Examples 1 in Circles

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Examples 1 in Circles

Put in a graph each of the equations in each example below.

0. $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $x^2 + y^2 = 2$,
and $x^2 + y^2 = c^2$ where c is constant, and $c > 0$.

1. $(x - 1)^2 + y^2 = 1$, $x^2 + (y - 1)^2 = 1$, $(u - 2)^2 + v^2 = 3$, and $x^2 + (y + \frac{3}{2})^2 = 16$

2. $(s + 1)^2 + (t + 2)^2 = 1$, and $(x + \frac{1}{3})^2 + (y - \frac{2}{5})^2 = 3$

3 $4a^2 + 4b^2 = 1$, $9x^2 + 9(y - 1)^2 = 4$, and $7(x - 1)^2 + 7(y - 2)^2 = 3$

4. $(x - a)^2 + (y - b)^2 = c^2$ where a, b, and c are constant, and $c \neq 0$.

5. $d(x - a)^2 + d(y - b)^2 = c^2$ where a, b, c, and d are constant, $c \neq 0$, and $d > 0$.

6. $x^2 + y^2 - 4x - 6y + 9 = 0$

7. $t^2 + s^2 + 6t + 2s + 9 = 0$

8. $4x^2 + 4y^2 + 6x + 2y - \frac{13}{4} = 0$

9. $14x + 17y - 2x^2 - 2y^2 = -24$

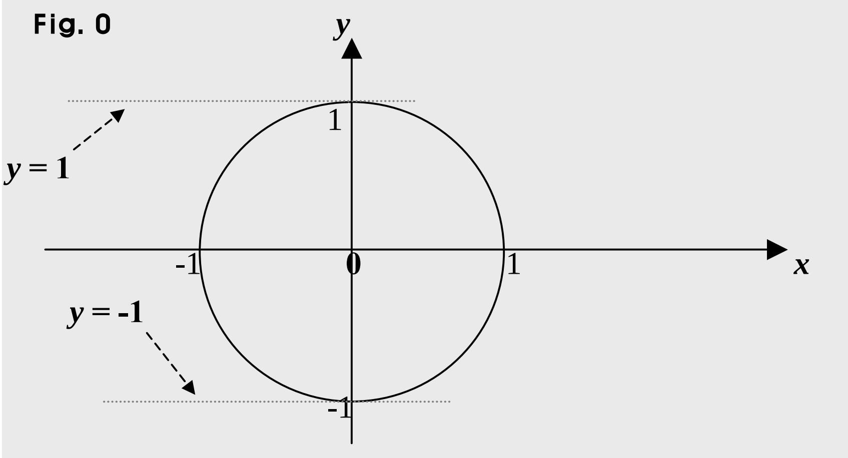
A. $1.9u + 3.6v - \frac{1}{3}u^2 - \frac{1}{3}v^2 + 0.3 = 0$

Suggestions or Solutions
To the Problems in the Example 0

To begin with, the first equation is $x^2 + y^2 = 1$.

The equation given indicates a unit circle centered at the origin **(0,0)**. So it is a circle of radius 1, and is the simplest and the most basic circle.

Fig. 0



And of the unit circle $x^2 + y^2 = 1$, changing the center or radius, or both, we can readily get the circle we want.

So for instance, moving the center to $(a, 0)$, we get
$(x + a)^2 + y^2 = 1$, which is another circle of radius 1, which is a unit circle, too.

Moving the center to $(0, b)$, we get
$x^2 + (y - b)^2 = 1$, which is another unit circle, too.

Moving the center to (a, b), we get
$(x - a)^2 + (y - b)^2 = 1$, which is a unit circle, also.

Changing the radius to r, and keeping the center intact, we get
$x^2 + y^2 = r^2$, which is a circle centered at the origin $(0, 0)$.

Changing the radius to r, and moving the center to (a, b), we get
$(x - a)^2 + (y - b)^2 = r^2$, which is often called a standard equation for circles.

Next, the second equation is $x^2 + y^2 = 4$.

We can put the equation above this way, too: $x^2 + y^2 = 2^2$.

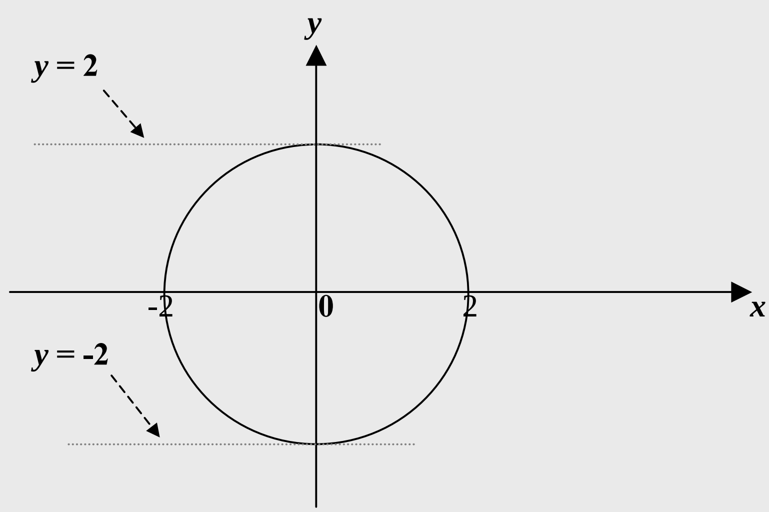
The equation is called a connective equation connecting coordinates of any point that is a particular distance away from the origin **(0, 0)** in the ***x*-*y*** plane, and the particular distance is 2.

So the equation is the connective equation between the coordinates of an arbitrary point **(*x*, *y*)** representing all points in a curve, and 2 is the distance from the arbitrary point to the origin in the ***x*-*y*** plane.

So the curve is a circle where the center is the origin and the radius is 2.

Therefore, the equation given is the equation of the circle below.

Fig. 1

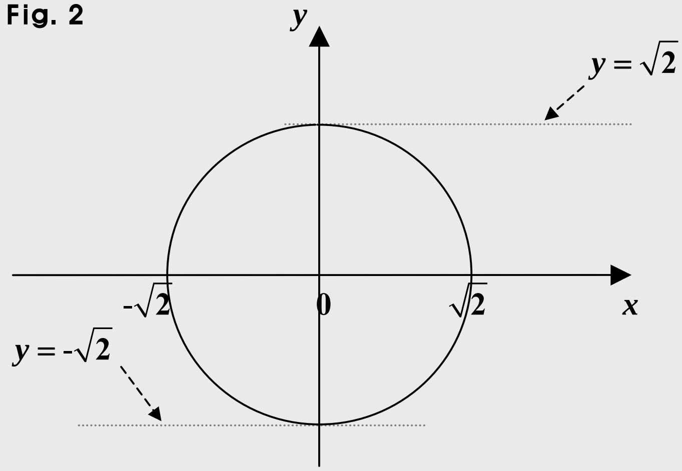


Next, the third equation is $x^2 + y^2 = 2$.

We can put it this way, too: $x^2 + y^2 = (\sqrt{2})^2$.

So the equation indicates a circle where the center is the origin, and the radius is $\sqrt{2}$.
And we can put the circle in a graph the way below.

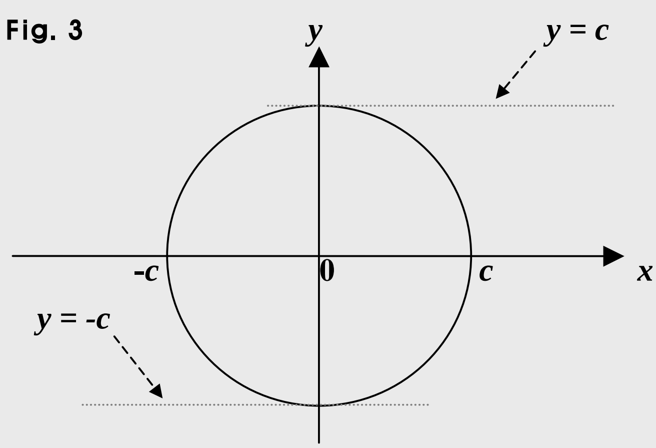
Fig. 2



And by the same token, $x^2 + y^2 = c^2$ indicates a circle of radius c centered at the origin.

And we can put the circle in a graph the way below.

Fig. 3

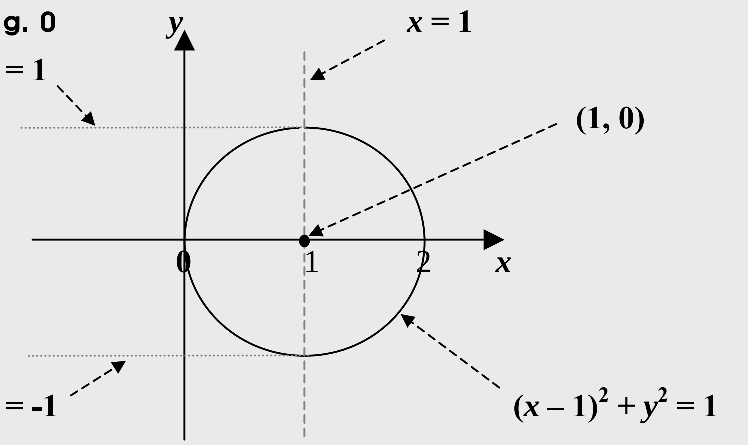


Suggestions or Solutions
To the Problems in the Example 1

To begin with, the first equation is $(x-1)^2 + y^2 = 1$.

It is the equation of a unit circle centered at **(1, 0)** in the ***x*-*y*** plane.

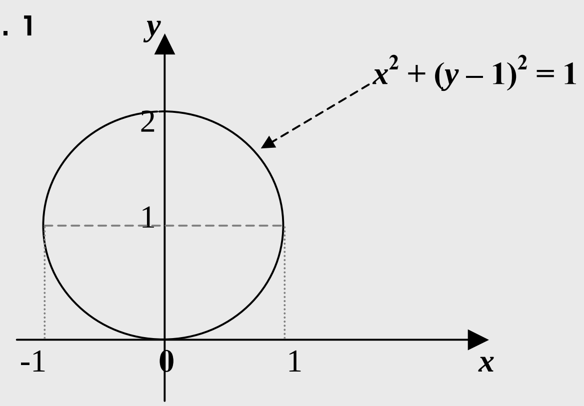
Fig. 0



Next, the second equation is $x^2 + (y-1)^2 = 1$.

And it is the equation of a unit circle centered at **(0, 1)**.
So we can put it in a graph the way below.

Fig. 1

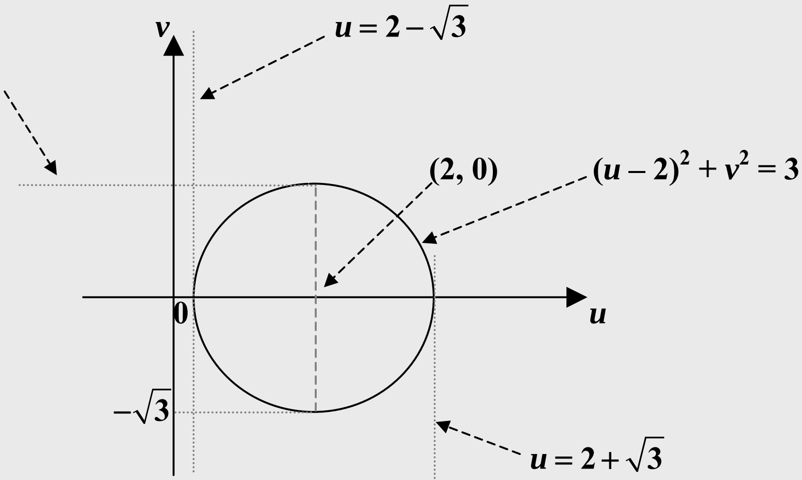


Next, the third equation is $(u-2)^2+v^2=3$.

And we can put the equation this way, too: $(u-2)^2+v^2=(\sqrt{3})^2$.

So it is a circle in the ***u-v*** plane, and is centered at **(-1, -2)**, and the radius is $\sqrt{3}$.
And thus, we can put it in a graph the way blow.

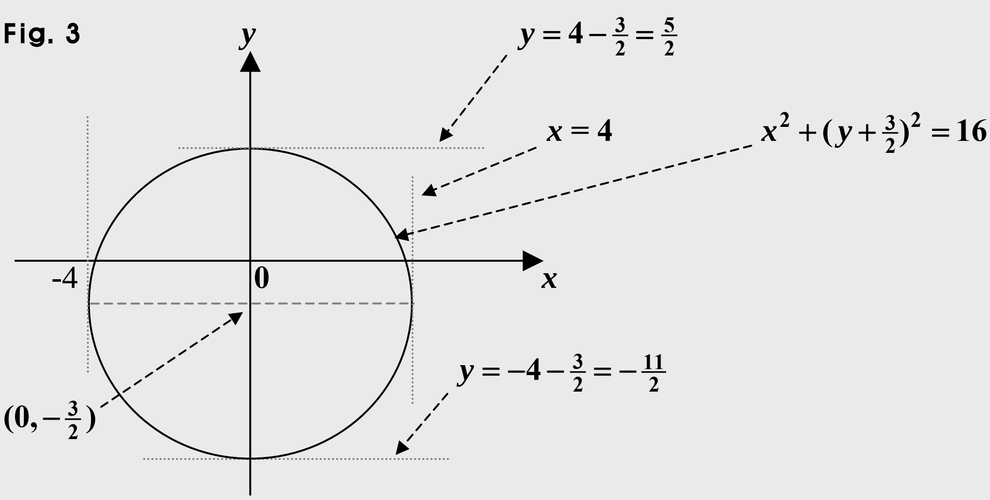
Fig. 2



And next, the last equation is $x^2+(y+\frac{3}{2})^2=16$.

We can put the equation this way, too: $x^2+(y+\frac{3}{2})^2=4^2$, so it is a circle where the center is $(0,-\frac{3}{2})$, and the radius is 4. Thus, the graph is as follows.

Fig. 3

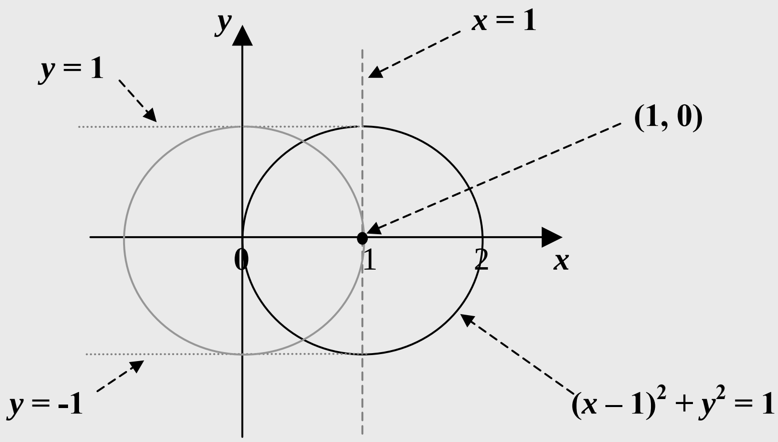


And as shown in the graph below, we can get the circle $(x-1)^2 + y^2 = 1$, too, moving a unit circle centered at (0, 0). Moving the unit circle in the amount of 1 in the direction of the ***x***-axis, we get the circle in black, which is the curve of the equation above.

So the circle in black below is of radius 1 and centered at **(1, 0)**, and thus, its equation is

$(x-1)^2 + y^2 = 1$.

Fig. 4



Let's take another look at the graph of the equation from another different perspective.

Suppose we keep intact the unit circle centered at the origin, and move the ***y***-axis in the amount of –1 in the direction of the ***x***-axis.

That is, we move only the ***y***-axis to the left in the amount of 1.

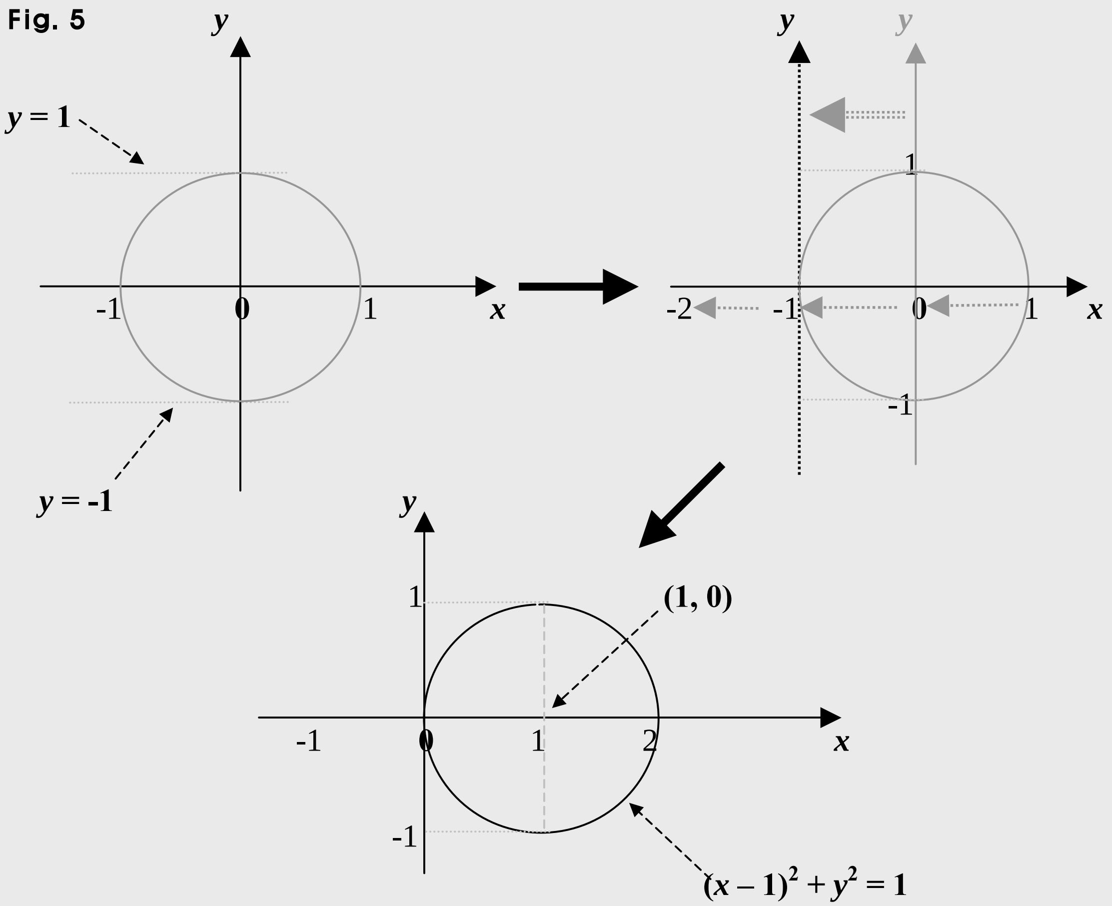
Then, the origin gets moved in the same amount and direction, too.
And so do all the numbers in the ***x***-axis.

Then, we get a new circle, and the new circle is a unit circle centered at **(1, 0)**.

Thus, the equation of the new circle is $(x-1)^2 + y^2 = 1$.

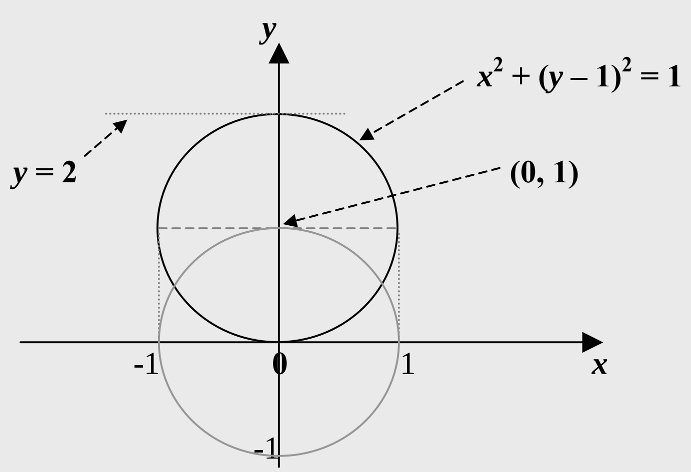
And putting the ideas in graph, we can put them the way below.

Fig. 5



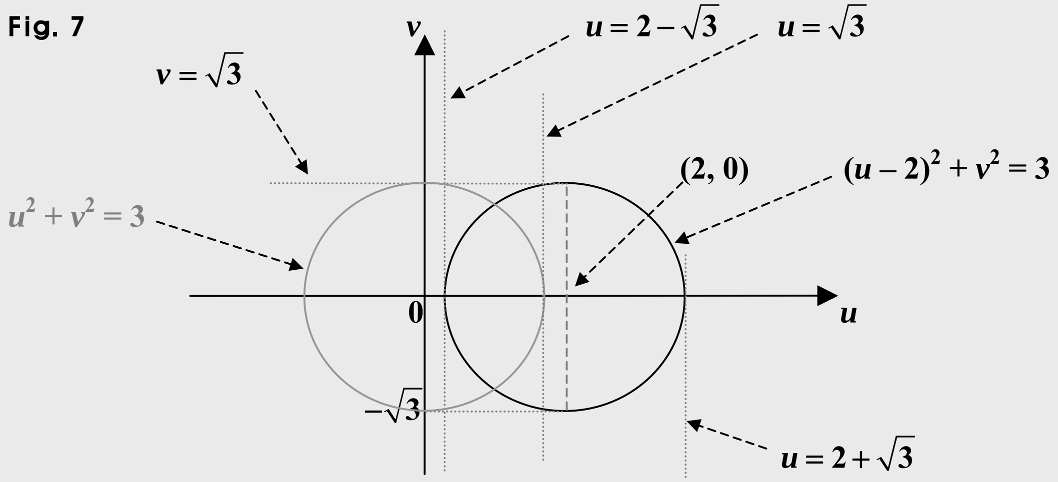
Next, we can get the circle $\boldsymbol{x^2 + (y - 1)^2 = 1}$, moving the unit circle centered at the origin in the amount of 1 in the direction of the ***y***-axis only.

Fig. 6



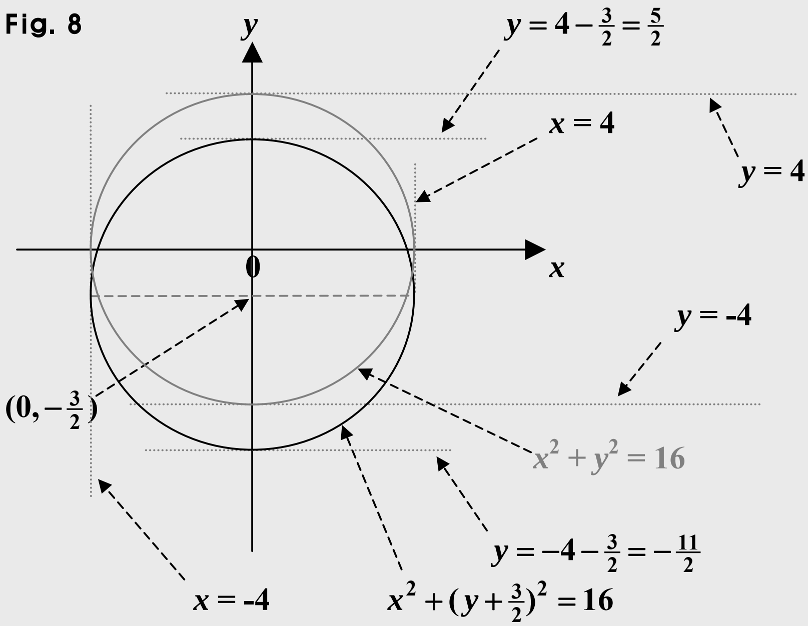
Next, moving a circle $u^2 + v^2 = 3$ by 2 along the ***u***-axis, we get $\boldsymbol{(u - 2)^2 + v^2 = 3}$.

Fig. 7



And next, moving a circle $x^2 + y^2 = 16$ by $-\frac{3}{2}$ along the ***y***-axis, that is, in the amount of $-\frac{3}{2}$ in the direction of the **y-axis,** we get $\boldsymbol{x^2 + (y + \frac{3}{2})^2 = 16}$.

Fig. 8

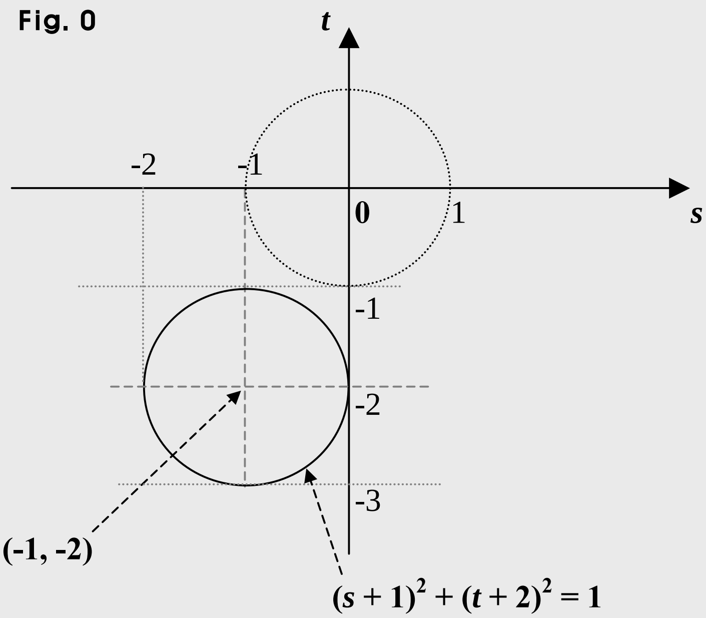


Suggestions or Solutions
To the Problems in the Example 2

To begin with, the first equation is $(s + 1)^2 + (t + 2)^2 = 1$.

It is the equation of a unit circle centered at **(-1, -2)** in the ***s*-*t*** plane.
Also, we can get the circle above if moving to **(-1, -2)** the center of the unit circle centered the origin.

Fig. 0



By the way, in math, if two circles are the same, both circles are in the same plane, and have not only the same radii but the same centers, too.

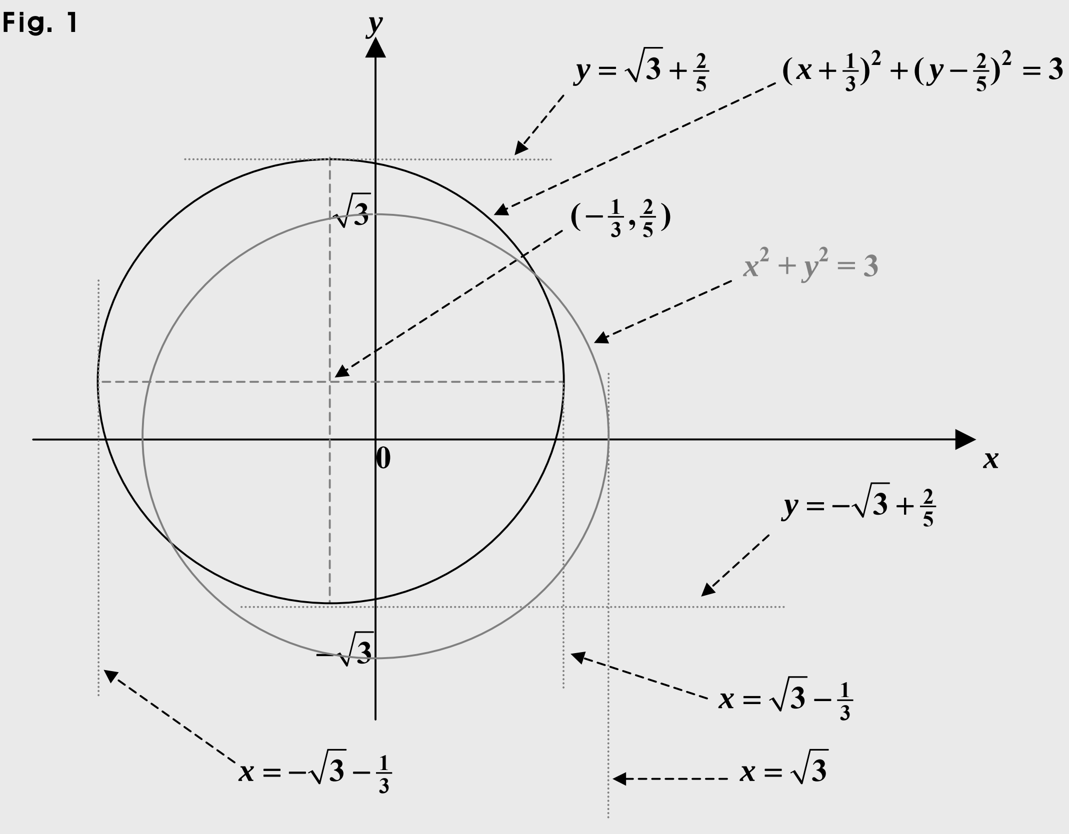
And next, the second equation is $(x + \frac{1}{3})^2 + (y - \frac{2}{5})^2 = 3$.

We can put the equation this way, too: $(x + \frac{1}{3})^2 + (y - \frac{2}{5})^2 = (\sqrt{3})^2$, which is a circle where the center is $(-\frac{1}{3}, \frac{2}{5})$, and the radius is $\sqrt{3}$.

And also, we can get the circle above moving a circle $\boldsymbol{x^2+y^2=3}$ by $-\frac{1}{3}$ in the direction of the $\boldsymbol{x}$-axis and by $\frac{2}{5}$ in the direction of the $\boldsymbol{y}$-axis. In the circle $\boldsymbol{x^2+y^2=3}$, the center is the origin, and the radius is $\boldsymbol{\sqrt{3}}$.

So putting the two circles in a graph, we can put them the way below.

Fig. 1



Suggestions or Solutions
To the Problem in the Example 3

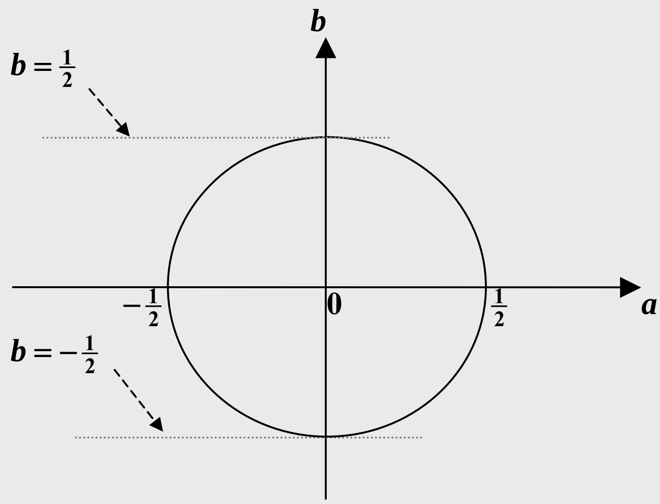
To begin with, the first equation is: $\mathbf{4a^2 + 4b^2 = 1.}$

The equation indicates a circle in the ***a-b*** plane.

Converting the equation the way below, we can see the center and radius, which specify the circle, of course.

$\mathbf{4a^2 + 4b^2 = 1 \Rightarrow a^2 + b^2 = \frac{1}{4} = (\frac{1}{2})^2}$. So the center is the origin, and the radius is $\frac{1}{2}$.

Fig. 0



Next, the second equation is $\mathbf{9x^2 + 9(y-1)^2 = 4}$.

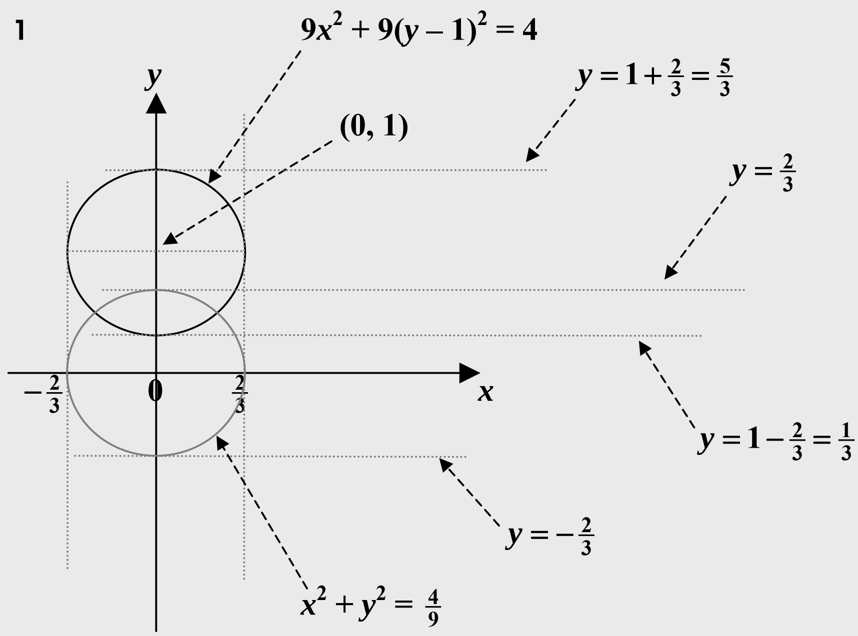
The equation is not in the standard form, and is not in the general form, either. It looks more like though, in the standard form. So it can be said to be in a semi standard form.

Thus, converting it into the complete standard equivalent, we get

$\mathbf{9x^2 + 9(y-1)^2 = 4 \Rightarrow x^2 + (y-1)^2 = \frac{4}{9} = (\frac{2}{3})^2}$. So the center is **(0, 1)**, and the radius is $\frac{2}{3}$.

Also, translating a circle $9x^2 + 9y^2 = 4$, that is, $x^2 + y^2 = \frac{4}{9}$, by 1 in the direction of the y-axis, we can get the same circle, too, that is, the circle $9x^2 + 9(y-1)^2 = 4$, which can be put this way: $x^2 + (y-1)^2 = \frac{4}{9}$.

Fig. 1



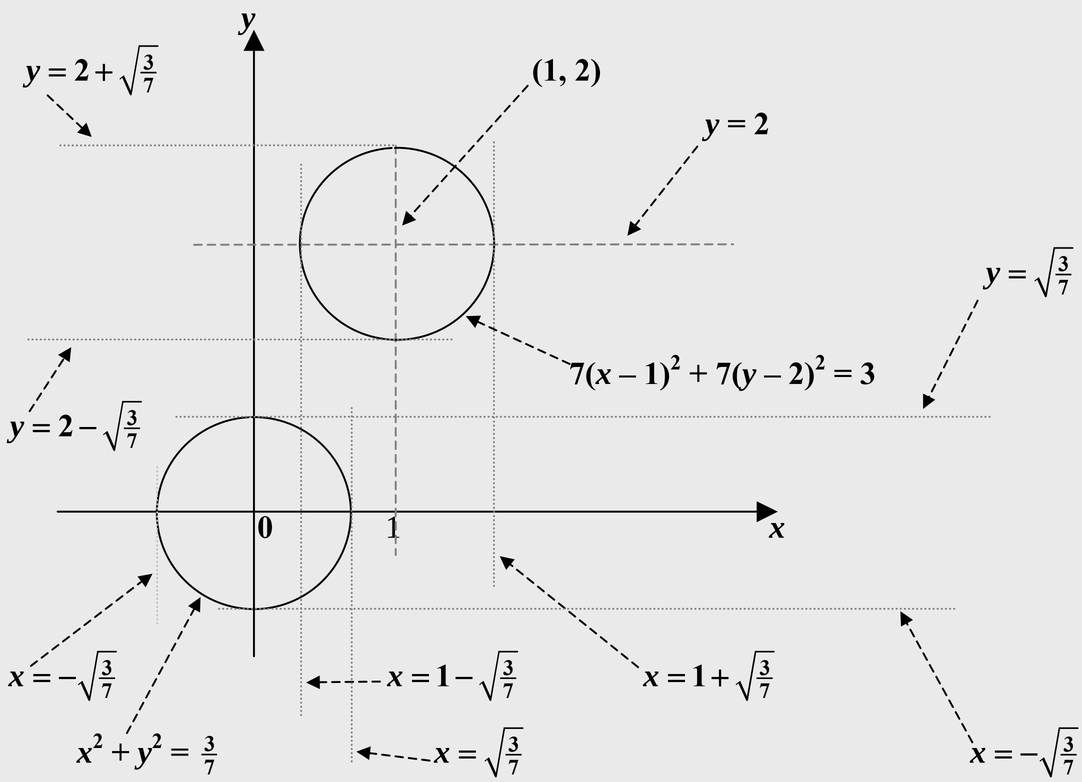
And next, the last equation is $7(x-1)^2 + 7(y-2)^2 = 3$.

As the two equations above, this equation is not in the standard form, and is not in the general form, either. It looks more like the standard form, so it can be said to be quasi-standard form. And putting the equation in the standard form, we get

$$7(x-1)^2 + 7(y-2)^2 = 3 \Rightarrow (x-1)^2 + (y-2)^2 = \frac{3}{7} = \left(\sqrt{\frac{3}{7}}\right)^2$$

So the center is **(1, 2)**, and the radius is $\sqrt{\frac{3}{7}} \approx 0.655$. Also, we can get the same circle as the one above moving a circle $x^2 + y^2 = \frac{3}{7}$ by 1 in the direction of the x-axis and by 2 in the direction of the y-axis. In the circle $x^2 + y^2 = \frac{3}{7}$, the center is the origin, and the radius is $\sqrt{\frac{3}{7}}$. So putting them both in one graph, we can put them the way below.

Fig. 0



y
$y = 2 + \sqrt{\frac{3}{7}}$
(1, 2)
$y = 2$
$y = \sqrt{\frac{3}{7}}$
$7(x - 1)^2 + 7(y - 2)^2 = 3$
$y = 2 - \sqrt{\frac{3}{7}}$
0
1
x
$x = -\sqrt{\frac{3}{7}}$
$x = 1 - \sqrt{\frac{3}{7}}$
$x = 1 + \sqrt{\frac{3}{7}}$
$x^2 + y^2 = \frac{3}{7}$
$x = \sqrt{\frac{3}{7}}$
$x = -\sqrt{\frac{3}{7}}$

Suggestions or Solutions
To the Problem in the Example 4

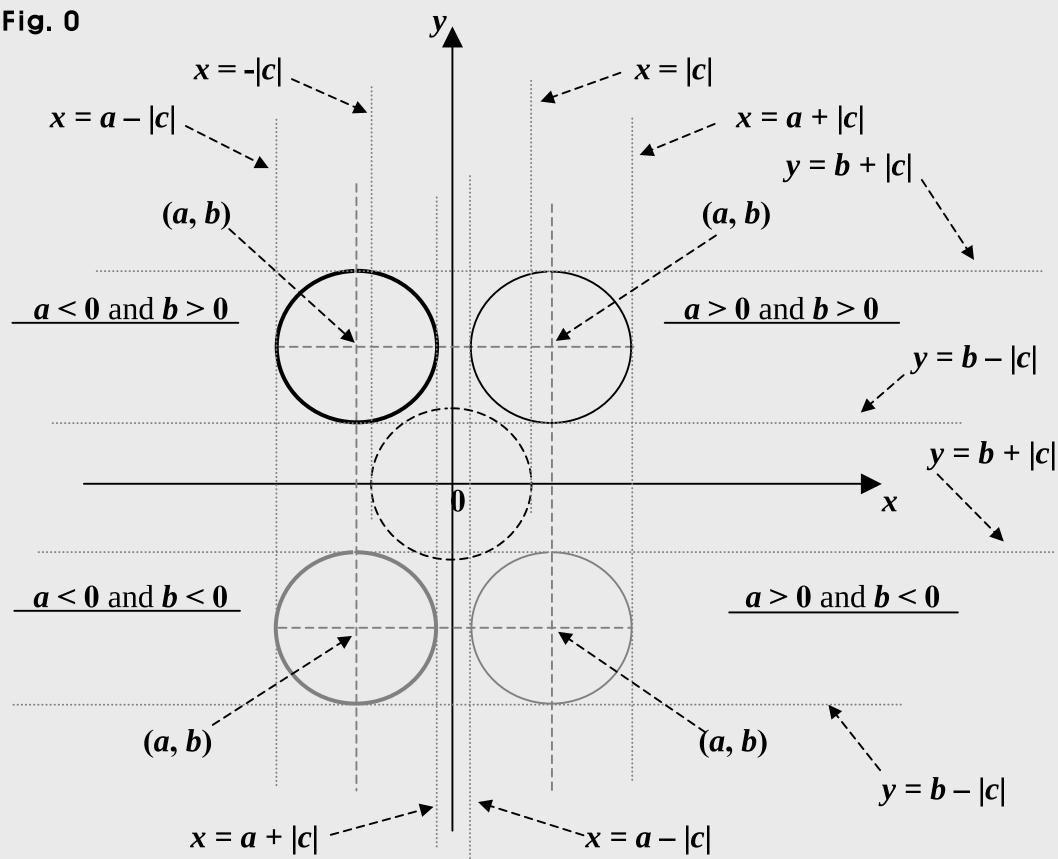
Put in a graph the equation as follows: $(x - a)^2 + (y - b)^2 = c^2$ where a, b, and c are constants and $c \neq 0$.

Note that in $|x|$, the pair of the vertical bars can be called an absolute sign, and $|x|$ means the absolute value of x, and thus, is the magnitude of x. For instance, $|-2| = 2$.

We have: $(x - a)^2 + (y - b)^2 = c^2$. So the center is at (a, b), and the radius is $|c|$.

Depending on the signs of a and b, we can put the circle four different ways. Technically though, we have 5, because if a and b both are 0, the center is at the origin.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 5

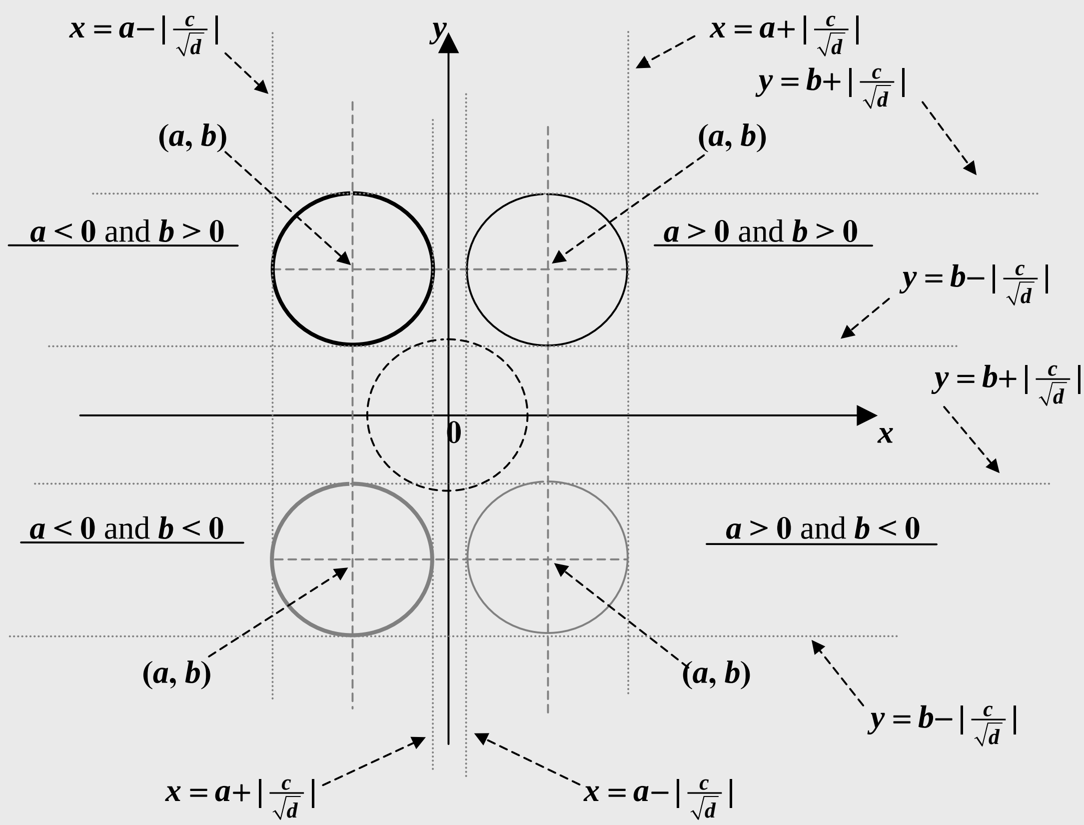
Put in a graph the equation as follows: $d(x - a)^2 + d(y - b)^2 = c^2$ where a, b, c, and d are constants, $c \neq 0$, and $d > 0$.

Putting the equation in the standard form, we get $(x - a)^2 + (y - b)^2 = \frac{c^2}{d} = (\frac{c}{\sqrt{d}})^2$.

So the center is at (a, b), and the radius is $|\frac{c}{\sqrt{d}}|$.

And we can have five different cases as below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 6

Put in a graph the equation as follows: $x^2 + y^2 - 4x - 6y + 9 = 0$.

The equation indicates a circle, and is in the general form.
How do we know though, if it really does indicate a circle?

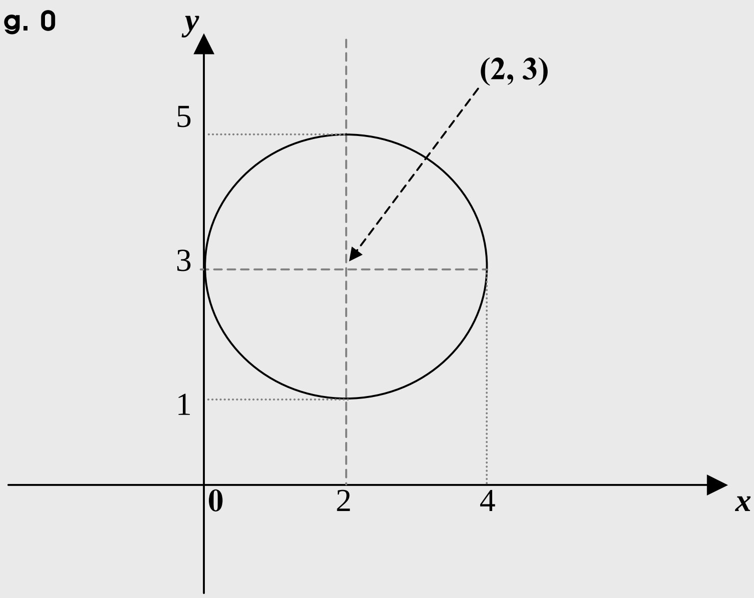
Putting the equation in the standard form, we can actually confirm that it indicates a circle. And if it is the case, we can quickly see the center and radius. So we can readily put the circle in a graph.

$$x^2 + y^2 - 4x - 6y + 9 = x^2 - 4x + y^2 - 6y + 9 = x^2 - 4x + 4 - 4 + y^2 - 6y + 9$$

$$= (x - 2)^2 - 4 + (y - 3)^2 = 0 \Rightarrow (x - 2)^2 + (y - 3)^2 = 4 = 2^2.$$

So it indicates a circle, and we can see that the center is **(2, 3)**, and the radius is **2**.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 7

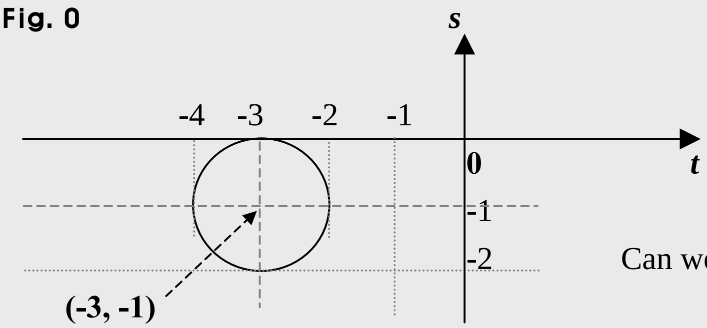
Put in a graph the equation as follows: $t^2 + s^2 + 6t + 2s + 9 = 0$.

The equation represents a circle in the general form.
Putting the equation in the standard form, we can confirm that it indicates a circle. And if it is the case, we can readily see the center and the radius.

$$t^2 + s^2 + 6t + 2s + 9 = t^2 + 6t + s^2 + 2s + 9 = t^2 + 6t + 9 - 9 + s^2 + 2s + 1 - 1 + 9$$
$$= (t + 3)^2 + (s + 1)^2 - 9 - 1 + 9 = (t + 3)^2 + (s + 1)^2 - 1 = 0 \Rightarrow (t + 3)^2 + (s + 1)^2 = 1.$$

So it is a unit circle centered at **(-3, -1)** in the ***t-s*** plane.

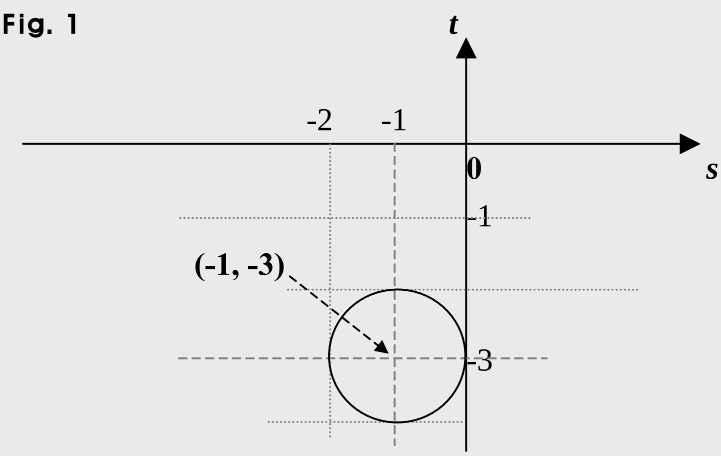
Fig. 0



Can we put it in the ***s-t*** plane, though?

Of course, we can. We have $(t + 3)^2 + (s + 1)^2 = 1$. So we can get $(s + 1)^2 + (t + 3)^2 = 1$. Putting thus, the circle in the ***s-t*** plane, we can say that the center is at **(-1, -3)**.

Fig. 1



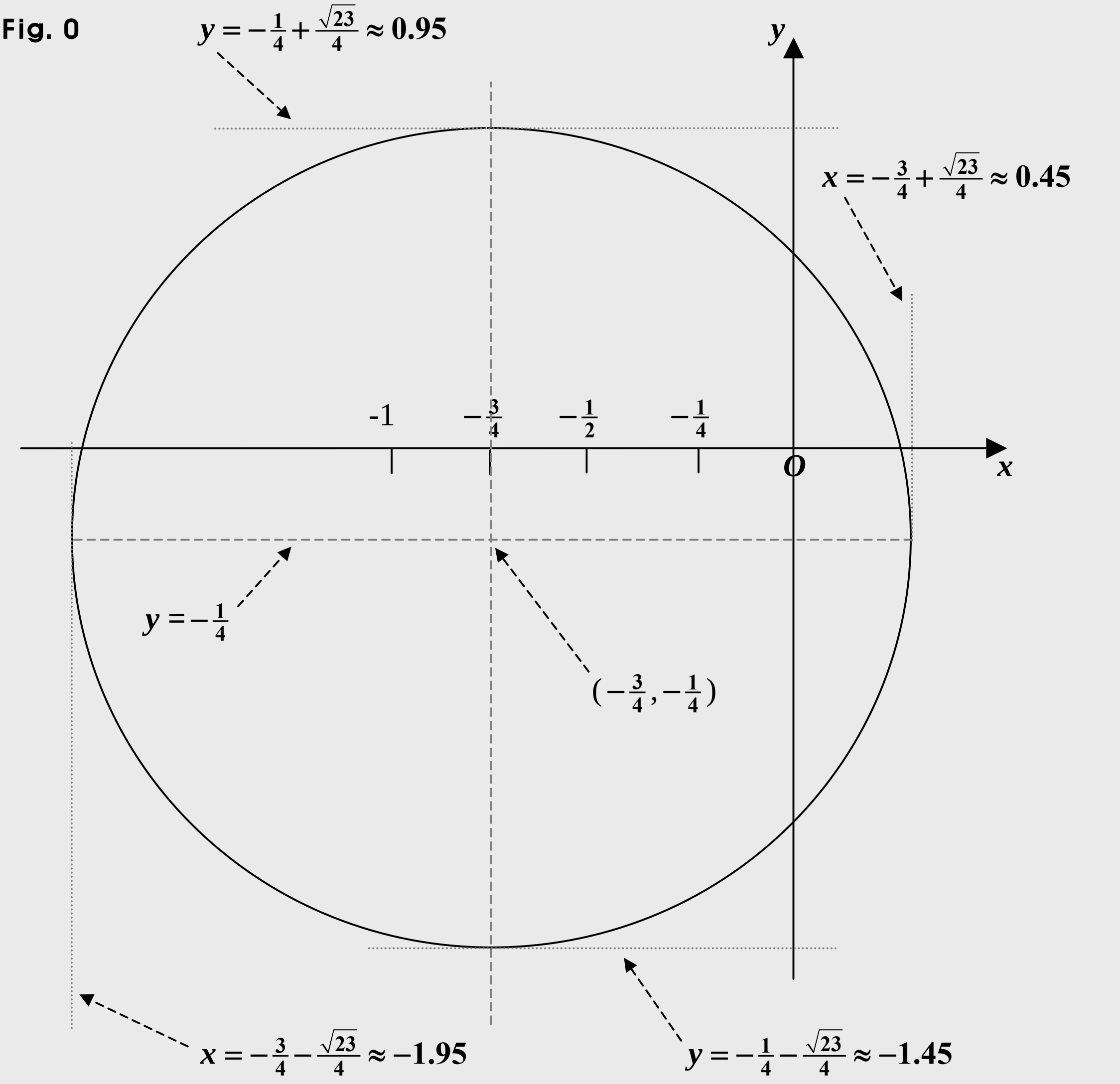
Suggestions or Solutions
To the Problem in the Example 8

Put in a graph the equation as follows: $4x^2 + 4y^2 + 6x + 2y - \frac{13}{4} = 0$.

$$4x^2 + 4y^2 + 6x + 2y - \tfrac{13}{4} = 0 \Rightarrow x^2 + y^2 + \tfrac{3}{2}x + \tfrac{1}{2}y - \tfrac{13}{16} = 0$$
$$\Rightarrow x^2 + \tfrac{3}{2}x + \tfrac{9}{16} - \tfrac{9}{16} + y^2 + \tfrac{1}{2}y + \tfrac{1}{16} - \tfrac{1}{16} - \tfrac{13}{16} = (x + \tfrac{3}{4})^2 + (y + \tfrac{1}{4})^2 - \tfrac{23}{16} = 0$$
$$\Rightarrow (x + \tfrac{3}{4})^2 + (y + \tfrac{1}{4})^2 = \tfrac{23}{16} = (\tfrac{\sqrt{23}}{4})^2$$

So it is a circle, where the center is at $(-\frac{3}{4}, -\frac{1}{4})$, and the radius = $\frac{\sqrt{23}}{4} \approx 1.199$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 9

Put in a graph the equation as follows: 14*x* + 17*y* – 2*x*² – 2*y*² = -24.

$$14x + 17y - 2x^2 - 2y^2 = -24 \Rightarrow 2x^2 + 2y^2 - 14x - 17y - 24 = 0.$$

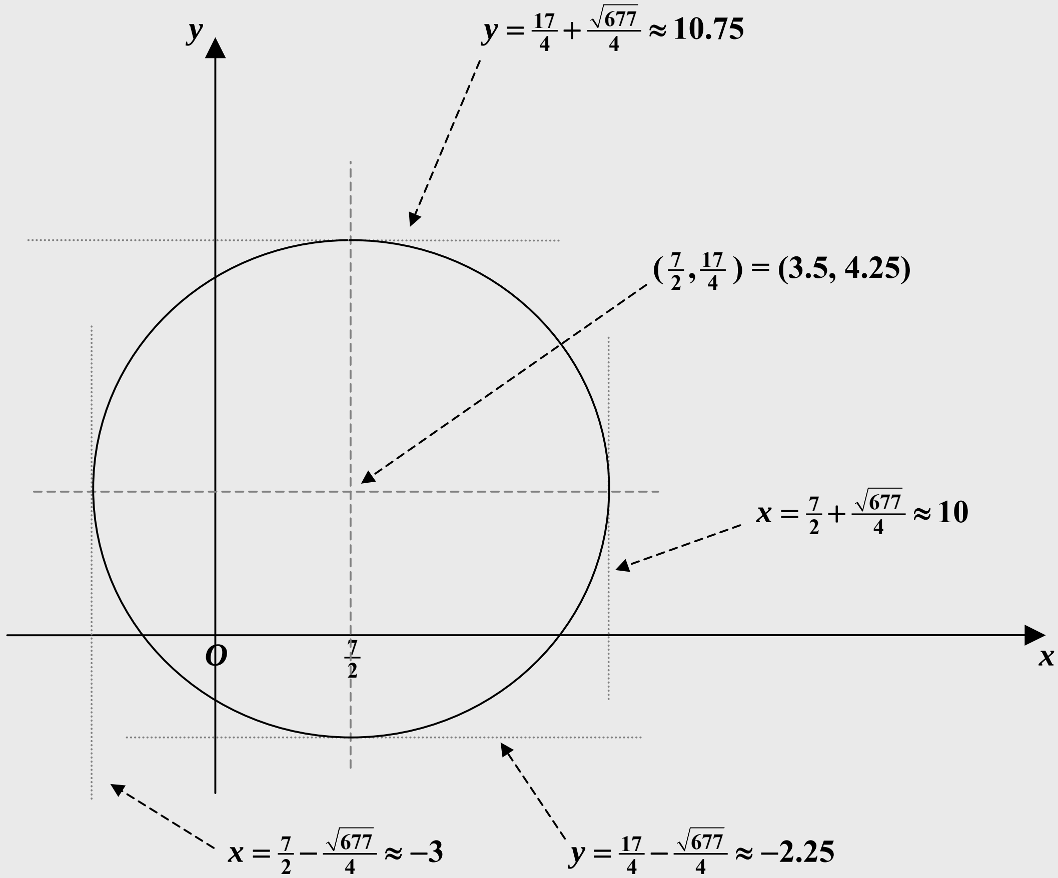
$$\Rightarrow x^2 - 7x + y^2 - \tfrac{17}{2}y - 12 = 0 \Rightarrow x^2 - 7x + (\tfrac{7}{2})^2 - (\tfrac{7}{2})^2 + y^2 - \tfrac{17}{2}y + (\tfrac{17}{4})^2 - (\tfrac{17}{4})^2 - 12 = 0$$

$$\Rightarrow (x - \tfrac{7}{2})^2 + (y - \tfrac{17}{4})^2 - \tfrac{49}{4} - (\tfrac{17}{4})^2 - 12 = 0 \Rightarrow (x - \tfrac{7}{2})^2 + (y - \tfrac{17}{4})^2 = \tfrac{196+289+192}{16} = \tfrac{677}{16}$$

$$\Rightarrow (x - \tfrac{7}{2})^2 + (y - \tfrac{17}{4})^2 = \tfrac{677}{16} = (\tfrac{\sqrt{677}}{4})^2$$

So it is a circle where the center is $(\frac{7}{2}, \frac{17}{4})$, and the radius is $\frac{\sqrt{677}}{4} \approx \mathbf{6.5}$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example A

Put in a graph the equation as follows: $1.9u + 3.6v - \frac{1}{3}u^2 - \frac{1}{3}v^2 + 0.3 = 0$.

$$1.9u + 3.6v - \tfrac{1}{3}u^2 - \tfrac{1}{3}v^2 + 0.3 = 0 \Rightarrow \tfrac{1}{3}u^2 + \tfrac{1}{3}v^2 - 1.9u - 3.6v - 0.3 = 0$$
$$\Rightarrow u^2 + v^2 - 5.7u - 10.8v - 0.9 = 0 \Rightarrow u^2 - 5.7u + v^2 - 10.8v - 0.9 = 0$$
$$\Rightarrow u^2 - 5.7u + \tfrac{5.7^2}{4} - \tfrac{5.7^2}{4} + v^2 - 10.8v + 5.4^2 - 5.4^2 - 0.9 = 0$$
$$\Rightarrow (u - \tfrac{5.7}{2})^2 + (v - 5.4)^2 - \tfrac{5.7^2}{4} - 5.4^2 - 0.9 = 0 \Rightarrow (u - \tfrac{5.7}{2})^2 + (v - 5.4)^2 = 38.1825$$

So it is a circle, the center is $(\frac{5.7}{2}, 5.4) = (2.85, 5.4)$, and the radius is $\sqrt{38.1825} \approx 6.18$.

Fig. 0