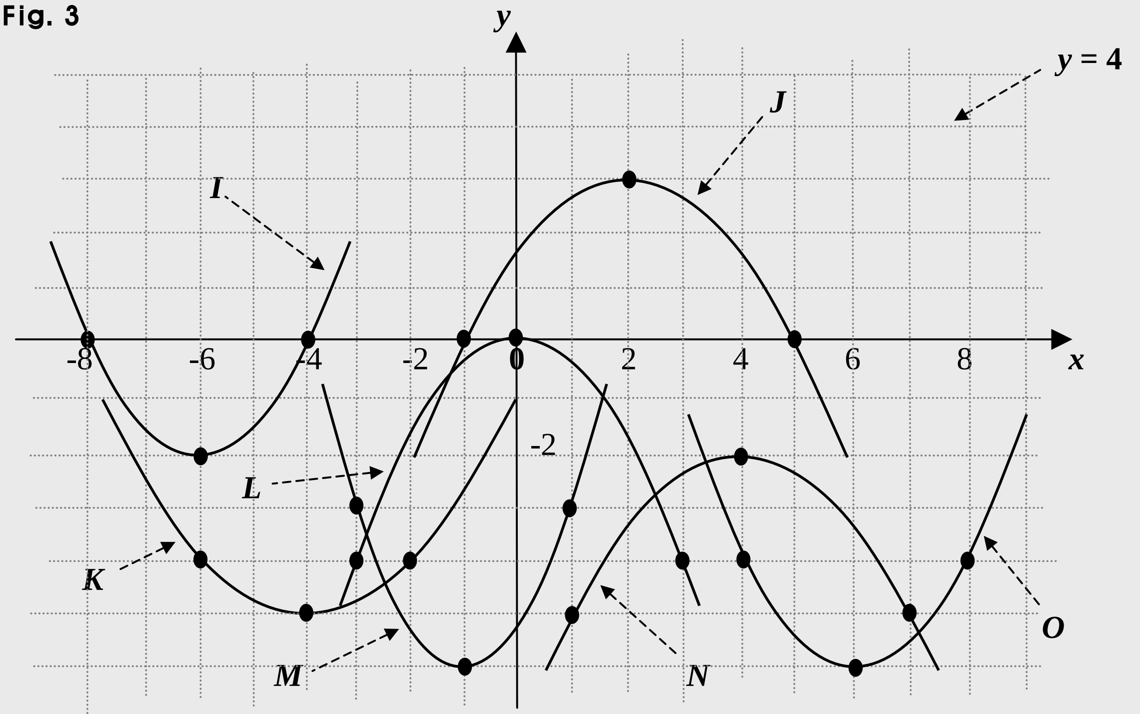
Fig. 3



And if a parabola is vertical, and we know the vertex and a point in the parabola, we can get the equation of the parabola using the vertex form as follows.

$y - v = m(x - u)^2$, or $y = m(x - u)^2 + v$, where (u, v) is the vertex.

Fig. 3



So beginning with the parabola ***I***, since the vertex is at (-6, -2), we can get first,

$\boldsymbol{y = m(x - u)^2 + v \Rightarrow y = m(x + 6)^2 - 2}$.

And next, since the parabola ***I*** has a point at (-8, 0), we can get

$\boldsymbol{y = m(x + 6)^2 - 2 \Rightarrow 0 = m(-8 + 6)^2 - 2 = 4m - 2 \Rightarrow 4m = 2 \Rightarrow m = 1/2}$.

So we get $\boldsymbol{y = (1/2)(x + 6)^2 - 2}$, which is the equation of the parabola ***I***, and can be put this way, too: $\boldsymbol{y = (x + 6)^2/2 - 2}$, or $\boldsymbol{y + 2 = (x + 6)^2/2}$.

Next, moving on to the parabola ***J***, since the vertex is at (2, 3), we can get first,

$\boldsymbol{y = m(x - u)^2 + v \Rightarrow y = m(x - 2)^2 + 3}$.

And next, since the parabola ***I*** has a point at (5, 0), we can get

$\boldsymbol{y = m(x - 2)^2 + 3 \Rightarrow 0 = m(5 - 2)^2 + 3 = 9m + 3 \Rightarrow 9m = -3 \Rightarrow m = -1/3}$.

So we get $\boldsymbol{y = (-1/3)(x - 2)^2 + 3}$, which is the equation of the parabola ***J***, and can be put this way, too: $\boldsymbol{y = -(x - 2)^2/3 + 3}$, $\boldsymbol{y = 3 - (x - 2)^2/3}$, or $\boldsymbol{y - 3 = -(x - 2)^2/3}$.

Next, looking at the parabola ***K***, we can see the vertex is at (-4, -5), so we can get first, $\boldsymbol{y = m(x - u)^2 + v \Rightarrow y = m(x + 4)^2 - 5}$.

And next, the parabola ***K*** has a point at (-2, -4), so we can get
$\boldsymbol{y = m(x + 4)^2 - 5 \Rightarrow -4 = m(-2 + 4)^2 - 5 = 4m - 5 \Rightarrow 4m = 5 - 4 = 1 \Rightarrow m = 1/4}$.

So we get $\boldsymbol{y = (1/4)(x + 4)^2 - 5}$, which is the equation of the parabola ***K***, and can be put this way, too: $\boldsymbol{y = (x + 4)^2/4 - 5}$, or $\boldsymbol{y + 5 = (x + 4)^2/4}$.

Next, examining the parabola ***L***, we can notice that the vertex is the origin (0, 0), so we can get first, $\boldsymbol{y = m(x - u)^2 + v \Rightarrow y = m(x + 0)^2 - 0 \Rightarrow y = mx^2}$.

And next, the parabola ***L*** has (-3, -4), so we can get $\boldsymbol{y = mx^2 \Rightarrow -4 = 9m \Rightarrow m = -4/9}$.

So we get $\boldsymbol{y = (-4/9)x^2}$, which is the equation of the parabola ***L***, and of course, can be put this way, too: $\boldsymbol{y = -4x^2/9}$.

Examining next, the parabola ***M***, we can see its vertex is at (-1, -6), and it has a point at (1, -3), so we get $\boldsymbol{y = m(x - u)^2 + v \Rightarrow -3 = m(1 + 1)^2 - 6 = 4m - 6 \Rightarrow 4m = 6 - 3 = 3 \Rightarrow m = 3/4 \Rightarrow y = (3/4)(x + 1)^2 - 6}$, which is the equation of the parabola ***M***, and of course, can be put this way, too: $\boldsymbol{y = 3(x + 1)^2/4 - 6}$, or $\boldsymbol{y + 6 = 3(x + 1)^2/4}$.

Looking at next, the parabola ***N***, we can see the vertex is at (4, -2), and it has a point at (1, -5), so we get $\boldsymbol{y = m(x - u)^2 + v \Rightarrow -5 = m(1 - 4)^2 - 2 = 9m - 2 \Rightarrow 9m = 2 - 5 = -3 \Rightarrow m = -1/3 \Rightarrow y = (-1/3)(x - 4)^2 - 2}$, which is the equation of ***N***, and of course, can be put this way, too: $\boldsymbol{y = -(x - 4)^2/3 - 2}$, $\boldsymbol{y = 2 - (x - 4)^2/3}$, or $\boldsymbol{y - 2 = -(x - 4)^2/3}$.

And next, looking at the parabola ***O***, we can see the vertex is at (6, -6), and it has a point at (8, -4), so we get $\boldsymbol{y = m(x - u)^2 + v \Rightarrow -4 = m(8 - 6)^2 - 6 = 4m - 6 \Rightarrow 4m = 6 - 4 = 2 \Rightarrow m = 1/2 \Rightarrow y = (1/2)(x - 6)^2 - 6}$, which is the equation of ***O***, and of course, can be put this way, too: $\boldsymbol{y = (x - 6)^2/2 - 6}$, or $\boldsymbol{y + 6 = (x - 6)^2/2}$.