Examples 3 in the Vertex Form

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Examples 3 in the Vertex Form

Assuming ***V*** is the vertex of a parabola, and ***F*** is the focus, find the equation of each of the parabolas as follows.

0. ***V*(0, 0)** and ***F*(3, 0)**

1. ***V*(0, -1)** and ***F*(3, -1)**

2. ***V*(1, 0)** and ***F*(3, 0)**

3. ***V*(1, -1)** and ***F*(3, -1)**

Suggestions or Solutions
To the Problem in the Example 0

The vertex is the origin, and the focus is (3, 0).

Suppose first, the directrix is a line $\boldsymbol{x = d}$.

Then, $\mathbf{0} = \frac{d+3}{2} \Rightarrow \boldsymbol{d} = \mathbf{-3}$, since the vertex is the midpoint between the directrix and focus.

Suppose next, that $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola.

Then, since $\boldsymbol{P}$ is the same distance away from the focus and directrix, we get

$\{x - (-3)\}^2 + (y - y)^2 = (x - 3)^2 + (y - 0)^2 \Rightarrow (x + 3)^2 = (x - 3)^2 + t^2$

$\Rightarrow t^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2x \cdot 6 = 12x$

$\Rightarrow y^2 = 12x$, which is the parabola.

If not quite sure of the idea behind the processes above, follow the steps below.

There can be many ways to find a parabola.

We can get the parabola using the definition for parabolas, too, because it says what a parabola is. The definition is saying that a parabola is a set of points, from each of which, <u>the distance to the focus is the same as the distance to the directrix</u>.

What then, do we need to begin with?

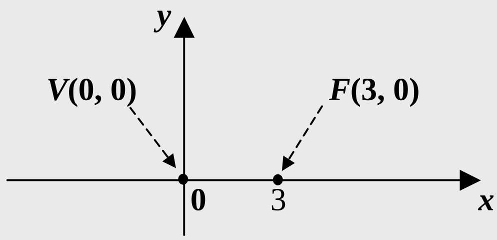
We need to find the directrix, first.   Why?

We need it if we use the definition. … Instead of just thinking, putting ideas in a graph, we can see better the solution’s whereabouts. So it’s a good idea to begin with a graph.

Making a graph though, you don’t have to make it complete at once. Just begin with the information given first, such as points and values. Then, add more points or values if relevant or helping, or add lines or curves that seem to fit the situation.

So first, you can begin a graph the way as follows.

Fig. 0



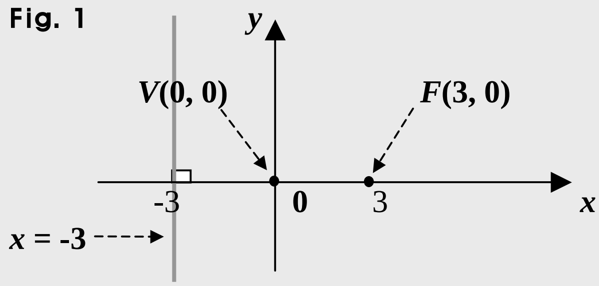
How then, can we get the directrix?

We can find the directrix using the facts below.

The directrix is perpendicular to the axis of symmetry.
The axis of symmetry has the focus and the vertex.
And the vertex is the midpoint between the focus and the directrix.

So to the graph above, we can add the directrix the way as follows.

Fig. 1



So the directrix is a line $\boldsymbol{x = -3}$.

What then, is the next? That is, what are we going to do with the directrix?

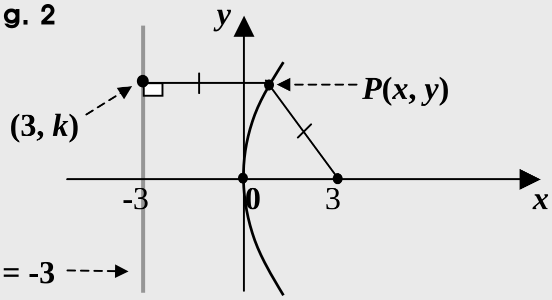
We can now use the definition, which is saying that each point in a parabola is the same distance away from the focus and the directrix. How then, can we use the definition?

In math, we have a point that is random, but represents every point in a curve.
And we call such a point an arbitrary point in the curve.

So we can put the definition this way, too: the arbitrary point in a parabola is the same distance away from the focus and the directrix.

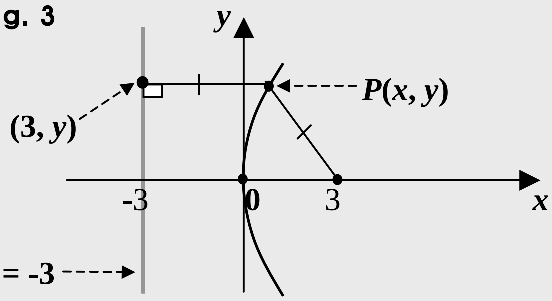
And we know a parabola is bent toward the focus.
So assuming now ***P*(*x*, *y*)** is an arbitrary point in the parabola, and using the fact above, we can put in the graph, the parabola and some relevant information the way below.

Fig. 2



Note that the two points **(3, *k*)** and ***P*(*x*, *y*)** are in a line.
So their ***y***-coordinates are the same, and thus, the ***y***-coordinate at the point **(3, *k*)** is ***y***, too.
So we can now put all the ideas in a graph the way below.

Fig. 3



Now, we can start taking advantage of the fact that the two distances from the arbitrary point to the directrix and the focus are the same.

So assuming first, ***U*** is the distance from ***P*(*x*, *y*)** to the directrix, ***V*** is the distance from ***P*(*x*, *y*)** to the focus, and using the distance formula, we get

$U^2 = \{x - (-3)\}^2 + (y - y)^2 = (x + 3)^2$, and $V^2 = (x - 3)^2 + (y - 0)^2 = (x - 3)^2 + y^2$.

And next, since the two distances are the same, we get $(x + 3)^2 = (x - 3)^2 + y^2$
$\Rightarrow y^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2y \cdot 6 = 12x \Rightarrow y^2 = 12x$.

Next, finding the equation of a curve, we find the connective equation between the coordinates of the arbitrary point in the curve. And we call the connective equation, the equation of the curve.

Now, $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, and thus, can represent all the points in the parabola. So the connective equation between the coordinates of $\boldsymbol{P(x, y)}$ explains each of all the points in the parabola.

And the connective equation is $\boldsymbol{y^2 = 12x}$, where $\boldsymbol{x}$ is the $\boldsymbol{x}$-coordinate at $\boldsymbol{P(x, y)}$, and $\boldsymbol{y}$ is the $\boldsymbol{y}$-coordinate at $\boldsymbol{P(x, y)}$.

Therefore, $\boldsymbol{y^2 = 12x}$ is the equation of the parabola in the figure above.

- What if we approach the solution without using a graph?

We can get the solution with no use of a graph, too, of course.
So let's get this time, the solution with reasoning processes only with no graph.

To begin with, we can get the parabola using the definition for parabolas, because it says what a parabola is. The definition is saying that a parabola is a set of points, from each of which, the distance to the focus is the same as the distance to the directrix.

What then, do we need to begin with?

We need to find the directrix, first.   Why?

We want to use the fact that each point in a parabola is the same distance from the directrix and the focus. We are given the focus, but not the directrix.
How then, can we get the directrix?

The directrix is perpendicular to the axis of symmetry, so finding the axis of symmetry, we can get the directrix. Where then, is the axis of symmetry?

We are given two points, which are the vertex and the focus, both of which have to be in the same line, which is called the axis of symmetry.

Now, the vertex is at the origin, so it is at **(0, 0)**.
And the focus is at **(3, 0)**. What then, is the axis of symmetry?

Looking at the coordinates at the vertex and focus, we can notice that the ***y***-coordinates are the same, and are 0, so what is the line the two points are located at?

In a line $\boldsymbol{y = 0}$, the ***y***-coordinate of every point is 0, so the two points are in the line $\boldsymbol{y = 0}$, which is therefore, the axis of symmetry, and is in fact, the ***x***-axis.

Now, the directrix is perpendicular to the axis of symmetry, which is a line $\boldsymbol{y = 0}$, so it is perpendicular to the ***x***-axis. Thus, the directrix meets the ***x***-axis at 90^{o}, and is parallel to the ***y***-axis, of course. How then, can we get the directrix?

The vertex of a parabola is the midpoint between the directrix and focus, and the vertex and focus are in the same line called the axis of symmetry.

And we know the directrix is a line perpendicular to the ***x***-axis.
So we can assume that the directrix is a line $\boldsymbol{x = d}$, and since the vertex is the midpoint between the directrix and the focus, we get

$\boldsymbol{0 = \frac{d+3}{2} \Rightarrow d = -3}$. Therefore, the directrix is a line $\boldsymbol{x = -3}$.

What then, is the next? That is, what are we going to do with the directrix?

We can now use the definition, saying that each point in a parabola is the same distance away from the focus and the directrix. How then, can we use the definition?

In math, we have a point that is random, but represents every point in a curve.
And we call such a point an arbitrary point in the curve.

So we can put the definition this way, too: the arbitrary point in a parabola is the same distance away from the focus and the directrix.

Now, we can start taking advantage of the fact that the distance from the arbitrary point to both the directrix and the focus is the same.

So assuming now $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, $\boldsymbol{U}$ is the distance from $\boldsymbol{P}$ to the directrix, and $\boldsymbol{V}$ is the distance from $\boldsymbol{P}$ to the focus, and using the distance formula, we get

$\boldsymbol{U^2 = \{x - (-3)\}^2 + (y - y)^2 = (x + 3)^2}$, and $\boldsymbol{V^2 = (x - 3)^2 + (y - 0)^2 = (x - 3)^2 + y^2}$.

And next, since the two distances are the same, we get $\boldsymbol{(x + 3)^2 = (x - 3)^2 + y^2}$
$\boldsymbol{\Rightarrow y^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2y \cdot 6 = 12x \Rightarrow y^2 = 12x}$.

Next, finding the equation of a curve, we find the connective equation between the coordinates of the arbitrary point in the curve. And we call the connective equation, the equation of the curve.

Now, $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, and thus, can represent all the points in the parabola. So the <u>connective equation</u> between the coordinates of $\boldsymbol{P(x, y)}$ explains each of all the points in the parabola.

And the connective equation is $\boldsymbol{y^2 = 12x}$, where $\boldsymbol{x}$ is the $\boldsymbol{x}$-coordinate at $\boldsymbol{P(x, y)}$, and $\boldsymbol{y}$ is the $\boldsymbol{y}$-coordinate at $\boldsymbol{P(x, y)}$.

Therefore, $\boldsymbol{y^2 = 12x}$ is the equation of the parabola in the figure above.

Suggestions or Solutions
To the Problem in the Example 1

The vertex is (0, -1), and the focus is (3, -1).

The vertex is the midpoint between the directrix and focus, so assuming the directrix is a line $x = d$, we get $\mathbf{0} = \frac{d+3}{2} \Rightarrow d = -3$.

Assuming next, $P(x, y)$ is an arbitrary point in the parabola, since P is the same distance away from the focus and directrix, we get

$$\{x - (-3)\}^2 + (y - y)^2 = (x - 3)^2 + \{y - (-1)\}^2 \Rightarrow (x + 3)^2 = (x - 3)^2 + (y + 1)^2$$

$$\Rightarrow (y + 1)^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2x \cdot 6 = 12x$$

$\Rightarrow (y + 1)^2 = 12x$, which is the parabola.

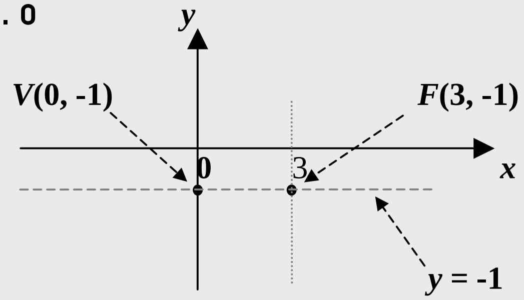
If not quite sure of the idea behind the processes above, follow the steps below.

We can get the parabola using the definition for parabolas. And the definition is saying that a parabola is a set of points, from each of which, <u>the distance to the focus is the same as the distance to the directrix</u>.

So we need the directrix. … Instead of just thinking, putting ideas in a graph, we can see better the solution's whereabouts. So it's a good idea to begin with a graph.

Making a graph, just start with the information given. It can be points, values, lines, etc. Then, try adding more if relevant or helping, or try adding lines or curves that seem to fit the situation.   So first, you can begin a graph the way as follows.

Fig. 0



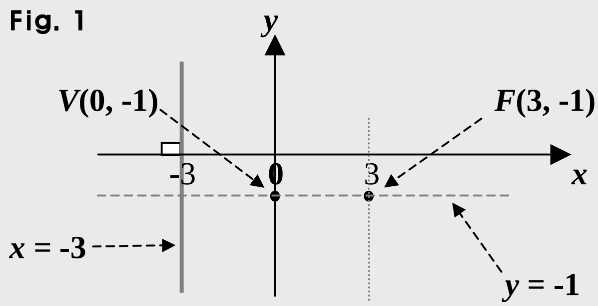
We know that the axis of symmetry has the focus and the vertex.
So we can see now, the axis of symmetry is a line $\boldsymbol{y = -1}$. Where then, is the directrix?

We can find the directrix using the facts below.

The directrix is perpendicular to the axis of symmetry.
And the vertex is the midpoint between the focus and the directrix.

So to the graph above, we can add the directrix the way as follows.

Fig. 1

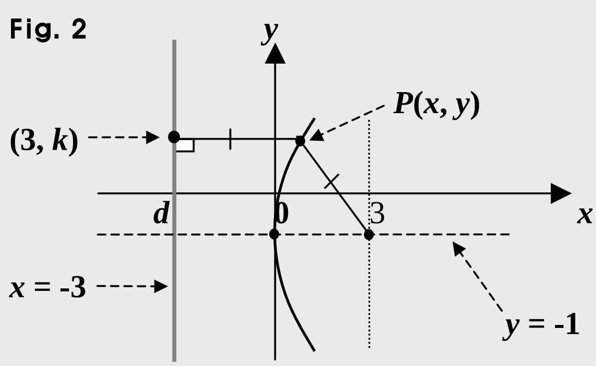


So the directrix is a line $\boldsymbol{x = -3}$.

We now we have the directrix, which is a line $\boldsymbol{x = -3}$, so we can now use the definition, which is saying that each point in a parabola is the same distance away from the focus and the directrix. And we can put the definition this way, too: the arbitrary point in a parabola is the same distance away from the focus and the directrix.

So assuming now $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, and using the fact above, we can put in the graph, the parabola and some relevant information the way below.

Fig. 2

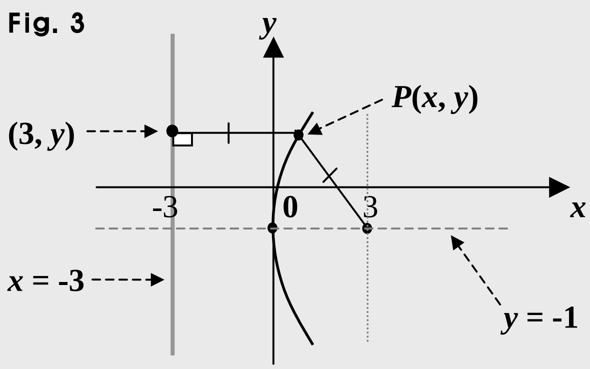


Note that the two points $\boldsymbol{(d, y)}$ ar
$\boldsymbol{P(x, y)}$ are in a line.

And the line $\boldsymbol{x = -1}$ is the directr

So we can now put all the ideas in a graph the way below.

Fig. 3



Now, we can start taking advantage of the fact that the two distances from the arbitrary point to the directrix and the focus are the same.

So assuming first, U is the distance from $P(x, y)$ to the directrix, V is the distance from $P(x, y)$ to the focus, and using the distance formula, we get

$U^2 = \{x - (-3)\}^2 + (y - y)^2 = (x + 3)^2$, and $V^2 = (x - 3)^2 + (y - 0)^2 = (x - 3)^2 + y^2$.

And next, since the two distances are the same, we get $(x + 3)^2 = (x - 3)^2 + y^2$

$\Rightarrow y^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2y \cdot 6 = 12x \Rightarrow y^2 = 12x$.

Now, $P(x, y)$ is an arbitrary point in the parabola, and thus, can represent all the points in the parabola. So the <u>connective equation</u> between the coordinates of $P(x, y)$ explains each of all the points in the parabola.

And the connective equation is $y^2 = 12x$, where x is the x-coordinate at $P(x, y)$, and y is the y-coordinate at $P(x, y)$.

Therefore, $y^2 = 12x$ is the equation of the parabola in the figure above.

• And we can get the solution with no use of a graph, too.
So for practice purposes, let's get this time, the solution with reasoning processes only with no graph.

To begin with, what is a parabola?

A parabola is a set of points, and each and each point in a parabola is the same distance away from the focus and directrix.

So using the fact above, we can get the parabola. We are given the focus, but not the directrix. So we need to find the directrix, first. How then, can we get the directrix?

The directrix is perpendicular to the axis of symmetry, so finding the axis of symmetry, we can get the directrix. Where then, is the axis of symmetry?

We are given two points, which are the vertex and the focus, both of which have to be in the same line, which is called the axis of symmetry.

Now, the vertex is **(0, -1)**, and the focus is **(3, -1)**.

So the two points share the same ***y***-coordinate, which is -1.
What then, is the line where the two points both are located?

It is a line $\boldsymbol{y = -1}$, since the ***y***-coordinate of every point in the line $\boldsymbol{y = -1}$ is -1.
And we know the two points are in the line called the axis of symmetry.

So the axis of symmetry is a line $\boldsymbol{y = -1}$, which is parallel to the ***x***-axis.
How then, can we get the directrix?

The directrix is perpendicular to the axis of symmetry.

And we know in this example, the axis of symmetry is parallel to the ***x***-axis.
So the directrix is perpendicular to the ***x***-axis.

So we can assume that the directrix is a line $\boldsymbol{x = d}$.

And the vertex is the midpoint between the directrix and the focus.

So since the directrix is $\boldsymbol{x = d}$, the vertex is **(0, -1)**, and the focus is **(3, -1)**, we get

$\mathbf{0} = \frac{d+3}{2} \Rightarrow \boldsymbol{d} = \mathbf{-3}$. Thus, the directrix is a line $\boldsymbol{x} = \mathbf{-3}$.

What then, are we going to do with the directrix?

We can now use it with the fact that the distance from an arbitrary point in a parabola to both the directrix and the focus is the same.

So assuming now, $\boldsymbol{U}$ is the distance from $\boldsymbol{P(x, y)}$ to the directrix, $\boldsymbol{V}$ is the distance from $\boldsymbol{P(x, y)}$ to the focus, and using the distance formula, we get

$U^2 = \{x - (-3)\}^2 + (y - y)^2 = (x + 3)^2$, and $V^2 = (x - 3)^2 + \{y - (-1)\}^2 = (x - 3)^2 + (y + 1)^2$.

And next, since the two distances are the same, we get $(x + 3)^2 = (x - 3)^2 + (y + 1)^2$

$\Rightarrow (y + 1)^2 = (x + 3)^2 - (x - 3)^2 = (x + 3 + x - 3)(x + 3 - x + 3) = 2x \cdot 6 = 12x$

$\Rightarrow (y + 1)^2 = 12x$.

Now, $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, and thus, can represent all the points in the parabola. So the connective equation between the coordinates of $\boldsymbol{P(x, y)}$ explains each of all the points in the parabola.

And the connective equation is $(y + 1)^2 = 12x$, where $\boldsymbol{x}$ is the $\boldsymbol{x}$-coordinate at $\boldsymbol{P(x, y)}$, and $\boldsymbol{y}$ is the $\boldsymbol{y}$-coordinate at $\boldsymbol{P(x, y)}$.

Therefore, $(y + 1)^2 = 12x$ is the equation of the parabola we want.

Suggestions or Solutions
To the Problem in the Example 2

The vertex is (1, 0), and the focus is (3, 0).

Assuming first, the directrix is a line $\boldsymbol{x = d}$, we get $\mathbf{1} = \frac{d+3}{2} \Rightarrow \boldsymbol{d} = \mathbf{-1}$.

Therefore, the directrix is a line $\boldsymbol{x} = \mathbf{-1}$.

Suppose now, $\boldsymbol{P(x, y)}$ is an arbitrary the point in the parabola.
Then, $\boldsymbol{P}$ is the same distance away from the focus and directrix, so we get

$$\{x - (-1)\}^2 + (y - y)^2 = (x - 3)^2 + (y - 0)^2 \Rightarrow (x + 1)^2 = (x - 3)^2 + y^2$$

$$\Rightarrow y^2 = (x + 1)^2 - (x - 3)^2 = (x + 1 + x - 3)(x + 1 - x + 3) = 2(x - 1)(4) = 8(x - 1)$$

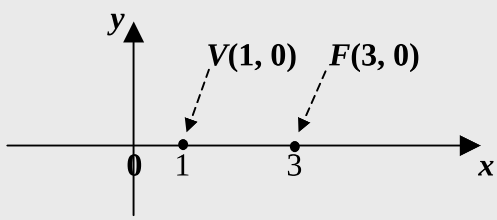
$\Rightarrow y^2 = 8(x - 1)$, which is the equation of the parabola.

If not quite sure of the idea behind the processes above, follow the steps below.

As mentioned in the previous examples, instead of just thinking, putting ideas in a graph, we can see better the solution's whereabouts. So it's a good idea to begin with a graph.

We can start with information as points or values given, or lines or curves, etc.
So first, we can begin a graph the way as follows.

Fig. 0

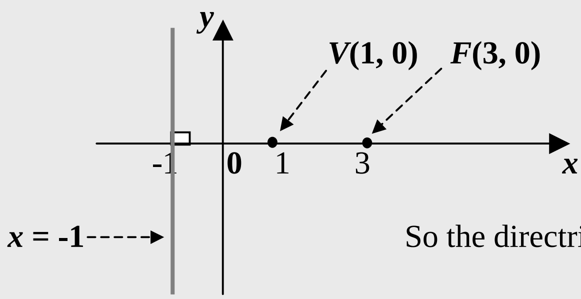


Then, we can now see that the axis of symmetry is the $\boldsymbol{x}$-axis, since the vertex and the focus are in the axis of symmetry.

We can get the parabola using the definition, which says each point in a parabola is the same distance away from the focus and the directrix. So we want to get the directrix.

We know the directrix meets the axis of symmetry at 90^{o}, and the vertex is the midpoint between the focus and the directrix.
So we can put the directrix in the graph, the way below.

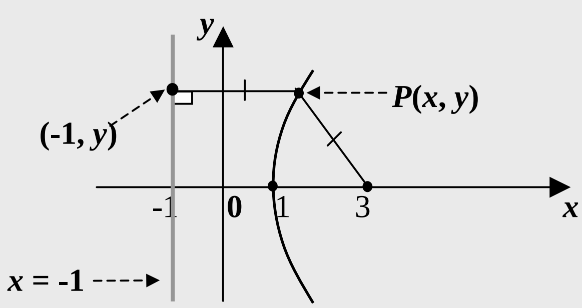
Fig. 1



So the directrix is a line $\boldsymbol{x = -1}$.

And assuming $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, we can add to the graph, the point $\boldsymbol{P}$ and the parabola the way below.

Fig. 2



We now have the directrix. So we can now get the parabola using the fact that each point in a parabola is the same distance away from the focus and the directrix.

So assuming now, $\boldsymbol{U}$ is the distance from $\boldsymbol{P(x, y)}$ to the directrix, $\boldsymbol{V}$ is the distance from $\boldsymbol{P(x, y)}$ to the focus, and using the distance formula, we get

$\boldsymbol{U^2 = \{x - (-1)\}^2 + (y - y)^2 = (x + 1)^2}$, and $\boldsymbol{V^2 = (x - 3)^2 + (y - 0)^2 = (x - 3)^2 + y^2}$.

And next, since the two distances are the same, we get $\mathbf{(x + 1)^2 = (x - 3)^2 + y^2}$

$\Rightarrow \mathbf{y^2 = (x + 1)^2 - (x - 3)^2 = (x + 1 + x - 3)(x + 1 - x + 3) = 2(x - 1)4 = 8(x - 1)}$

$\Rightarrow \mathbf{y^2 = 8(x - 1)}$.

And the equation above is the connective equation between the coordinates of $\boldsymbol{P(x, y)}$.

So $\mathbf{y^2 = 8(x - 1)}$ is the equation of the parabola in the figure above.

In short:

Assuming first, the directrix is a line $\boldsymbol{x = d}$, we get $\mathbf{1 = \frac{d+3}{2} \Rightarrow d = -1}$.

Therefore, the directrix is a line $\boldsymbol{x = -1}$.

Suppose now, $\boldsymbol{P(x, y)}$ is an arbitrary the point in the parabola.
Then, $\boldsymbol{P}$ is the same distance away from the focus and directrix, so we get

$\mathbf{\{x - (-1)\}^2 + (y - y)^2 = (x - 3)^2 + (y - 0)^2 \Rightarrow (x + 1)^2 = (x - 3)^2 + y^2}$

$\Rightarrow \mathbf{y^2 = (x + 1)^2 - (x - 3)^2 = (x + 1 + x - 3)(x + 1 - x + 3) = 2(x - 1)(4) = 8(x - 1)}$

$\Rightarrow \mathbf{y^2 = 8(x - 1)}$, which is the equation of the parabola.

Suggestions or Solutions
To the Problem in the Example 3

The vertex is (1, -1), and the focus is (3, -1).

Assuming first, the directrix is a line $\boldsymbol{x = d}$, we get $\mathbf{1 = \frac{d+3}{2} \Rightarrow d = -1}$.

So assuming next, $\boldsymbol{P(x, y)}$ an arbitrary point in the parabola, since $\boldsymbol{P}$ is the same distance away from the focus and directrix, we get

$$\{x-(-1)\}^2+(y-y)^2=(x-3)^2+\{y-(-1)\}^2 \Rightarrow (x+1)^2=(x-3)^2+(y+1)^2$$

$$\Rightarrow (y+1)^2=(x+1)^2-(x-3)^2=(x+1+x-3)(x+1-x+3)=2(x-1)(4)=8(x-1)$$

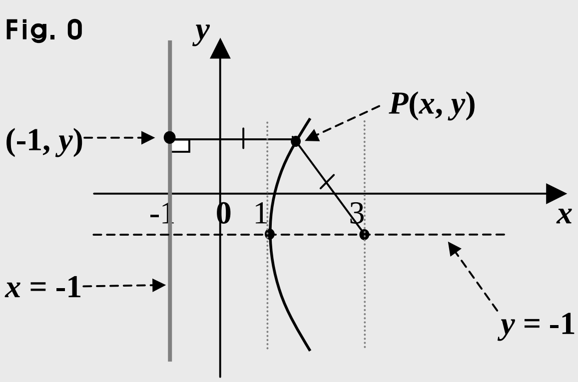
$\Rightarrow \boldsymbol{(y+1)^2 = 8(x-1)}$, which is the parabola.

If not quite sure of the idea behind the processes above, follow the steps below.

As mentioned in the previous examples, instead of just thinking, putting ideas in a graph, we can see better the solution's whereabouts. So it's a good idea to begin with a graph.

If getting used to this type of problems, and doing practices enough, you can quickly get to the graph as shown in the figure below, beginning with the vertex and the focus, of course.

Fig. 0



Note that the two points $\boldsymbol{(d, y)}$ and $\boldsymbol{P(x, y)}$ are in a line.

And the line $\boldsymbol{x = -1}$ is the directrix.

Not quite sure how to get it?

You can get the graph above using the facts below.

Each point in a parabola is the same distance away from the focus and the directrix.

Between the focus and the directrix, the midpoint is the vertex.

And the vertex and the focus are in the axis of symmetry perpendicular to the directrix.

And in fact, once you've got the directrix, using the definition for parabolas, we can get the parabola.

So assuming now, $\boldsymbol{P(x, y)}$ is an arbitrary point in the parabola, $\boldsymbol{U}$ is the distance from $\boldsymbol{P}$ to the directrix, and $\boldsymbol{V}$ is the distance from $\boldsymbol{P}$ to the focus, and using the distance formula, we get

$\boldsymbol{U^2 = \{x - (-1)\}^2 + (y - y)^2 = (x + 1)^2}$, and $\boldsymbol{V^2 = (x - 3)^2 + \{y - (-1)\}^2 = (x - 3)^2 + (y + 1)^2}$.

And next, since the two distances are the same, we get $\boldsymbol{(x + 1)^2 = (x - 3)^2 + (y + 1)^2}$

$\Rightarrow \boldsymbol{(y + 1)^2 = (x + 1)^2 - (x - 3)^2 = (x + 1 + x - 3)(x + 1 - x + 3) = 2(x - 1)(4) = 8(x - 1)}$

$\Rightarrow \boldsymbol{(y + 1)^2 = 8(x - 1)}$.

So $\boldsymbol{(y + 1)^2 = 8(x - 1)}$ is the equation of the parabola in the figure above.

• And of course, we can get the solution with no use of a graph, too.

So for practice purposes, let's get this time, the solution with reasoning processes only with no graph.

A parabola is symmetric about a line, so the line is called the axis of symmetry.

And the axis of symmetry passes through the vertex and the focus.

What then, is the axis of symmetry, in this example, of course?

The vertex is **(1, -1)**, and the focus is **(3, -1)**.

So both points share the same y-coordinate, which is -1.

Thus, both are in a line $y = -1$, since the y-coordinate of every point in the line is -1.

And the two points are located in the axis of symmetry.

So the axis of symmetry is a line $y = -1$, which is parallel to the x-axis.

Next, the directrix is perpendicular to the axis of symmetry, which is parallel to the x-axis in this case.

So the directrix is perpendicular to the x-axis, and thus, is passing through a point in the x-axis at 90^o. So we can assume that the directrix is a line $x = d$.

Since the vertex is the midpoint between the directrix and focus, we get

$1 = \frac{d+3}{2} \Rightarrow d = -1$. Thus, the directrix is a line $x = -1$.

So next, we want to use the fact that each point in a parabola is the same distance away from a particular point called a focus and a particular line called a directrix.

So assuming now, $P(x, y)$ is an arbitrary point in the parabola, U is the distance from P to the directrix, and V is the distance from P to the focus, and using the distance formula, we get

$U^2 = \{x - (-1)\}^2 + (y - y)^2 = (x + 1)^2$, and $V^2 = (x - 3)^2 + \{y - (-1)\}^2 = (x - 3)^2 + (y + 1)^2$.

And next, since the two distances are the same, we get $(x + 1)^2 = (x - 3)^2 + (y + 1)^2$

$\Rightarrow (y + 1)^2 = (x + 1)^2 - (x - 3)^2 = (x + 1 + x - 3)(x + 1 - x + 3) = 2(x - 1)(4) = 8(x - 1)$

$\Rightarrow (y + 1)^2 = 8(x - 1)$, which is the parabola.